Testing Cumulative Semivariogram Modelling Technique on Bouguer Anomaly

Rishabh Gupta, Utkarsh Rastogi

Abstract: In principle, spatial correlation is depicted using Semivariogram technique. Cumulative Semivariogramtechnique is the alternative for conventional Semivariogram technique, has various advantages but not widely used in gravity modelling. Our objective is to test the Cumulative Semivariogram technique in gravity modelling by estimating Bouguer Anomaly in Krishna Godavari (KG) basin area by fitting the experimental Cumulative Semivariogram against the theoretical Cumulative Semivariogram and fitting the experimental Semivariogram against theoretical Semivariogram. Hence, calculate the value obtained at the random point using appropriate kriging technique after Cumulative Semivariogram model fitting and Semivariogram model fitting. Finally calculate the percentage variation between the predicted values obtained from both the techniques with the observed Bouguer Anomaly. Percentage variation will give an insight whether Cumulative Semivariogram technique is the better alternative than Semivariogram technique for geostatistical modelling or not.

Index Terms: Bouguer Anomaly, Cumulative Semivariogram, Kriging, Semivariogram and spatial correlation

1 Introduction

Generally, Semivariogram technique proposed by [6] is used to determine spatial correlation between regionalized variables and hence, predict the value of regionalized variable at an unsampled location using Kriging.The very basic definition of the Semivariogram is the graph of half squared difference variation of regionalized variable with distance given by “(1)”. Where, k is the counter of the distance, n is the number of

$$\gamma(d) = \frac{1}{2n_d}\sum_{k=1}^{n_d}(Z_i - Z_{i+d})^2$$

sample points of observations of the values of attribute z separated by distance d. The Semivariogram thus obtained is called experimental Semivariogram which is then fitted to the preexisting models called theoretical Semivariogram [8]. The theoretical Semivariogram thus obtained is used for interpolation using appropriate Kriging techniques given by [4]. The Cumulative Semivariogram method proposed by [9] is an alternative to the classical Semivariogram technique. The Cumulative Semivariogram is the graph that shows the variation of successive half squared difference summations with distance hence a non-decreasing Cumulative Semivariogram curve is obtained as shown in Fig. 1, which is then fitted with the preexisting theoretical models given by [7]. In order to be able to apply Kriging estimation techniques to a regionalized variable, a functional relationship must be established between the distance and measure of regional dependence which herein is the Cumulative Semivariogram [10]. By considering basic definition of both classical Semivariogram and Cumulative Semivariogram, they may be related through an integration given by “(2)”. Where, $\gamma_c(d)$ = cumulative Semivariance at distance d, γ (u) = Semivariance at distance u. The Cumulative Semivariogram method has the various advantages over the classical Semivariogram technique as follows:

1. The Cumulative Semivariogram is a non-decreasing function; however, there may be local flat portions,

$$\gamma_c(d) = \int_0^d \gamma(u)\,du \tag{2}$$

implying constancy of the regionalized variables at certain distance i.e., the same values have been observed at two locations h apart.
2. The slope of the theoretical Cumulative Semivariogram at any distance is an indicator of the dependence between pairs of regionalized variables separated by that distance.
3. The sample Cumulative Semivariogram reflects even smaller dependencies between data pairs, which are not possible to detect with classical Semivariogram due to the averaging procedure.
4. Any classical Semivariogram model has a theoretical Cumulative Semivariogram counterpart, which can be obtained through an integration operation.

In this paper our objective is to test the whether Cumulative Semivariogram technique is the better alternative than classical Semivariogram technique to estimate Bouguer anomaly given by [3], by applying both the modelling techniques to predict the Bouguer anomaly values at an unsampled locations in part of KG Basin, India using ordinary kriging. Finally, calculation of percentage deviation between predicted values obtained by applying both the techniques and observed Bouguer anomaly atunsampled location will help us in determination of most appropriate modelling technique since, lesser the percentage deviation closer the value is predicted. In the next sections we have discussed about our area of interest, data sources, and methodology of how we predicted the value at an unsampled location using Ordinary Kriging, results and finally conclude our study.

2 Area of Interest

Our area of interest is a part of Krishna Godavari Basin. Extensive deltaic plain formed by two large east coastal rivers, Krishna and Godavari in the state of Andhra Pradesh, India and the adjoining areas of Bay of Bengal in which these rivers discharge their water is known as Krishna Godavari Basin. The detailed geology of KG Basin area is given by [2]. Fig. 2, shows the area of interest over which we carried out

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geostatistical modelling. The **spatial extent** of Area of Interest is as follows:

Top: 17.005523 DD
Left: 79.990032 DD Right: 82.999229 DD
Bottom: 15.058142 DD

Total Area: 39320.095549m^2, all data is taken in UTM coordinate system.

3 METHODOLOGY

113 sample points of Bouguer Anomaly, as shown in Fig. 3, is taken over the area of interest from International Gravimetric Bureau (BGI) which collects gravity data on a world-wide basis. Fig. 4, shows the frequency distribution of the sample points, which shows sample points are normally distributed over an area. Hence, no transformation technique is required prior to geostatistical modelling as given by [5]. To calculate percentage deviation between observed Bouguer anomaly and predicted Bouguer anomaly, we have taken a random point at location (16.270833DD latitude, 81.0625 DD longitude), the observed Bouguer anomaly at this location is -20mgal.

3.1 Semivariogram modelling of Sample data

To carry out Semivariogram modelling, we created software "SVBinner" using C# .Net with the help of [11], which takes sample points location (X, Y coordinates), Z value in the form of Excel worksheet as an input and lag value (i.e. distance interval within which all Semivariance values are averaged) and gives averaged distance and corresponding Semivariance as an output in new excel worksheet as shown in Fig. 5. In SVBinner we input worksheet consisting of location of 113 sample points and value of Bouguer anomaly at each point, lag value is taken to be 12020m. Of the output worksheet we obtained from SVBinner, we plot a graph with distance in X-axis and Semivariance in Y-axis using Excel chart tools [12]. Hence, we obtained experimental Semivariogram for Bouguer anomaly data. After obtaining experimental Semivariogram we need to fit a best theoretical model in order to carryout Kriging. Using Trendline tools [12], we found the best fitted theoretical model is Power model given by "(3)" with maximum regression of 0.6594 as shown in Fig. 6.

$$y = 5.3991x^{0.4369} \qquad (3)$$

Where, α= 5.3991 and β= 0.4369 are model parameters used for Kriging.

3.2 Cumulative Semivariogram modelling of Sample Data

To carry out cumulative Semivariogram modelling we created another software "CSVTool" using C# .Net with the help of [11].CSVTool takes worksheet with same attributes as of "SVBinner" (refer Sec. 3.1) as an input and gives distance between two sample points and their corresponding cumulativeSemivariance, log of distance and log of cumulative Semivariance as an output in new excel worksheet as shown in Fig. 7. We input the same sample point's worksheet to CSVTool as of SVBinner.The steps to calculate Cumulative Semivariogram is given by [7]. Of the output worksheet obtained we plot the graph using Excel chart tools of the following and hence, fitted with a linear trendline to obtain the best fitted Cumulative Semivariogram model as given by [9]:

1. For **Linear model**: distance on X-axis, cumulative Semivariance on Y-axis as shown in Fig. 8. The regression for the fitted model comes to be 0.9607.
2. For **Power model:** $\log_{10}$Distance on X-axis, $\log_{10}$CSV on Y-axis as shown in Fig. 9. The regression for the fitted model comes to be 0.9216.
3. For **Logarithmic model:** $\log_{10}$Distance on X-axis, Cumulative Semivariance on Y-axis as shown in Fig. 10. The regression for the fitted model comes to be 0.8449.
4. For **Exponential model:** distance on X-axis, $\log_{10}$CSV on Y-axis as shown in Fig. 11. The regression for the fitted model comes to be 0.5829.

From the above observations we can see that the maximum regression comes for the linear model. Hence, linear model fits the best with experimental Cumulative Semivariogram given by "(4)".

$$y = 50.218x – 876984 \qquad (4)$$

Where, α= -876984 and β= 50.218 are model parameters.

3.3 Kriging to predict values at unsampled location

In any geostatistical modelling there are two stages as the estimation of Semivariogram orCumulative Semivariogram from a given set of regionalized data for the extraction of spatial dependence and then Kriging methodology, which provides estimation for locations where no data is available. So far, we have obtained the best fitted Semivariogram and Cumulative Semivariogram model for our sample data. Kriging is used to estimate what weights should be applied to each surrounding point to estimate the value of the regionalized variable at an unknown location. We created another software "ODKrigingTool" using C# .Net tocarry out ordinary Kriging as shown in Fig. 12, which is the most commonly used Kriging technique on our sample data. The steps to carry out ordinary kriging is given by [1]. ODKrigingTool takes the following inputs:

1. Type of variogram modelling you are performing
2. Best fitted model
3. Model parameters
4. Nearby sample points in the form of worksheet, similar to input worksheet in SVBinner software (refer Sec. 3.1)
5. Location of unknown point at which value has to be predicted

The output is generated in the form of excel worksheet. Since, we have taken a random point at which we have to predict the value (refer Sec. 3). Using, ODKrigingTool we have generated data for kriging by entering model parameters α= 5.3991 and β= 0.4369 for power model which is best fitted in Semivariogram modelling (refer Sec. 3.1). Similarly, we have generated data for kriging by entering model parameters α= -876984 and β= 50.218 for linear model which is best fitted in Cumulative Semivariogram modelling (refer Sec. 3.2). Finally, we copied the output generated by ODKrigingTool to "Output generation excel workbook" which is created for calculation of the predicted value at a particular location as given by (Burrough & Mcdonnell, 2011).

4 Results and Discussions

Bouguer anomaly we obtained at the selected location through kriging by applying Semivariogram model is -13.25409 mgal and by applying Cumulative Semivariogram model is -19.07 mgal. As we know that observed Bouguer anomaly at the selected location is -20 mgal (refer Sec. 3). Hence, we have calculated the percentage deviation from the original value with respect to predicted value using “5”.

$$\%\, Deviation = \frac{Original\ Value\ -\ Predicted\ Value}{Original\ Value} * 100 \qquad (5)$$

The % deviation for predicted value using Semivariogram model came out to be 33.73% and for cumulative Semivariogram modelcame out to be 4.65%.

5 Conclusion

From our observations we found that Cumulative Semivariogram modelling method generates better output than Semivariogram modelling. The error margin for Cumulative Semivariogram model predicted value is 4.65 % and for Semivariogram predicted value it is 33.73% with respect to observed Bouguer anomaly at selected location. Taking our study into consideration we would suggest that Cumulative Semivariogram modelling generates better predicted output for Bouguer anomaly as compared to Semivariogram modelling.

Acknowledgment

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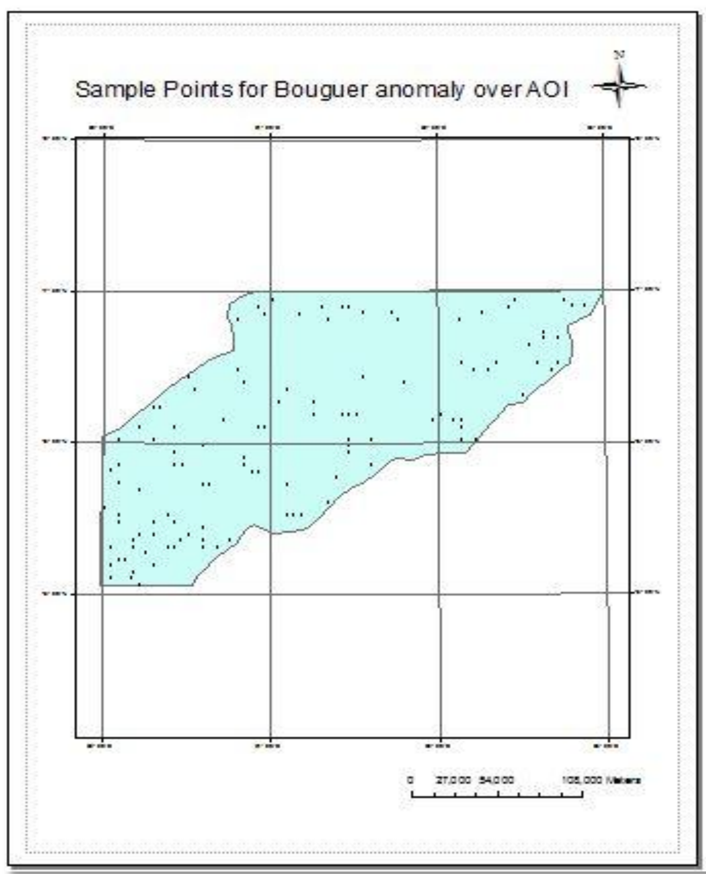

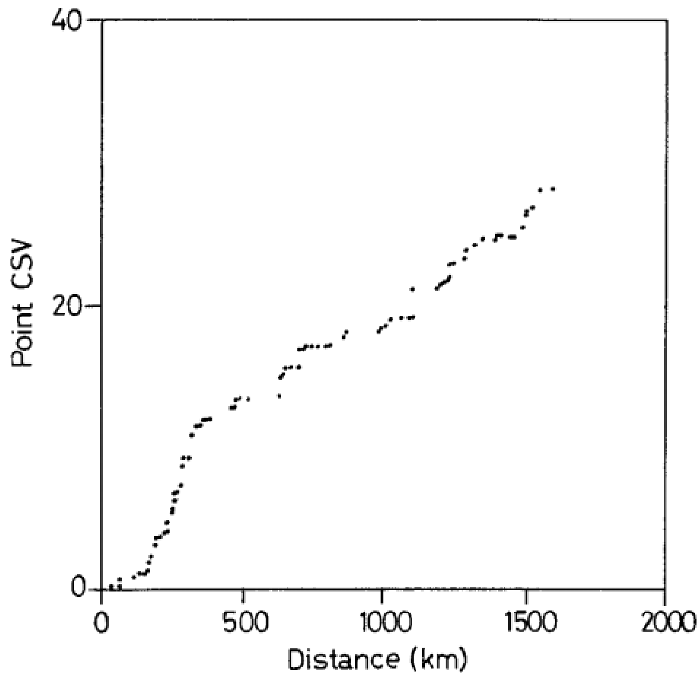

Fig. 2. Map of Area of Interest

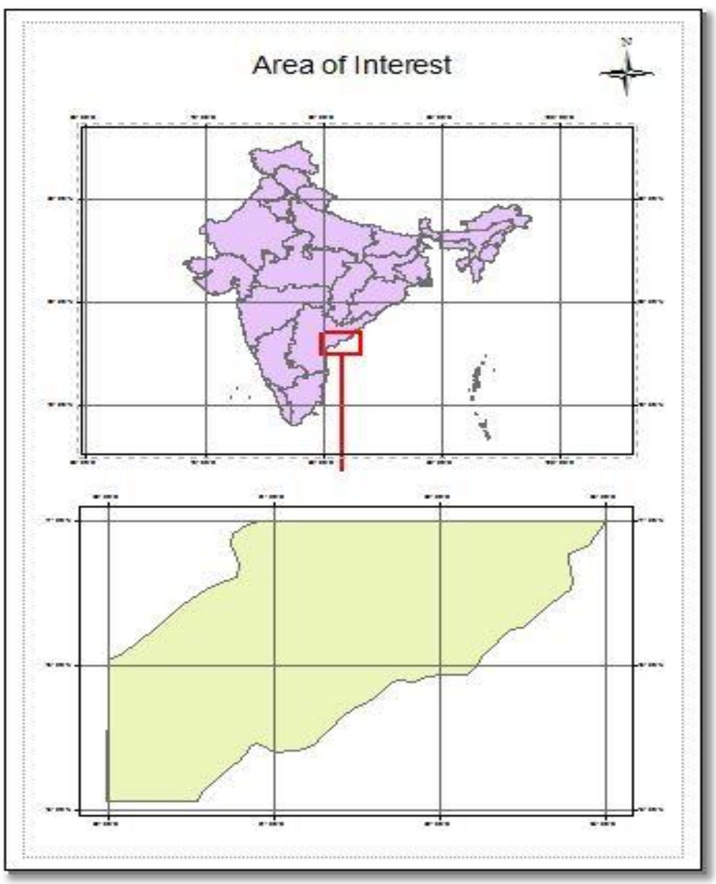

Fig. 3. Distribution of Sample points over Area of Interest

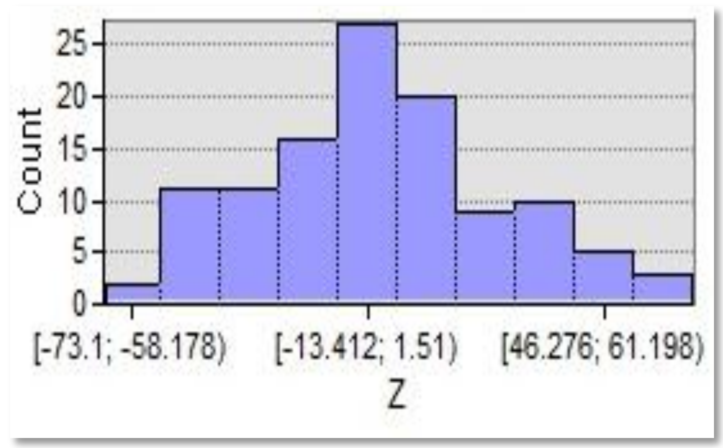

Fig. 4. Frequency distribution of sample points

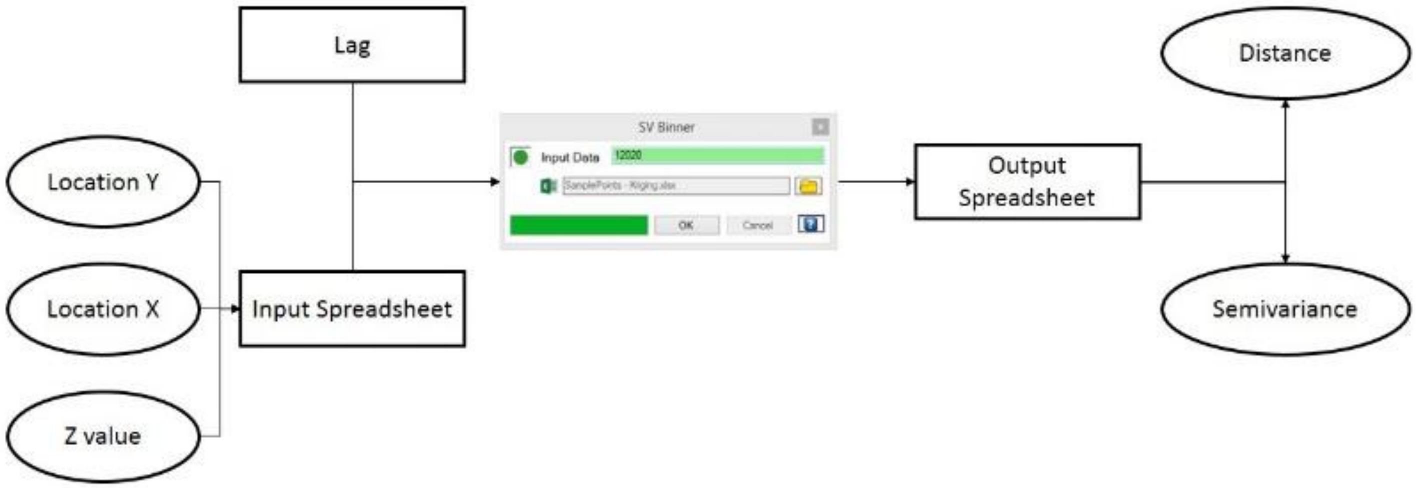

Fig. 5. Processing layout of SVBinner software

	A	B	C
1	Distance	Semivariance	Lag: 12020
2	10773.19	129.8678334	
3	22825.54	385.8077746	
4	34760.24	642.1127197	
5	46818.73	858.8714951	
6	59173.49	807.8886005	
7	71695.05	790.0528638	
8	83102.63	849.6065593	
9	95420.84	1052.576748	
10	107842.3	1328.344897	
11	119839	1392.926445	
12	132110.1	1248.352358	
13	143924.7	1303.646619	
14	156077.8	1149.344892	
15	168241	1117.254767	
16	179963.2	934.9911364	
17	192063.4	945.2474325	

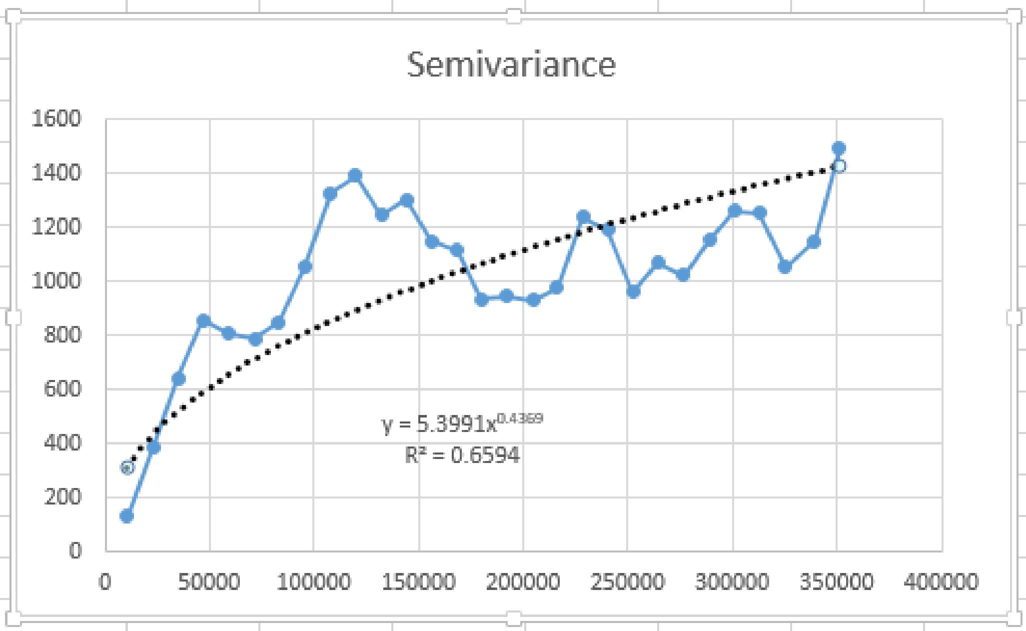

Fig. 6. Output of SVBinner software and Power model fitted to experimental variogram

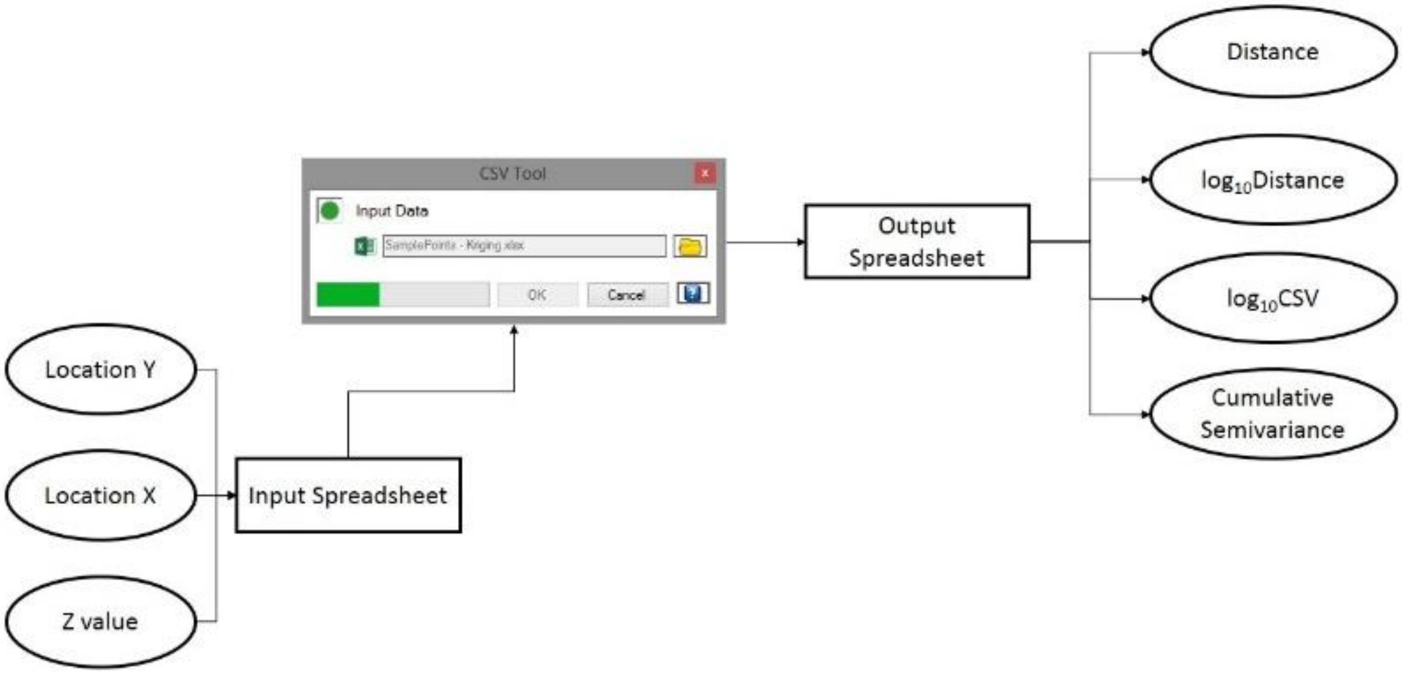

Fig. 7. Processing layout of CSVTool software

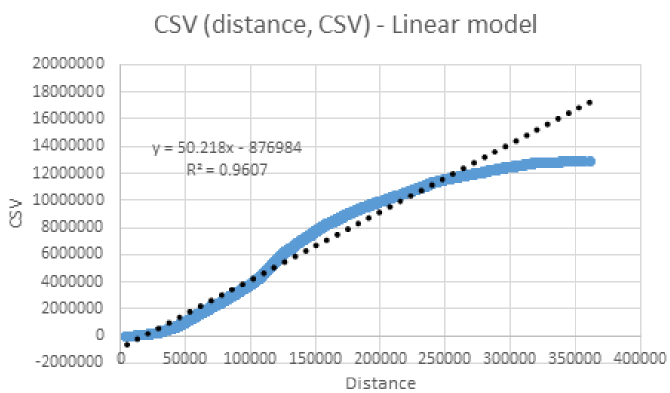

Fig. 8. Linear trendline fitted to CSV to verify linear model

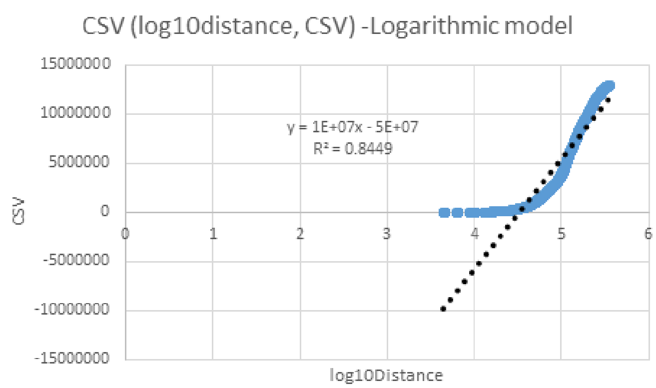

Fig. 10. Linear trendline fitted to CSV to verify logarithmic model

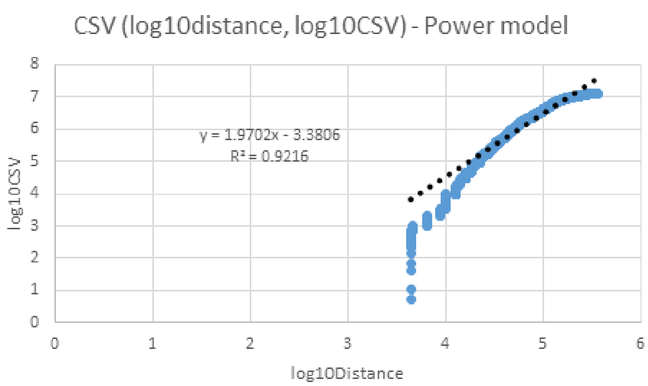

Fig. 9. Linear trendline fitted to CSV to verify power model

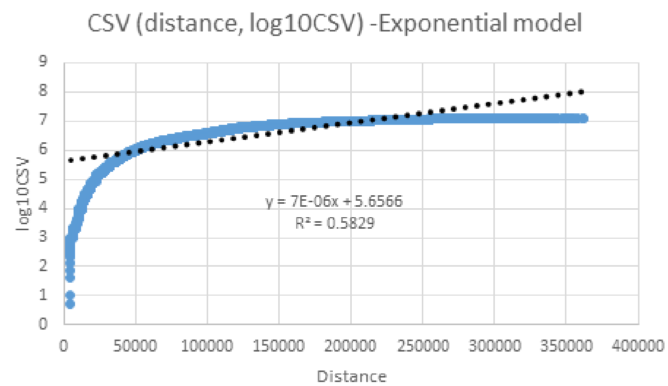

Fig. 11. Linear trendline fitted to CSV to verify exponential model

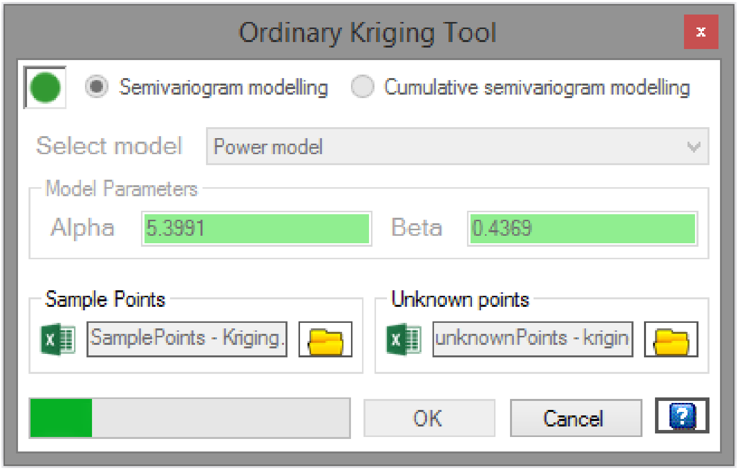

Fig. 12. ODKrigingTool software