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Development Of Linear Quadratic Regulator Design For Small UAV System

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Abstract: The aim of this paper is to know the importance role of stability analysis for both unmanned aircraft system and for all control system. The objective of paper is to develop a method for dynamic stability analysis of the design process. These are categorized into:To design, model and stability analysis of UAV based on the forces and moment equations of aircraft dynamic model, To choose the suitable controller for desired altitude of a particular UAV model, To analyze the stability condition for aircraft using mathematical modeling and MATLAB. In this paper, the analytical model of the longitudinal dynamic of flying wing UAV has been developed using aerodynamic data. The stability characteristics of UAV can be achieved from the system transfer function with LQR controller.

Index Terms: LQR, UAV, MATLAB, Stability Analysis, Control System Design.

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1 INTRODUCTION

UAVS are better suited for dull, dirty, or dangerous missions than manned aircraft. UAS are mainly used for; intelligence, surveillance and reconnaissance (ISR), border security, counter insurgency, attack and strike, target identification and designation, electronic attack, law enforcement and security applications, environmental monitoring and agriculture, remote sensing, aerial mapping and meteorology. It has the two control modes: longitudinal and lateral. Longitudinal flight control, the combination of elevator and throttle are used to control altitude and airspeed. Lateral flight control uses state estimate feedbacks to convert waypoint and loiter commands into aileron or rudder commands.UAV is the multi input multi output system. A basic of UAV control depends on the obtained dynamic model, which allows us to analyze the system behavior. The aerodynamic model is very important in the study of UAVs as small oscillations could lead the UAV to crash. Therefore, the controller must be applied to the UAV dynamic model.In our research work, we used the LQR technique to adjust the flight stability in longitudinal motion.

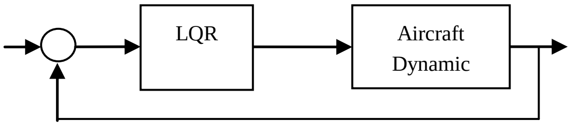

Figure.1. *LQR control system*

2 LONGITUDINAL FIXED-WING AIRCRAFT FLIGHT DYNAMICS

Analytical modeling of flight dynamics is used in development of modern aircraft. A flight dynamic model is a mathematical modeling of dynamic response and stability condition that is expected for the proposed aircraft. All flight dynamics are based on the Newton’s second law. An aircraft’s motion in its six-degrees-of-freedom (6DOF) can be described by a system of non-linear first order differential equations. The equations of motion are taken in the Earth coordinate system. The longitudinal dynamics require twelve states: three altitude angles (phi, θ, psi), three body-axis velocities(u, v, w), three body axis angular rates(p, q, r), three inertial positions(X, Y, Z).

$X - mg\sin(\theta) = m(\dot{u} + qw - rv)$
$Y + mg\cos(\theta)\sin(\theta) = m(\dot{v} + ru - pw)$
$Z + mg\cos(\theta)\cos(\theta) = m(\dot{w} + pv - qw)$
$L = I_x\dot{p} - I_{xz}\dot{r} + qr(I_z - I_y) - I_{xz}pq$
$M = I_y\dot{q} + rp(I_x - Iz) + I_{xz}(p^2 - r^2)$
$N = - I_{xz}\dot{p} + I_z\dot{r} + pq(I_y - I_x) + I_{xz}qr$

Where, m is the mass of the UAV, (X, Y, Z) are forces in the three directions (x, y, z) and (L, M, N) are three moments in the same axis coordinated. (phi, θ , psi) are three Euler angles of pitch, roll, yaw respectively. While (I_x, I_y, I_z) are moment of inertia in roll, pitch and yaw respectively, while (I_{xy}, I_{xz}, I_{yz}) are products of inertia in the appropriate axis. Also, we must assume these facts for model of flight; there is a flat earth , there is non-rotating earth , there is no rotation mass, there is a constant wind, the aircraft is rigid body , the aircraft is symmetric.

3 MATHEMATICAL MODEL OF THE UAV

In this study, the longitudinal motion of the Smart one-C UAV is investigated.The primary control surfaces in the longitudinal dynamic model are elevator deflection angle and throttle angle. We must define the positive elevator deflection angle to obtain the transfer function of the aircraft. For this study, the state space model of small UAV pitch model was obtained by using the analytical modeling. This gives

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{\omega} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_\omega & X_q + \omega_0 & -g\cos\theta_0 \\ Z_u & Z_\omega & Z_q + \omega_0 & -g\sin\theta_0 \; 0 \\ M_u & M_\omega & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta\omega \\ \Delta q \\ \Delta\theta \end{bmatrix} +$$

$$\begin{bmatrix} X_{\delta_e} & X_{\delta_T} \\ Z_{\delta_e} & 0 \\ M_{\delta_e} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_e \\ \Delta\delta_T \end{bmatrix} \qquad +++$$

$$= \begin{bmatrix} -0.0543 & -0.5332 & 0 & -9.7295 \\ -2.7791 & -10.3435 & 8.47 & -1.1732 \\ -0.3403 & -2.0302 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta\omega \\ \Delta q \\ \Delta\theta \end{bmatrix} +$$

$$\begin{bmatrix} 2.4224 & 0.0224 \\ -20.2054 & 0 \\ -18.4384 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_e \\ \Delta\delta_T \end{bmatrix}$$

The longitudinal transfer function is given below

$$\frac{\overline{\Delta u}}{\overline{\overline{\Delta\delta}}} = \frac{2.422s^3 + 35.83s^2 + 304.9s + 1439}{s^4 + 10.4s^3 + 16.36\,s^2 - 6.299\,s + 20.73}$$

$$\frac{\overline{\Delta\omega}}{\overline{\overline{\Delta\delta}}} = \frac{-20.21\,s^3 - 164.7\,s^2 + 6.096\,s - 429.5}{s^4 + 10.4s^3 + 16.36\,s^2 - 6.299\,s + 20.73}$$

$$\frac{\overline{\Delta\theta}}{\overline{\overline{\Delta\delta}}} = \frac{-18.44\,s^2 - 151.5s + 20.67}{s^4 + 10.4s^3 + 16.36\,s^2 - 6.299\,s + 20.73}$$

Where, $\frac{\overline{\Delta u}}{\overline{\overline{\Delta\delta}}}, \frac{\overline{\Delta\omega}}{\overline{\overline{\Delta\delta}}}, \frac{\overline{\Delta\theta}}{\overline{\overline{\Delta\delta}}}$ are the forward velocity, vertical velocity , pitch angle transfer function with elevator deflection angle input respectively

4 STABILITY ANALYSIS

We can analyze the stability conditions of a control system for aircraft by placing the poles locations and transient response of the system. The pole values of the aircraft from the transfer function are -8.3017 , -2.7965 , 0.3491 + j0.8781, 0.3491 - j0.8781.The complex conjugate poles exit on the right half of the s-plane gives the dynamically unstable with the sinusoidal oscillations in exponentially increasing response. So,The root locus and transient response of unstable system for aircraft are presented by using MATLAB simulation.

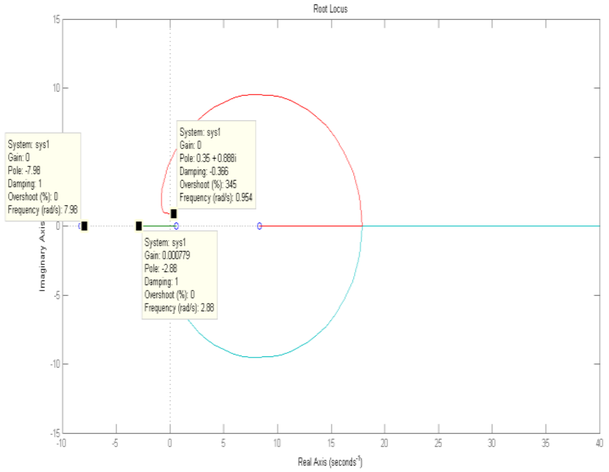

Figure.2. *Root locus for selected UAV*

$$T1 = \frac{2.482s^3 + 36.53s^2 + 315.4s + 1432}{s^4 + 10.22s^3 + 16.72s^2 - 6.456s + 21.33}$$

The response of the total transfer function is described as follow: Maximum percent overshoot:

$$M_p = 100e^{-\xi\pi/\sqrt{1-\xi^2}} = 345.35$$

Settling time: $T_s = \dfrac{4}{\xi\,\omega_n} = -11.42$

Rise time: $T_r = \dfrac{\pi-\beta}{\omega_d} = \dfrac{\pi-\beta}{\omega_n\sqrt{1-\xi^2}} = 1.35$

Damping ratio, ξ = -0.367

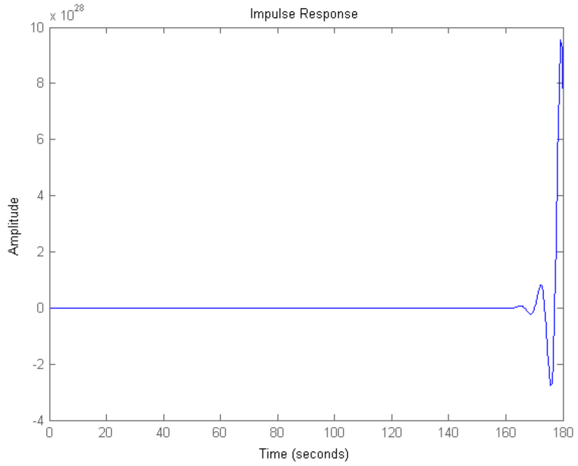

Figure.3.*Impulse response for selected UAV*

5 LQR APPLICATION FOR THE UAV

In this research work, LQR is used as the optimal control technique for the UAV. The system gives $\dot{x}$ =Ax + Bu where (A, B) is stabilizable, we want to design state feedback control u=K x to stabilize the system while meeting the transient response specifications. The following cost function has to be described

$$J = \int_0^\infty [x^T Qx(t) + u^T Ru(t)]\,dt$$

Where, Q is state cost and R is control cost

The following procedures are described for using the Linear Quadratic Regulator:

- Choose Q and R matrix such that Q=C_2^T and R >0
- Solve the Riccati equation

PA+A^TP+Q-PB$R^{-1}B^T$P= 0 and compute K=-$R^{-1}B^T$P

- Simulate the initial response for different initial conditions
- If the transient response specifications are not met , go to step1 and re-choose R.

The effectiveness matrices used in LQR controller for the system are given below:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 1.17 \\ 0 & 1 & 0 & -9.7 \\ 0 & 0 & 0 & 0 \\ 1.17 & -9.7 & 0 & 95.5 \end{bmatrix}$$

$$R = \begin{bmatrix} 5 & 0 \\ 0 & 0.1 \end{bmatrix}$$

Experience has shown that aircraft exhibits two different types of longitudinal oscillations: short period mode with relatively heavy damping and long period mode with very light damping. In this study, we should choose the short mode because this mode is more important for longitudinal motion of small UAV.

This mode is required little damped period back to the stable conditions than long period. Typical values for the damped period of short mode in the range of 2 seconds to 5 seconds and 45 to 90 seconds for long period mode. Now we get the desired poles location in order to achieve the stable system with heavy damped mode by the controller improvement is as follow: -2.2490, -0.1968 0.5289 +j 1.4581, 0.5289 - j1.4581 Therefore the system gives the response with dynamically stable in exponentially decaying envelope.

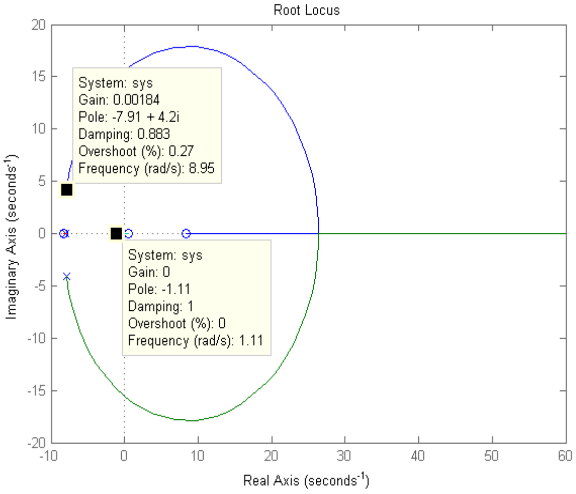

Figure.4. *Root locus with LQR controller*

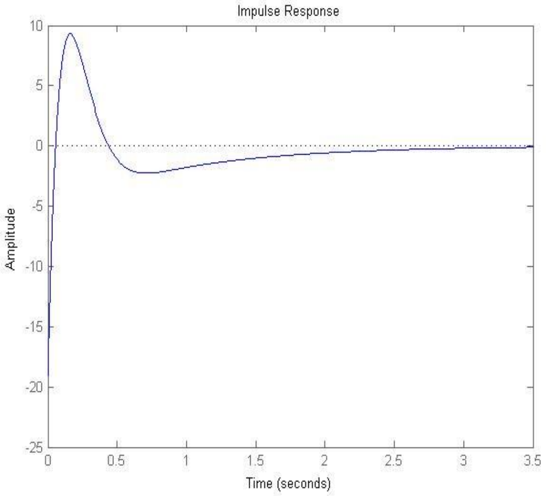

Figure.6. *Impulse response with LQR controller*

The transient response specifications for the system are presented as below:

Maximum percent overshoot:

$$M_p = 100\,e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.2698$$

Settling time: $T_s = \dfrac{4}{\xi\,\omega_n} = 0.505$

Rise time: $T_r = \dfrac{\pi-\beta}{\omega_d} = \dfrac{\pi-\beta}{\omega_n\sqrt{1-\xi^2}} = 0.631$

Damping ratio, $\xi = 0.883$

As seen from the figure, the system approach to the stability condition as the poles location exit on the left half of the s-plane and the dynamically stable response. Moreover, the controller gives the acceptable transient response specifications

6 CONCLUSION

In this paper, the analytical model of the longitudinal dynamic of flying wing UAV has been developed using aerodynamic data. The stability characteristics of UAV can be achieved from the system transfer function with LQR controller. We can used the various method to gain the desired pole location such as change the control input signal, pole placement and LQR. In this research, LQR is more effective than the other to get the stable system with the desired response for small UAV.

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