The R-Dynamic Local Irregularity Vertex Coloring Of Graph

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Abstract: We define the r-dynamic local irregularity vertex coloring. Suppose $\lambda : V(G) \rightarrow \{1,2, \dots , k\}$ is called vertex irregular k-labeling and $w : V(G) \rightarrow N$ where $w(u) = \sum_{v \in N(u)} \lambda(v)$. λ is called r-dynamic local irregular vertex coloring, if: (i) opt(λ) = min{max{λi}; λ_i vertex irregular k-labeling}, (ii) for every $uv \in E(G)$, $w(u) \neq w(v)$, and (iii) for every $v \in V(G)$ such that $|w(N(v))| \geq \min\{r, d(v)\}$. The chromatic number r-dynamic local irregular denoted by $\chi_{lis}^{r}(G)$, is minimum of cardinality r-dynamic local irregular vertex coloring. We study the r-dynamic local irregularity vertex coloring of graph and we have found the exact value of chromatic number r-dynamic local irregularity of some graph.

Index Terms: r-dynamic coloring, local irregularity, vertex coloring.

1 Introduction

GRAPH in this paper are simple and finite. For $v \in V(G)$, let $N(v)$ denote the set of vertices adjacent to v in G and $d(v) = |N(v)|$. Vertices in $N(v)$ are neighbors of v. Montgomery [3] introduced the r-dynamic coloring. Let r be a positive integer. An r-dynamic k-coloring is a proper vertex k-coloring such that every vertex v receives at least $\min\{r, d(v)\}$. Furthermore Lai defined r-dynamic chromatic number that the minimum k, which G admits an r-dynamic k-coloring and is denoted $\chi_r(G)$.

Kristiana, et.al [1] defined local irregularity vertex coloring. Suppose $l : V(G) \rightarrow \{1, 2, \dots ,k\}$ is called vertex irregular k-labeling and $w : V(G) \rightarrow N$ where $w(u) = \sum_{v \in N(u)} l(v)$, l is called local irregularity vertex coloring, if (i) max(l) = min{max{l_i} and (ii) for every $uv \in E(G)$, $w(u) \neq w(v)$. Furthermore Kristiana, et.al [2] founded chromatic number local irregularity of path graph, cycle graph, complete graph, bipartite complete graph, star graph, and friendship graph. In this paper, we combine r-dynamic coloring and local irregularity vertex coloring.

2 Result

In this paper, we present new definition of the r-dynamic local irregularity vertex coloring of graph and the chromatic number r-dynamic local irregular. We study the exact value of chromatic number r-dynamic local irregular of some graphs.

Definition 1

Let $\lambda : V(G) \rightarrow \{1,2, \dots , k\}$ is called vertex irregular k-labeling and $w : V(G) \rightarrow N$ where $w(u) = \sum_{v \in N(u)} \lambda(v)$. λ is called r-dynamic local irregular vertex coloring, if:

i. opt(λ) = min{max{λi}; λ_i vertex irregular k-labeling}
ii. For every $uv \in E(G)$, $w(u) \neq w(v)$
iii. For every $v \in V(G)$ such that $|w(N(v))| \geq \min\{r, d(v)\}$.

Definition 2

The chromatic number r-dynamic local irregular denoted by $\chi_{lis}^{r}(G)$, is minimum of cardinality r-dynamic local irregular vertex coloring.

For $r = 1$ is called chromatic number local irregular and for $r = 2$ is called chromatic number dynamic local irregular.

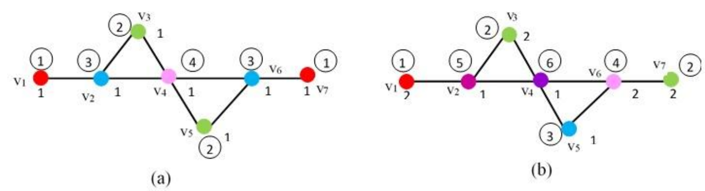

Figure 1. An example of local irregularity r-dynamic

Illustration of local irregularity r-dynamic vertex coloring is presented in Figure 1.

Observation 1

Let be graph G, where $N(u) = N(v)$, graph G doesn't have local irregularity vertex coloring for $r \geq 2$.

Based on Observation 1, some graph don't have local irregularity r-dynamic for $r \geq 2$, namely star graph, path graph with order 3 and bipartite graph.

Observation 2

Let be G connected graph, local irregularity r-dynamic vertex coloring for $r \geq 2$ have opt(λ) ≥ 3.

Lemma 1

Graph connected G, $\chi_{lis}^{r}(G) \geq \chi_{lis}(G)$

Proof: Let $b : V(G) \rightarrow N$ be local irregularity vertex coloring, for $uv \in E(G)$, $b(u) \neq b(v)$.

$$\chi_{lis}(G) = \min\{|b(V(G))|; b \text{ local irregularity vertex coloring}\}$$

Based on Definition 1, b is vertex irregular k-labeling such that $\chi_{lis}(G) \leq |b(V(G))|$.

Thus, $\chi_{lis}(G) \leq \min\{|b(V(G))|\} = \chi_{lis}^{r}(G)$. □

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Theorem 1
Let P_n be path graph, $\chi_{lis}^r(P_n) = 4$ where $n \geq 6$
Proof: $V(P_n) = \{a_i, 1 \leq i \leq n\}$ and $E(P_n) = \{a_i a_{i+1};\ 1 \leq i \leq n\}$
Based on Observation 1, opt(λ) = 3 and Based on Lemma 1, the lower bound chromatic number r-dynamic is $\chi_{lis}^r(Pn) \geq \chi_{lis}(P)$. Further, it will be shown the upper bound, we define $\lambda : V(P_n) \rightarrow \{1, 2, 3\}$.
Case 1. For $n \equiv 0 \pmod 3$

$$\lambda(a_i) = \begin{cases} 1, & i \equiv 1\ (mod\ 3), 1 \leq i \leq n \\ 2, & i \equiv 2\ (mod\ 3), 1 \leq i \leq n \\ 3, & i \equiv 0\ (mod\ 3), 1 \leq i \leq n \end{cases}$$

It easy to see opt(λ) = 3 and weight function as follows:

$$w(a_i) = \begin{cases} 2, & i = 1, n \\ 3, & i \equiv 0\ (mod\ 3), 2 \leq i \leq n-1 \\ 4, & i \equiv 2\ (mod\ 3), 2 \leq i \leq n-1 \\ 5, & i \equiv 1\ (mod\ 3), 2 \leq i \leq n-1 \end{cases}$$

Case 2. For $n \equiv 1 \pmod 3$

$$\lambda(a_i) = \begin{cases} 1, & i \equiv 1\ (mod\ 3), 1 \leq i \leq n-2 \\ 2, & i = n-1, n \text{ or } i \equiv 2\ (mod\ 3), 1 \leq i \leq n-2 \\ 3, & i \equiv 0\ (mod\ 3), 1 \leq i \leq n-2 \end{cases}$$

It easy to see opt(λ) = 3 and weight function as follows:

$$w(a_i) = \begin{cases} 2, & i = 1, n \\ 3, & i = n-2 \text{ or } i \equiv 0\ (mod\ 3), 2 \leq i \leq n-2 \\ 4, & i = n-1 \text{ or } i \equiv 2 (mod\ 3), 2 \leq i \leq n-2 \\ 5, & i \equiv 1 (mod\ 3), 2 \leq i \leq n-2 \end{cases}$$

Case 3. For $n \equiv 2 \pmod 3$

$$\lambda(a_i) = \begin{cases} 1, & i = n \text{ or } i \equiv 0\ (mod\ 3), 1 \leq i \leq n-1 \\ 2, & i \equiv 2\ (mod\ 3), 1 \leq i \leq n-1 \\ 3, & i \equiv 1\ (mod\ 3), 1 \leq i \leq n-1 \end{cases}$$

It easy to see opt(λ) = 3 and weight function as follows:

$$w(a_i) = \begin{cases} 2, & i = 1, n-1 \\ 3, & i = n \text{ or } i \equiv 1\ (mod\ 3), 2 \leq i \leq n-1 \\ 4, & i \equiv 2\ (mod\ 3), 2 \leq i \leq n-1 \\ 5, & i \equiv 0\ (mod\ 3), 2 \leq i \leq n-1 \end{cases}$$

For every $uv \in E(P_n)$, $u = a_i$, $v = a_{i+1}$, $1 \leq i \leq n-1$ obtained $w(a_i) \neq w(a_{i+1})$. For $a_i \in V(P_n)$ such that $|w(a_i)| \geq \min\{r, d(a_i)\}$. Based on Definition 1, w is called local irregularity r-dynamic. Weight function obtain $|w(V(P_n))| = 4$. Thus, $\chi_{lis}^r(P_n) \leq 4$.
Hence, $\chi_{lis}^r(P_n) = 4$. The proof is complete. □
Illustration the r-dynamic local irregularity vertex coloring of path graph is presented in Figure 2.

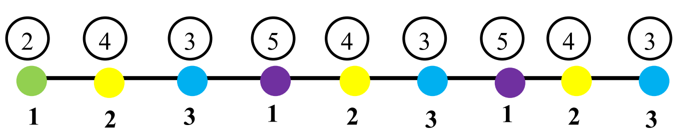

Figure 2. The 2-dynamic local irregularity of P_9

Theorem 2
Let C_n be cycle graph, for $n \geq 5$

$$\chi_{lis}^r(C_n) = \begin{cases} 3, & n \equiv 0\ (mod\ 3) \\ 4, & n = 7 \\ 5, & n \equiv 1,2\ (mod\ 3), n \neq 7 \end{cases}$$

Proof: $V(C_n) = \{a_i, 1 \leq i \leq n\text{-}1\}$ and $E(C_n) = \{a_i a_{i+1};\ 1 \leq i \leq n\text{-}1\} \cup \{a_n a_1\}$. Based on Observation 1, opt(λ) = 3 and Based on Lemma 1, the lower bound chromatic number r-dynamic is $\chi_{lis}^r(C_n) \geq \chi_{lis}(C_n) = 3$. Further, it will be shown the upper bound, we define $\lambda : V(C_n) \rightarrow \{1, 2, 3\}$.
Case 1. For $n \equiv 0 \pmod 3$

$$\lambda(a_i) = \begin{cases} 1, & i \equiv 1\ (mod\ 3), 1 \leq i \leq n \\ 2, & i \equiv 2\ (mod\ 3), 1 \leq i \leq n \\ 3, & i \equiv 0\ (mod\ 3), 1 \leq i \leq n \end{cases}$$

It easy to see opt(λ) = 3 and weight function as follows:

$$w(a_i) = \begin{cases} 3, & i \equiv 0\ (mod\ 3), 1 \leq i \leq n \\ 4, & i \equiv 2\ (mod\ 3), 1 \leq i \leq n \\ 5, & i \equiv 1\ (mod\ 3), 1 \leq i \leq n \end{cases}$$

For every $uv \in E(C_n)$, $u = a_i$, $v = a_{i+1}$, $1 \leq i \leq n\text{-}1$ obtained $w(a_i) \neq w(a_{i+1})$ and $u = a_n$, $v = a_1$ obtained $w(a_n) \neq w(a_1)$. For $a_i \in V(C_n)$ such that $|w(N(a_i))| \geq \min\{r, 2\}$. Based on Definition 1, w is called local irregularity r-dynamic. Weight function obtain $|w(V(C_n))| = 3$. Thus, $\chi_{lis}^r(C_n) \leq 3$.
Hence $\chi_{lis}^r(C_n) = 3$
Case 2. For n = 7

$$\lambda(a_i) = \begin{cases} 1, & n = 2,4 \\ 2, & n = 5,6,7 \\ 3, & n = 1,3 \end{cases}$$

It easy to see opt(λ) = 3 and weight function as follows:

$$w(a_i) = \begin{cases} 2, & i = 3 \\ 3, & i = 1,5 \\ 4, & i = 2,6 \\ 5, & i = 4,7 \end{cases}$$

For every $uv \in E(C_n)$, $u = a_i$, $v = a_{i+1}$, $1 \leq i \leq n\text{-}1$ obtained $w(a_i) \neq w(a_{i+1})$ and $u = a_n$, $v = a_1$ obtained $w(a_n) \neq w(a_1)$. For $a_i \in V(C_7)$ such that $|w(N(a_i))| \geq \min\{r, 2\}$. Based on Definition 1, w is called local irregularity r-dynamic. Weight function obtain $|w(V(C_n))| = 4$. Thus, $\chi_{lis}^r(C_n) \leq 4$.
Hence $\chi_{lis}^r(C_n) = 4$
Case 3. For $n \equiv 1,2 \pmod 3$, $n \neq 7$
Subcase 1. For $n \equiv 1 \pmod 3$, $n \neq 7$

$$\lambda(a_i) = \begin{cases} 1, & i = 2, n-1 \text{ or } i \equiv 1\ (mod\ 3), 4 \leq i \leq n-3 \\ 2, & i = n \text{ or } i \equiv 2\ (mod\ 3), 4 \leq i \leq n-3 \\ 3, & i = 1, n-2 \text{ or } i \equiv 0\ (mod\ 3), 3 \leq i \leq n-3 \end{cases}$$

It easy to see opt(λ) = 3 and weight function as follows:

$$w(a_i) = \begin{cases} 2, & i = 3, n-2 \\ 3, & i = 1 \text{ or } i \equiv 0\ (mod\ 3), 4 \leq i \leq n-4 \\ 4, & i = n \text{ or } i \equiv 2\ (mod\ 3), 4 \leq i \leq n-4 \\ 5, & i = n-1 \text{ or } i \equiv 2\ (mod\ 3), 4 \leq i \leq n-4 \\ 6, & i = 2, n-3 \end{cases}$$

Subcase 2. For $n \equiv 2 \pmod 3$

$$\lambda(a_i) = \begin{cases} 1, & i \equiv 1\ (mod\ 3), 1 \leq i \leq n-1 \\ 2, & i \equiv 2\ (mod\ 3), 1 \leq i \leq n-1 \\ 3, & i = n \text{ or } i \equiv 0\ (mod\ 3), 1 \leq i \leq n-1 \end{cases}$$

It easy to see opt(λ) = 3 and weight function as follows:

$$w(a_i) = \begin{cases} 2, & i = n \\ 3, & i \equiv 0\ (mod\ 3), 1 \leq i \leq n-2 \\ 4, & i \equiv 2\ (mod\ 3), 1 \leq i \leq n-2 \\ 5, & i \equiv 1\ (mod\ 3), 1 \leq i \leq n-2 \\ 6, & i = n-1 \end{cases}$$

For every $uv \in E(C_n)$, $u = a_i$, $v = a_{i+1}$, $1 \leq i \leq n\text{-}1$ obtained $w(a_i) \neq w(a_{i+1})$ and $u = a_n$, $v = a_1$ obtained $w(a_n) \neq w(a_1)$. For $a_i \in V(C_n)$ such that $|w(N(a_i))| \geq \min\{r, 2\}$. Based on Definition 1, w is

called local irregularity r-dynamic. Weight function obtain $|w(V(C_n))| = 5$. Thus, $\chi_{lis}^r(C_n) \leq 5$.
Hence $\chi_{lis}^r(C_n) = 5$.
The proof is complete. □
Illustration the *r*-dynamic local irregularity vertex coloring of cycle graph is presented in Figure 3.

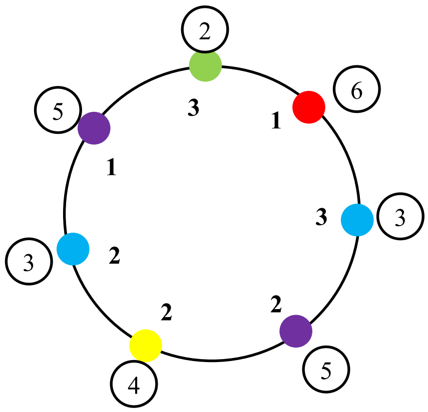

Figure 3. The 2-dynamic local irregularity of C_7

Theorem 3
Let K_n be complete graph, $\chi_{lis}^r(K_n) = n$
Proof: $V(K_n) = \{ a_i, 1 \leq i \leq n \}$. Suppose $u,v \in V(K_n)$, Based on Observation 3, $N(u)$ - $\{v\}$ = $N(v)$ - $\{u\}$ so that $\lambda(u) \neq \lambda(v)$. It show labeling of every vertex in complete graph as different. So opt(λ) = n. Based on Lemma 1, $\chi_{lis}^r(K_n) \geq \chi_{lis}(K_n) = n$
Further, to show the upper bound, we define $\lambda : V(K_n) \rightarrow \{1, 2, \ldots, n\}$ where $\lambda(a_i) = i$, $1 \leq i \leq n$. weight function is $w(a_i) = \frac{n(n+1)}{2} - i$. Because $i = 1, 2, \ldots, n$, where $|w(V(K_n))| = n$ so that $n = \chi_{lis}(K_n) \leq \chi_{lis}^r(K_n) \leq |w(V(K_n))| = n$. Thus, $\chi_{lis}^r(K_n) = n$.
The proof is complete. □

3 Conclusion

In this paper we have studied the *r*-dynamic local irregularity vertex coloring. We have concluded the exact value of the chromatic number *r*-dynamic local irregular of some graphs, namely path graph, cycle graph and complete graph.

Acknowledgment

We gratefully acknowledgment the support from University of Jember of 2019.

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