Effect Of The Changes In The Weights On The Solution Of The Preemptive Weighted Linear Goal Programming Problems

M. O. Wankhade, H. S. Lunge

Abstract: The purpose of this paper is to study the effect of the changes in the weights (w^{+}_{ik}, w^{-}_{ik}) of the deviational variables (n_i, p_i, n_i+p_i) in the achievement function of the Weighted Linear Goal Programming Problem (GPP). If the weights of the deviational variables associated with priority levels are increased/decreased Lee's Modified Simplex method showed that an achievement level of unachieved priorities are decreased/increased respectively and the values of decision variables remain unchanged. While the standard simplex method showed that if the weights of the deviational variables are increased/decreased, an achievement level of the priorities are decreased/increased respectively, the values of decision variables remain unchanged for some weighting structures and there exist some other weighting structures for which the solution of GPP is changed. Thus Standard simplex method can be used for solution and sensitivity analysis of a particular class of WGPP.

Key words: Linear Goal Programming, Preemptive weighted goal programming, achievement function.

1 Introduction

Goal Programming (GP) is one of the most practical descriptive approaches to the search for good decisions in the multi-objective optimization environment, within the limited information, resources and cognitive capabilities of the decision maker. A goal programming was conceived by Charnes A and Cooper W [1] nearly a half century ago. The tool was extended and enhanced by their students and, later by other investigators, most notably Ijiri Y, Ignizio J P, Gass S, Romero C, Lee S M[7] and many others. The basic structure of the additive weighted GP is to minimize the objective function, which takes a mathematical expression of the sum of the weighted undesired deviations and the preemptive priority factors implies the higher order goals must be optimized before lower order goals[8]. Ignizio J P and Romero C [2] proposed the generalized GP technique, a sequential algorithm, the concept of the multidimensional dual, providing goal programming with an effective economic interpretation of its results as well as a means to support sensitivity and post optimality analysis. Latinopoulos D and Mylopoulos Y [6] observed one weakness of GP is that it could lead to non-rational decisions in cases when the decision maker is not fully informed or confident about the targets, the weights, the priority levels and the ordering of preferences. They performed two different types of sensitivity analysis i.e. by increasing the priority levels and re-arrangement of the priorities.

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The decision maker may not be able to set his own weighting scheme. A recent weight sensitivity algorithm for investigating a portion of weight space of interest to the decision maker in goal programming has been presented by Jones D [4]. Iskander M G [3] proposed the approach to provide a solution in which the goals achievements are proportionally related to the relative weights. The next section of this paper describes the Weighted GP Model and Preemptive Weighted GP models. The results obtained by Lee's modified simplex method and standard simplex method to study the effect of changes in weights of the deviational variables in the achievement function on the solution of the Preemptive WGP Problems are presented in the third section. Finally, Section 4 draws conclusions.

2 Weighted GP Model

Weighted GP, sometimes termed non preemptive GPP, is as shown below;

Minimize $$z = \sum_{i=1}^{m} (w_i^{+} p_i + w_i^{-} n_i) \quad (1)$$

Subject to $$\sum_{j=1}^{n} a_{ij} x_j + n_i - p_i = b_i \quad (2)$$

$$n_i, p_i, x_j \geq 0 \quad (3)$$

i =1, 2, , ,m; j = 1, 2, , , n

w_i^{+} & $w_i^{-} \geq 0$ represents the relative weights to be assigned to deviational variables in the achievement function.

2.1 Preemptive Weighted GP model

By combining the idea of preemptive priorities and weighting Charnes A and Cooper W [1] suggested the GP model, termed as Preemptive Weighted Goal Programming model, which is as shown below;

Minimize $z = \sum_{i=1}^{m} P_i \left[\sum_{k=1}^{K} (w_{ik}^{+} p_i + w_{ik}^{-} n_i) \right]$

(4)

Subject to $\sum_{j=1}^{n} a_{ij} x_j + n_i - p_i = b_i$

(5)

$n_i, p_i, x_j \geq 0$

(6)

i = 1, 2, , , m; j = 1, 2, , , n

$w_{ik}^{+}, \&\ w_{ik}^{-} \geq 0$ represents the relative weights to be assigned to each of the k = 1, 2, , , K different classes within i^{th} category to which the non Archimedean transcendental value of P_i is assigned. (P_i – Preemptive priority factors) $P_1 > P_2 > P_3 > , , , > P_i > P_{i+1} > > > >$ so on.

2.2 Preemptive Weighted GP to Weighted GP

The Preemptive Weighted Goal Programming problem can be rewritten as Weighted GP Problem. The procedure is described below;

- **i.** Rewrite the achievement function without priorities i.e. in the form of equation (1).
- **ii.** The deviational variables which appear in the achievement function of the original Preemptive WGP problem should be assigned the given weights.
- **iii.** The deviational variables which do not appear in the achievement function of the original Preemptive WGP problem should be assigned 0 weights.
- **iv.** Keep the goals and constraints unchanged and solve the resultant problem by standard simplex method.

3 Numerical Example

Consider a Preemptive WGP problem; find (x_1, x_2, x_3, x_4) so as to

Minimize $\bar{a} = P_1\{5.4n_4+1.25p_4\}+P_2\{2n_3+6p_3\}+P_3\{n_1\}+P_4\{3p_2\}$

subject to
$x_1+2x_2+1.5x_3+3x_4+n_1-p_1=50;$
$2x_1+x_2-x_3+7x_4+n_2-p_2 = 10;$
$4x_1+3x_2+x_3+5x_4+n_3-p_3 =40;$
$4x_1-2x_2+3x_3-x_4+n_4-p_4 = 25;$
$x_j \geq 0$; $n_i, p_i \geq 0$ j = 1, 2, 3, 4; i = 1, 2, 3, 4

An optimal solution by Lee's[7] Modified Simplex Method is**;** $x_1 = 0$, $x_2 =8.6364$, $x_3=14.091$, $n_1=11.591$, $n_2=n_3=n_4=0$, $p_1=15.455$, $p_2=p_3=p_4=0$

Achievement function $\bar{a} = \{0, 0, 11.591, 0\}$.

Priority levels P_1, P_2 & P_3 are fully achieved. The third priority level P_3 cannot be achieved completely. The solutions of the problem by changing the weights of deviational variables of the achievement function using Lee's [7] Modified Simplex Method are presented in the table No.1. The WGP form of the given problem is as under;

Minimize $\bar{a} = n_1+3p_2+ 2n_3+6p_3+ 5.4n_4+1.25p_4$

subject to
$x_1+2x_2+1.5x_3+3x_4+n_1-p_1=50;$
$2x_1+x_2-x_3+7x_4+n_2-p_2 = 10;$
$4x_1+3x_2+x_3+5x_4+n_3-p_3 =40;$
$4x_1-2x_2+3x_3-x_4+n_4-p_4 = 25;$
$x_j \geq 0$; $n_i, p_i \geq 0$ j = 1, 2, 3, 4; i = 1, 2, 3, 4

An optimal solution by Standard Simplex Method is**;** $x_1 = 0$, $x_2 =8.6364$, $x_3=14.091$, $n_1=11.591$, $n_2=n_3=n_4=0$, $p_1=15.455$, $p_2=p_3=p_4=0$

$\bar{a} = \{0, 0, 11.591, 0\}$.

Priority levels P_1, P_2 & P_3 are fully achieved. The third priority level P_3 cannot be achieved completely. The solutions of the WGP problem by changing the weights of deviational variables of the achievement function using Standard Simplex Method are presented in the **table No.2**

4 Conclusion

From the columns of n_1, p_1 and priority P_3 of table No.1, we observed that if the weights of the deviational variables associated with priority levels of the Preemptive WGPP are increased/decreased Lee's Modified Simplex method showed that an achievement level of unachieved priorities are decreased/increased respectively, the values of decision variables and fully achieved priorities remain unchanged. From the columns of n_1, p_1 and priority P_3 of table No.2, we found that if the weights of the deviational variables associated with priority levels of the Preemptive WGPP are increased/decreased Standard Simplex method showed that an achievement level of unachieved priorities are decreased/increased respectively, the values of decision variables and fully achieved priorities remain unchanged for some weighting structures but there exist some other weighting structures which affect the values of the decision variables and achievement level of the priorities. And it was also observed that the weighing structures which affect the values of the decision variable gives inferior solutions as far as priority preferences of objective functions/goals are concerned. Thus the Standard simplex method can be used for solution and sensitivity analysis of a particular class of WGPP.

Sr No	Weights in priority						Decision variables		deviational variables		Achievement of priorities			
	P_1		P_2		P_3	P_4	x_2	x_3	n_1	p_1	P_1	P_2	P_3	P_4
	w_5	w_6	w_3	w_4	w_1	w_2								
	n_4	p_4	n_3	p_3	n_1	p_2								
1	1.25	5.4	6	2	1	3	8.636	14.091	11.591	15.455	0	0	11.591	0
2	1	1	1	1	1	1	8.636	14.091	11.591	15.455	0	0	11.591	0
3	3	2.5	8	9.2	2	4	8.636	14.091	11.591	15.455	0	0	23.182	0
4	67	85	52	29	4	48	8.636	14.091	11.591	15.455	0	0	46.364	0
5	189	175	200	323	10	89	8.636	14.091	11.591	15.455	0	0	115.91	0
6	919	641	740	514	20	411	8.636	14.091	11.591	15.455	0	0	231.82	0
7	989	887	845	799	100	990	8.636	14.091	11.591	15.455	0	0	1159.1	0
7	565	451	600	245	0.5	750	8.636	14.091	11.591	15.455	0	0	5.7955	0
8	123	653	299	156	0.4	272	8.636	14.091	11.591	15.455	0	0	4.6364	0
9	99	239	88	89	0.25	301	8.636	14.091	11.591	15.455	0	0	2.8976	0
10	439	339	196	509	0.2	67	8.636	14.091	11.591	15.455	0	0	2.3182	0
11	≤ 0.818	1	9	4	1	2	8.636	14.091	11.591	15.455	0	0	11.591	0
12	≥ 0.001	1	9	4	0.0001	2	8.636	14.091	11.591	15.455	0	0	0.0012	0
13	1	≤ 0.227	1	1	1	1	8.636	14.091	11.591	15.455	0	0	11.591	0

Table No.1. Solutions obtained by Lee's Method.
(The value of the variable/priority is equal to 0 if not shown in the table)

Sr No	Weights in priority						Decision variables		deviational variables		Achievement of priorities			
	P_1		P_2		P_3	P_4	x_2	x_3	n_1	p_1	P_1	P_2	P_3	P_4
	w_5	w_6	w_3	w_4	w_1	w_2								
	n_4	p_4	n_3	p_3	n_1	p_2								
2	1	1	1	1	1	1	8.636	14.091	11.591	15.455	0	0	11.591	0
3	3	2.5	8	9.2	2	4	8.636	14.091	11.591	15.455	0	0	23.182	0
4	67	85	52	29	4	48	8.636	14.091	11.591	15.455	0	0	46.364	0
5	189	175	200	323	10	89	8.636	14.091	11.591	15.455	0	0	115.91	0
6	919	641	740	514	20	411	8.636	14.091	11.591	15.455	0	0	231.82	0
7	989	887	845	799	100	990	8.636	14.091	11.591	15.455	0	0	1159.1	0
7	565	451	600	245	0.5	750	8.636	14.091	11.591	15.455	0	0	5.7955	0
8	123	653	299	156	0.4	272	8.636	14.091	11.591	15.455	0	0	4.6364	0
9	99	239	88	89	0.25	301	8.636	14.091	11.591	15.455	0	0	2.8976	0
10	439	339	196	509	0.2	67	8.636	14.091	11.591	15.455	0	0	2.3182	0
11	≤ 0.818	1	9	4	1	2	12.5	16.167	n_1= 0 n_3=14.167	14.167	0	>0	0	0
12	≥ 0.001	1	9	4	0.0001	2	12.5	16.167	n_1=0 n_3=14.167	14.167	0	>0	0	0
13	1	≤ 0.227	1	1	1	1	4	28	0	p_1=34 p_4=51	>0	0	0	0

Table No.2. Solutions obtained by Simplex Method.
(The value of the variable/priority is equal to 0 if not shown in the table)

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