Development Of Translational Motion Of Unmanned Aerial Vehicle Using MATLAB

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Abstract: This research work describes the translational motion analysis of unmanned aerial vehicle (UAV). Since the center of mass of the receiver is time–varying, the equations are written in a reference frame that is geometrically fixed in the aircraft. Due to the fact that aerial vehicle simulation and control deal with the position and orientation of the UAV, the equations of motion are derived in terms of the translational and rotational position and velocity with respect to the aircraft location. The formation relative motion control is a challenging problem due to the coupled translational and rotational dynamics. As the translational vector depends on the current attitude and its angular velocity, and some of the attitude constraints also couple the position and attitude of the spacecraft, it makes the formation control problem high dimensional. This work develops UAV stability conditions, including translational vector maneuverability condition and included angle condition between the translational and the rotational motion of UAV system, and then presents two methods to calculate the UAV attitude. Both of the two methods need first design the optimal trajectory of the translational vector, and then use geometric and nonlinear programming methods to calculate the target trajectory. The validity of the proposed approach is demonstrated in a UAV by using MATLAB. The performance of the translational motion control is evaluated by the simulated results.

Index Terms: UAV, Aerospace Vehicle, Translational Motion, MATLAB, Stability Analysis.

1 INTRODUCTION

THIS complete description of aerospace vehicle dynamics consists of the translational motion of a point on the vehicle, and the rotational motion of the vehicle about that point[1-2]. When there are merely interested in the trajectory (or flight path) of a vehicle, it can disregard the rotational motion of the vehicle and confine our attention to translation. Reducing the vehicle dynamics to the motion of a specific point on the vehicle is tantamount to approximating the aerospace vehicle by a point mass, or a particle. A particle can be defined as an object of infinitesimal dimensions, which occupies a point in space. Consequently, the position, velocity, and acceleration of a particle are each determined by only three scalar quantities; thus a particle has three degrees of freedom. No physical object fulfills the precise definition of a particle[3-4]. The particle is thus a mathematical abstraction, which is used whenever there are interested in studying the path of a physical object, ignoring its size and rotational (or angular) motion. A baseball can be regarded as a particle, if there want to study its trajectory from the time it leaves the hand of the pitcher, and prior to its reaching the bat, provided that ignore the dimensions of the ball and its spin. However, the particle approximation of an object gives an incomplete description of its motion. For example, ignoring the size of the baseball will prevent there from studying how closely the bat misses the ball, or whether the bat hits the ball squarely in its middle. Furthermore, the ignored spin of the baseball would be quite important not only in its interaction with the bat, but also in the deviation of its trajectory caused by aerodynamic forces. Hence, a particle approximation will be inadequate if the baseball's motion is to be accurately simulated[5-6]..

2 ATMOSPHERIC AND SPACE FLIGHT

The study of flight is traditionally divided into two categories: atmospheric and space flight mechanics. The two have evolved separately over the last century. The advent of sustained, powered flight through the air began in 1903 with the Wright Flyer, whose main purpose was to fight gravity through the thrust of its engine and the lift produced by its wings—both aerodynamic in nature—in a controllable fashion. As atmospheric flight progressed over the decades, a new methodology was developed for its analysis, largely based on the study of aerodynamic forces and moments. In contrast, space flight, which required neither lift, nor aerodynamic thrust, was contemplated using the theories of astronomy and ballistics[7-8].

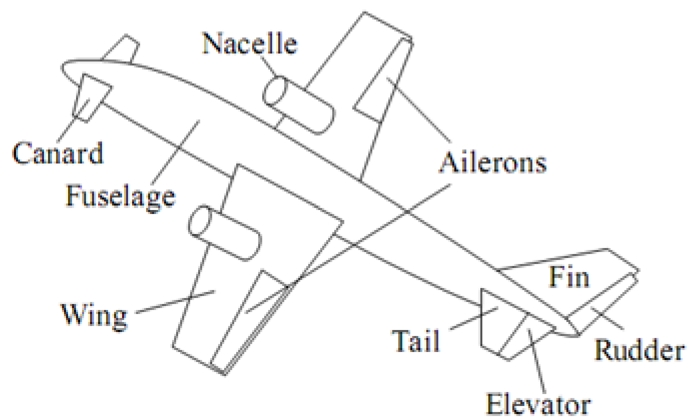

Fig.1. *Elements of Airplane Configuration*

The atmospheric flight vehicles are especially adapted for low aerodynamic drag and can be classified into lifting vehicles (or aircraft) and non-lifting (or ballistic) vehicles. Lifting vehicles derive their support (lift) in air using either static or dynamic interaction with the atmosphere. In the former the aerostatic category lie the hot-air balloons, blimps, and dirigibles, while in the aerodynamic lift category there have the airplanes, gliders, and rotorcraft (or helicopters). The airplane is a versatile atmospheric vehicle, consisting of fixed wings, fuselage, nacelles, and empennage (or stabilizing and control surfaces such as tail , canards, and fins), elevator , ailerons, and rudder, as depicted in Fig.1. While the wings produce the aerodynamic lift, the payload, crew, power plants, and fuel are housed in the fuselage and nacelles, and the stabilizing surfaces maintain the vehicle in a stable equilibrium, and provide control for maneuvering[9]. An airplane possesses all the features that are found piecemeal in other atmospheric flight vehicles. For example, a glider is an airplane without a power plant, while a helicopter has rotating—rather than fixed—wings. The ballistic category of atmospheric vehicles includes missiles, launch vehicles, and entry capsules. Some missiles and launch vehicles incorporate fins as aerodynamic stabilizing and control surfaces. The spacecraft are also categorized according to their missions, such as low-earth orbit, medium-earth orbit, geosynchronous orbit, lunar, and interplanetary spacecraft. Each mission is defined by the payload, and orbital elements of the final orbit. Of course, a reusable launch vehicle, such as the space shuttle, is also a spacecraft with a unique mission[10]. Traditionally, a flight vehicle is considered as being either atmospheric, or space vehicle depending upon the instantaneous location of the craft. For example, flight above an altitude of 100 km over the earth is generally regarded as space flight. However, when modeling trans-atmospheric flight (such as the ascent of a rocket into space, and an atmospheric entry), it is necessary that the artificial distinction between space and atmospheric flight be removed, such that a smooth, continuous trajectory is generated from the governing equations of motion[11].

3 Modeling and Simulation

Modeling of flight dynamics consists of idealization, selection of a reference coordinate frame, and derivation of governing equations of motion consistent with the idealization. Idealization is the process whereby necessary simplifying assumptions are made for studying the relevant dynamics. For example, in modeling the translational motion, it is often sufficient to ignore the size and mass distribution, and consider the vehicle as a point mass (or, particle). This is called the particle idealization of the vehicle. In this process, the distinct ways in which the vehicle can move, i.e., its degrees of freedom, are reduced to only three. Similarly, it is a common practice to treat the vehicle as a rigid body when considering its rotational motion, thereby reducing the degrees of freedom, from infinite (for a flexible body) to only six. The idealization must be carefully carried out so that the essential characteristics of the motion under study are not lost[12]. When the degrees of freedom are known, the next step is to select a set of motion variables (two for each degree of freedom), and a reference frame for expressing the equations of motion. For example, when studying high-altitude trajectories, such as those of spacecraft and atmospheric entry vehicles, the reference frame is usually fixed to the planet at its center, and the motion variables are spherical coordinates of position and velocity. On the other hand, low-altitude flight, such as that of an airplane, usually employs a flat, non rotating planet idealization, with the reference frame fixed to the planet's surface, and motion variables are expressed in Cartesian coordinates. The rotat[13]ional motion is generally described in reference to a coordinate frame fixed to the vehicle at its center of mass. The equations of motion can be divided into two categories: (a) kinematic equations, which only consider the geometric relationships among the motion variables, and (b) dynamic (or kinetic) equations, that are derived by taking into account the physical laws of motion. The fundamental physical laws pertinent to flight dynamics are Newton's laws of motion and gravitation, as well as the aerothermodyamic principles by which the aerodynamic and propulsive force and moment vectors are derived. Fig.2 depicts the various idealizations and reference frames employed in aerospace flight dynamics[14].

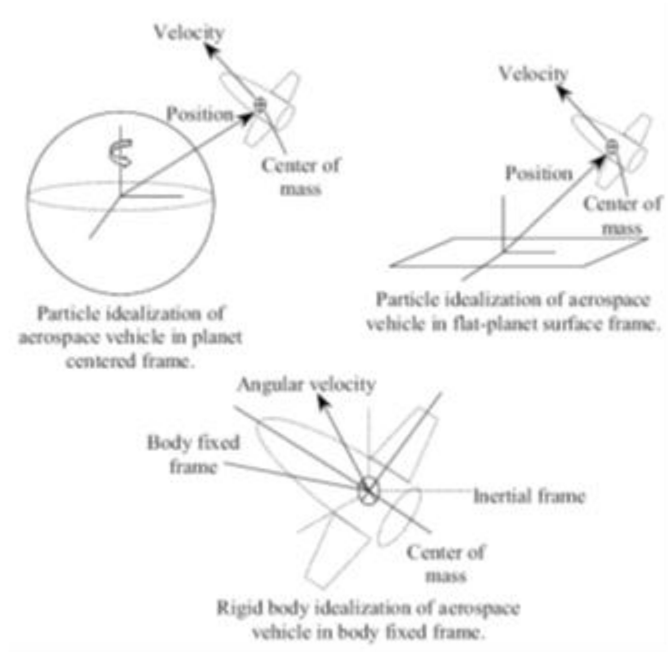

Fig.2. *Modeling of UAV*

Simulation is the task of solving the governing equations of motion in such a manner that a good approximation of the actual vehicle's motion is attained. Since the governing equations are generally nonlinear, coupled, ordinary differential equations, their solution in a closed form is seldom possible, and a numerical integration subject to appropriate initial condition is often the only alternative. Therefore, simulation of flight dynamics essentially consists of numerical integration of a set of nonlinear, ordinary differential equations. The accuracy attained in the solution depends primarily upon the numerical procedure, and to some extent on the latter's implementation in a computer algorithm. All numerical schemes employ varying degrees of approximation, wherein the derivatives are evaluated by Taylor series expansion. The number of terms retained in such a series is a rough indicator of the scheme's accuracy. The neglected terms of the series are grouped into the truncation error of the numerical scheme. Since the neglected higher-order terms must be relatively smaller in size, it is necessary that the numerical integration be performed over steps of small intervals. Therefore, truncation error accumulates as the number of steps required in the integration increases. Generally, a fine balance must be struck between the reduction of the total truncation error, and the number of terms that must be retained in the memory for each computational step. Since the nonlinear numerical integration procedure has to be iterative in nature, one must also look at its stability and convergence properties. Stability of a numerical scheme allows the truncation error to remain bounded, while convergence implies that the numerical solution reaches essentially a steady state, and does not keep oscillating forever.

4 Position and Angular Velocity of UAV Relative to Ground Station

The proposed system block diagram is illustrated in Fig.3. An aircraft whose position and angular velocity relative to a ground station, (SXYZ), with unit vectors **I**, **J**,**K**, are R0 =

$-0.2t^2\mathbf{I} + 0.5t^2\mathbf{J} + 30t\mathbf{K}$ m and $\boldsymbol{\omega} = 0.02\mathbf{J} - 0.01\mathbf{K}$ rad/s, respectively. The orientation of a coordinate frame (oxyz) fixed to the aircraft with unit vectors **i**, **j**, **k** is defined by Euler angles, θ (pitch angle), φ (roll angle), and ψ (yaw angle), as shown in Fig. 4. A restless passenger is walking up and down the aisle such that her position relative to the point, o, in the aircraft is $\mathbf{r} = \cos(t/10)\mathbf{i} - \sin(t/10)\mathbf{j}$ m. The total velocity and acceleration are found of the passenger at time t = 100 s if the initial orientation of the aircraft at t = 0 is given by θ = 0, φ = 0, and ψ = 0. The rotation from (SXYZ) to (oxyz) in terms of the Euler angles of Fig.4 is the following:

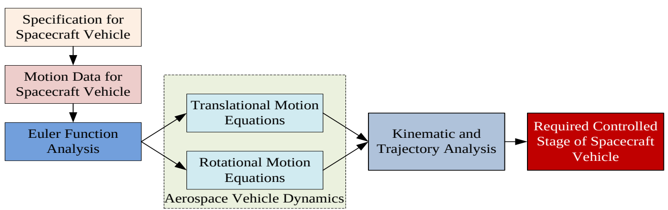

Fig.3. *Proposed System Block Diagram*

$$\begin{Bmatrix} i \\ j \\ k \end{Bmatrix} = \begin{pmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\varphi\sin\psi+\sin\theta\sin\varphi\cos\psi & \cos\varphi\cos\psi+\sin\theta\sin\varphi\sin\psi & \cos\theta\sin\varphi \\ \sin\varphi\sin\psi+\sin\theta\cos\varphi\cos\psi & -\sin\varphi\cos\psi+\sin\theta\cos\varphi\sin\psi & \cos\theta\cos\varphi \end{pmatrix} \times \begin{Bmatrix} I \\ J \\ K \end{Bmatrix}$$

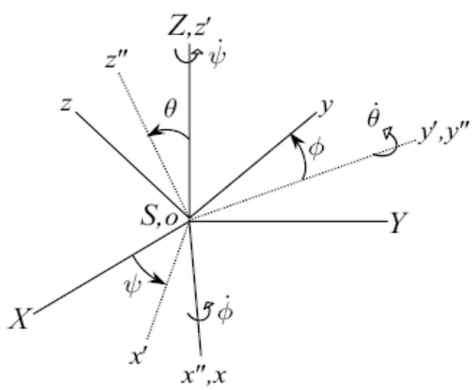

Fig.4. *The Orientation of Aircraft Axes, (oxyz), Relative to Ground Station, (SXYZ)*

The evolution of Euler angles is related to the aircraft's angular velocity as follows : $\boldsymbol{\omega} = \dot{\varphi}\mathbf{i} + \dot{\theta}\mathbf{j} + \dot{\psi}\mathbf{K}$

$= \dot{\varphi}(\cos\theta\cos\psi\mathbf{I} + \cos\theta\sin\psi\mathbf{J} - \sin\theta\mathbf{K}) + \dot{\theta}(-\sin\psi\mathbf{I} + \cos\psi\mathbf{J}) + \dot{\psi}\mathbf{K}$

$= (\dot{\varphi}\cos\theta\cos\psi - \dot{\theta}\sin\psi)\mathbf{I} + (\dot{\varphi}\cos\theta\sin\psi + \dot{\theta}\cos\psi)\mathbf{J} + (\dot{\psi} - \dot{\varphi}\sin\theta)\mathbf{K}$

$= 0.02\mathbf{J} - 0.01\mathbf{K}$

The above equation results in the following differential equations for the Euler angles:

$\dot{\varphi}\cos\theta\cos\psi - \dot{\theta}\sin\psi = 0,$

$\dot{\varphi}\cos\theta\sin\psi + \dot{\theta}\cos\psi = 0.02,$

$\dot{\psi} - \dot{\varphi}\sin\theta = -0.01$

In order to numerically integrate the nonlinear differential equations of Euler Angles, it can be resorted to a fourth-order, variable time step, Runge–Kutta algorithm (see Appendix A), as programmed in the MATLAB routine ode45. For this purpose, an M-file named eulerevolve.m is written to calculate the Euler angle derivatives, at each time step, using equations of Euler Angles. The point of singularity of Euler angle representation (θ = 90◦) would render this procedure useless (in such a case, a different representation, e.g., quaternion, should be used). The following MATLAB commands are used to solve this condition:

```
[t,x]=ode45(@eulerevolve, [0 100], [0 0 0]');%
plot(t,x*180/pi),xlabel('Time (s)'),ylabel('Euler Angles (deg.)') %

size(x) % size of solution vector

phi=x(61,1),thet=x(61,2),psi=x(61,3) %Euler angles at t=100 s

C=[cos(thet)*cos(psi) cos(thet)*sin(psi) -sin(thet);

-cos(phi)*sin(psi)+sin(thet)*sin(phi)*cos(psi)
cos(phi)*cos(psi)+sin(phi)*sin(thet)

*sin(psi)...cos(thet)*sin(phi);

sin(phi)*sin(psi)+sin(thet)*cos(phi)*cos(psi)-
sin(phi)*cos(psi)+sin(thet)*cos(phi)

*sin(psi)...cos(thet)*cos(phi)] % Rotation matrix at t=100 s

R0=[-0.2*100^2;0.5*100^2;30*100];omega=[0;0.02;-
0.01];%Origin;angular velocity

v0=[-40;100;30];a0=[-0.4;1;0]; %

r=C'*[cos(10);-sin(10);0] % Relative position

rdot= C'*[-sin(10)/10;-cos(10)/10;0]%Relative velocity

v=v0+rdot+cross(omega,r) %Total velocity of passenger

rdotdot=C'*[-cos(10)/100;sin(10)/100;0]%Relative acceleration

a=a0+rdotdot+2*cross(omega,rdot)+cross(omega,cross(omeg
a,r))%Total acceleration.
```

5 Flowchart for Translational Motion Animation of UAV

The flowchart of translational motion animation of aerospace vehicle is illustrated in Fig.5. The size of aerospace vehicle and motion data are inputted to the animation function and then the propagation of animation vertices and patches are computed. The Euler function is called from the main function and the animation of translational and rotational motion of aerospace vehicle is displayed on the command window.

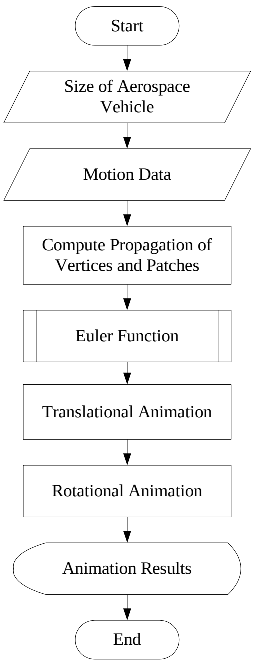

Fig.5. *Flowchart of Translational Motion Animation.*

6 Simulation Results of Translational Motion of UAV

He simulation result for initial condition of translational motion of aerospace vehicle with animation is illustrated in Fig.6. In this Figure, the aerospace vehicle is moved forwards direction according to the three dimensional position of x,y, and z coordinate in space. At first, the object is translated by following the motion equations of aerospace vehicle. Based on the theoretical background of the translational motion equations, the space vehicle is travelled to the translated parameters of Euler angle.

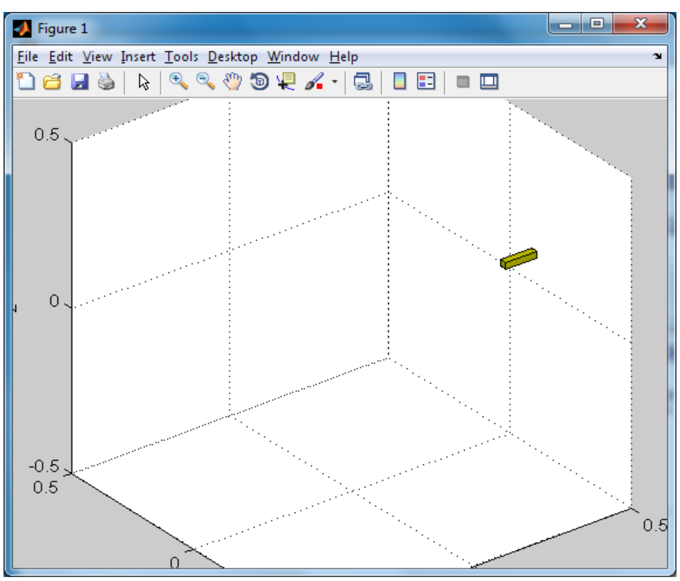

Fig.6. *Screenshot Result for Initial condition of Translational Motion of Aerospace Vehicle*

The simulation result for final condition of translational motion of aerospace vehicle with animation is illustrated in Fig.7. In this Figure, the aerospace vehicle is moved backwards direction according to the three dimensional position of x,y, and z coordinate in space. The space object is converted by following the motion equations of aerospace vehicle. Based on the conjectural environment of the translational motion equations, the space vehicle is travelled to the translated parameters of Euler angle.

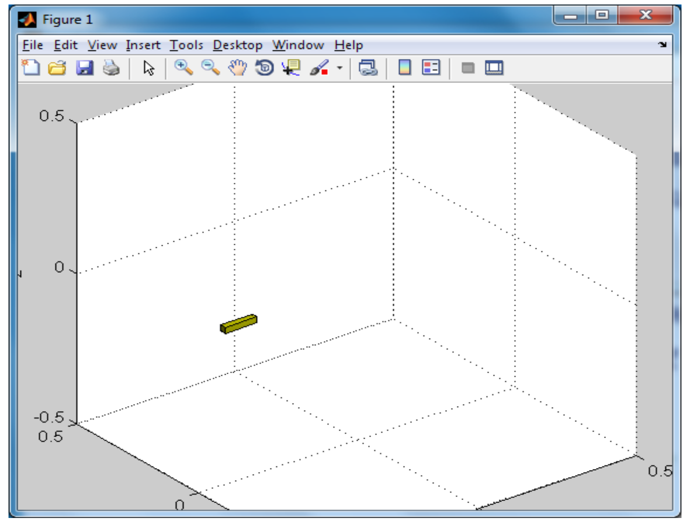

Fig.7. *Screenshot Result for final condition of Translational Motion of Aerospace Vehicle.*

7 Simulation Results of Rotational Motion of UAV

The simulation result for initial condition of rotational motion of aerospace vehicle with animation is illustrated in Fig.9. In this Figure, the aerospace vehicle is moved upwards direction according to the three dimensional position of x,y, and z coordinate in space. At first, the object is rotated by following the motion equations of aerospace vehicle.

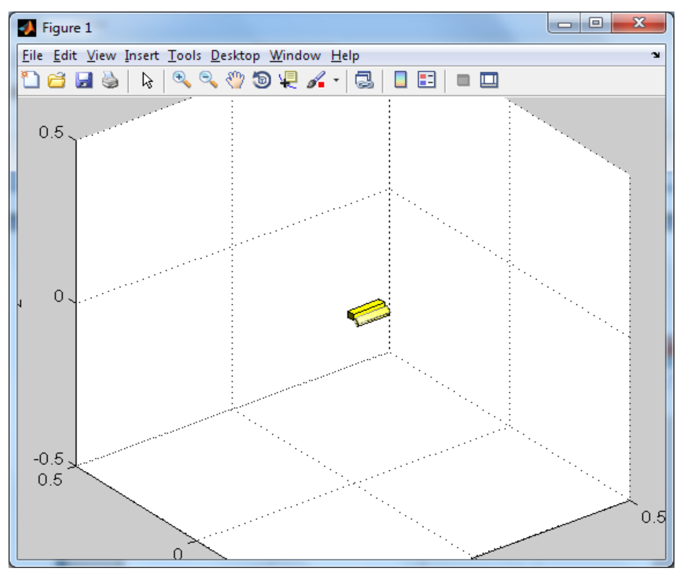

Fig.9. *Screenshot Result for Initial condition of Rotional Motion of Aerospace Vehicle*

Based on the theoretical background of the rotational motion equations, the space vehicle is travelled to the rotated parameters of Euler angle. The simulation result for final condition of rotational motion of aerospace vehicle with animation is illustrated in Fig.10. In this Figure, the aerospace vehicle is moved downwards direction according to the three dimensional position of x,y, and z coordinate in space. At initial, the object is rotated by following the motion equations of aerospace vehicle. Based on the speculative conditions of the rotational motion equations, the space vehicle is travelled to the rotated parameters of Euler angle.

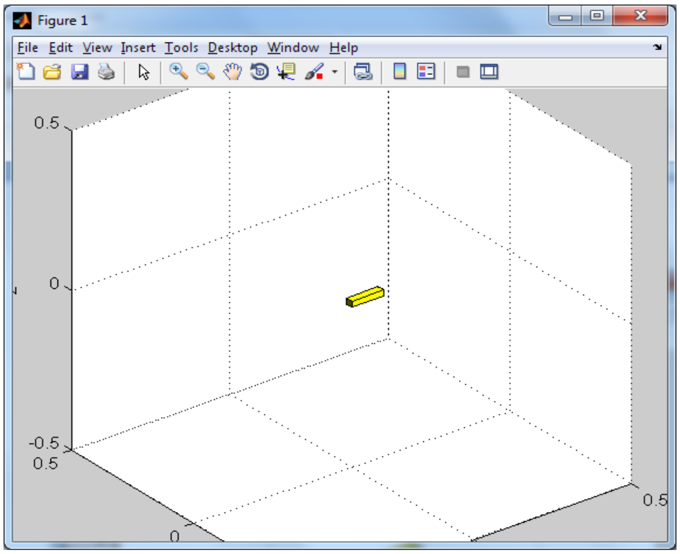

Fig.10. *Screenshot Result for Final condition of Rotational Motion of Aerospace Vehicle*

7.1 Simulation Results of Euler Angle of Translational Motion

Of course, the point of singularity of Euler angle representation (θ = 90◦) would render this procedure useless (in such a case, a different representation, e.g., quaternion, should be used). From the resulting plot of the Euler angles (Fig. 11), it is evident that the singularity has not been encountered for the given angular velocity and initial condition..

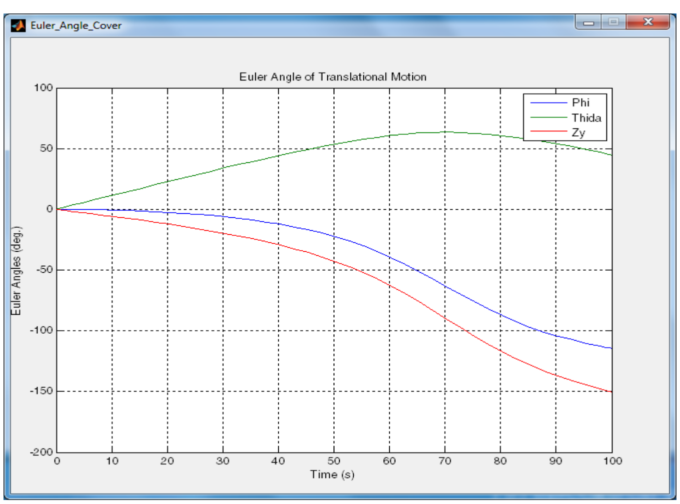

Fig.11. *Screenshot Result of Euler Angle of Translational Motion*

Hence, the total velocity and acceleration vectors of the passenger at t =100 s in the chosen stationary frame are v = −39.946I+100.0275J+29.8872K m/s and a = −0.4136I+ 0.9924J− 0.0043K m/s2.

7.2 Simulation Results of Lift Vector of Translational Motion

The resulting plots of L and μ for a complete turn are shown in Fig. 12. Note that the lift magnitudes are nearly double the weight for the given turn rate and wind velocity. Also, the maximum bank angle and lift are required when the aircraft is banking into the wind, which happens in this case for ψ = 45◦ (i.e., southwest orientation of ox) at t = 31.5 s.

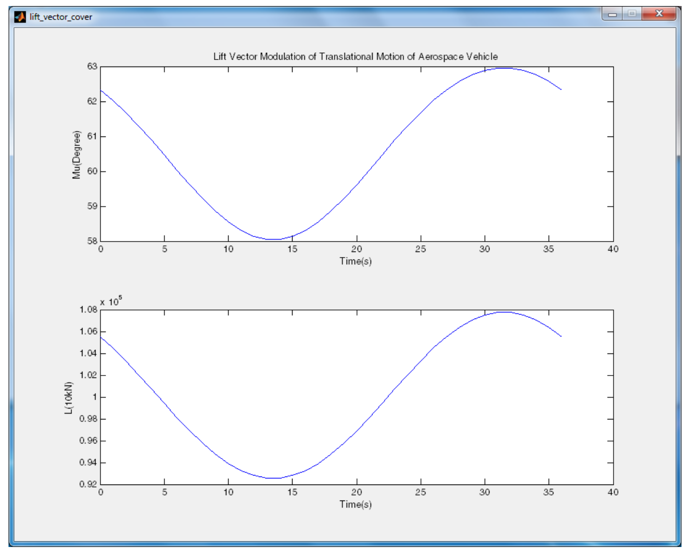

Fig.12. *Screenshot Result of Lift Vector of Translational Motion*

7.3 Simulation Results of Flight Path Angle of Aerospace Vehicle

The specified tolerance, δ = 1× 10^{-10}, is thus met in two iterations, with the eccentric anomaly calculated to be E = 3.82491121296282 rad (219.15127◦). The true anomaly is then calculated to be θ = 3.74977535060384 rad (214.84630◦). The resulting plots are shown in Fig.13.

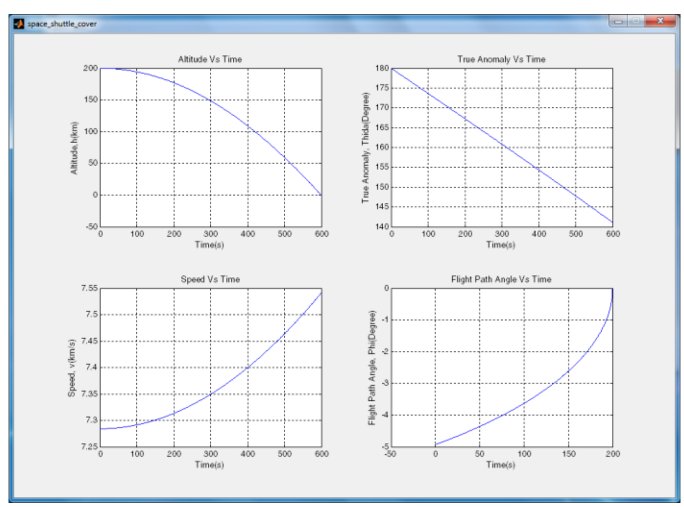

Fig.13. *Screenshot Result of Flight Path Angle of Aerospace Vehicle*

According to this response curves, the altitude, true anomaly, speed, and flight path for space vehicle based on the translational motion with respect to time are described and the space vehicle is unconditionally stable at the specified range of the space shuttle analysis. In these response curves from de-boost until earth impact, neglecting the effects of the earth's atmosphere, and assuming no further maneuvering.

4 Discussions

Atmospheric and space flight vehicles, although having evolved separately, obey the same physical principles and share the same modeling and simulation concepts. The categorization of flight vehicles into spacecraft (satellites, lunar, and interplanetary craft) and aircraft (balloons, airplanes, gliders, missiles, launch, and entry vehicles) is by mission rather than by physical distinction, and includes vehicles such as the space shuttle. Modeling of any flight vehicle involves idealization, selection of appropriate coordinate frames, and derivation of the governing equations of motion. Simulation refers to the task of accurately integrating the governing differential equations of motion in time, while including appropriate environmental and control effects. The ready-made modeling and simulation tools, such as the MATLAB/SIMULINK software, enable the analysis of most flight situations with ease. However, a successful simulation with even such versatile tools requires a correct problem formulation and a suitable mathematical model.

5 Conclusion

A trajectory model is based upon the particle assumption of a flight vehicle where the rotational motion is disregarded. The resulting translational model has only three degrees of freedom. While the laws of motion are valid in an inertial frame, it is often necessary to express the velocity and acceleration in a moving frame, because the force acting on the vehicle is generally resolved in such a frame. Acceleration resolved in a moving frame consists of linear, angular, centripetal, and Coriolis accelerations, as well as the acceleration of the frame's origin. The flight dynamic equations motions in a moving frame are inherently nonlinear and require an iterative numerical solution procedure. A vehicle with variable mass is capable of producing thrust due to reaction of the ejected mass by Newton's third law of motion. The general motion of a body can be described by the translation of the center of mass and the rotation of the body about its center of mass. Energy and angular momentum are very useful in describing the general motion of a body. The work done by a conservative force is independent of the path the body follows. A system of N bodies in mutual gravitational attraction is a conservative system, whose analytical solution is possible only if N = 2. The integrals of the two-body problem satisfy Kepler's laws of elliptical planetary orbits, but are more general in that there can also describe open trajectories. While the shape of a two-body trajectory can be obtained in a closed form, the relative position as a function of time generally requires an iterative solution. Lagrange's coefficients provide a compact representation of both relative position and velocity of the two-body motion.

References

[1] Whittaker, E.T.: Analytical Dynamics of Particles and Rigid Bodies. Cambridge University Press, Cambridge (1965).

[2] Wertz, J.R. (ed.): Spacecraft Attitude Determination and Control. Kluwer Academic Publishers, Dordrecht (1978).

[3] Shuster, M.D.: A survey of attitude representations. J. Astronautical Sci., **41**,439–517 (1993).

[4] Kreyszig, E.: Advanced Engineering Mathematics. Wiley, New York (2001).

[5] Abramowitz, M., and Stegun, I.A.: Handbook of Mathematical Functions. Dover Publications, New York (1974).

[6] Lemoine, F.G., Smith, D.E., Rowlands, D.D., Zuber, M.T., Neumann, G.A.,and Chinn, D.S.: An improved solution of the gravity field of Mars (GMM-2B) from Mars Global Surveyor. J. Geophysical Res., **106(E10)**, 23359–23376(2001).

[7] Britting, K.R.: Inertial Navigation Systems Analysis. Wiley Interscience, Somerset, NJ (1971).

[8] Vallado, D.A.: Fundamentals of Astrodynamics and Applications. Kluwer Academic Press, Dordrecht (2001).

[9] Chobotov, V.A.: Orbital Mechanics. American Institute of Aeronautics and Astronautics (AIAA), Education Series, Washington, DC (1996).

[10] Battin, R.H.: Astronautical Guidance. McGraw-Hill, New York (1964).

[11] Battin, R.H.: An Introduction to the Mathematics and Methods of Astrodynamics. AIAA Education Series, Reston, VA (1999).

[12] Dunham, D.W., and Davis, S.A.: Optimization of a multiple Lunar–Swingby trajectory sequence. J. Astron. Sci., **33**, 275–288 (1985).

[13] Well, K.H.: Use of the method of particular solutions in determining periodic orbits of the earth-moon system. J. Astronautical Sci., **19**, 286–296 (1972).

[14] Byrnes, D.V.: Application of the pseudostate theory to the three-body Lambert problem. J. Astronautical Sci., **37**, 221–232 (1989).