Optimization Of Ring Stiffener Of A Missile

Sheikh Naunehal Ahamed, Mohammed Mushraffuddin

Abstract: In general thick cylindrical structures have radial, axial, circumferential and extensional modes and some of these free vibrational modes exists within the machine operating frequency range and can lead to potential resonance. One way to avoid the resonance is to shift the system natural frequencies away from the machine operating range. In case of turbo-generators the forcing function, which is combination of various deformation stresses that deforms the structure into an oval shape. Our research explains how to shift oval mode frequencies using topology optimization scheme in the context of a finite element (FE) approach. The key challenge involved in FE is that one should be able to retain the mode of interest throughout the cycle of optimization. During the optimization scheme, there will be a progressive change in the geometry and material, which may cause removal/ shifting of the mode of interest. The optimization is carried out using conventional artificial boundary condition of a missile ring stiffeners oval mode.

Index Terms: circumferential nodes, hoop’s stress, mesh, resonance, topology optimization, von misses stress

1 INTRODUCTION

A number of theories for the prediction of the natural frequencies of cylinders have been developed and used over the years. as mentioned in Guan, W. “Modal Analysis of a Thick-Walled Cylinder. MSc. Thesis, University of Saskatchewan, Saskatoon, Canada, 1993. Because the solutions for the vibrational behavior of a cylinder can not be exactly obtained by the linear elastic theory, people have tried to create various theories to solve the problem in an approximate way. The differences between these approximate methods are due to the various assumptions for the displacement components which are used in the analysis. Some of the theories are capable of dealing with finite length free hollow cylinders, while others are only appropriate for solid cylinders of infinite length. Among these researchers, only a few give a complete description of the mode shapes. The most recent work on the thick-walled cylinder is described in Singal and Williams’ paper. Based on the three-dimensional theory of elasticity, the well-known energy method was used in the derivation of the frequency equation of the cylinder. The frequency equation yields natural frequencies for all the circumferential modes of vibration, including the breathing and beam-type modes. Experimental investigations carried out on several models, showed very close agreement between the theoretical and experimental values of the natural frequencies. The load-carrying action of a plate is similar, to a certain extent, to that of beams thus, plates can be approximated by a gridwork of an infinite number of beams or by a network of an infinite number of cables, depending on the flexural rigidity of the structures. This two-dimensional structural action of plates results in lighter structures, and therefore offers numerous economic advantages. The plate, being originally flat, develops shear forces, bending and twisting moments to resist transverse loads.

Because the loads are generally carried in both directions and because the twisting rigidity in isotropic plates is quite significant, a plate is considerably stiffer than a beam of comparable span and thickness. So, thin plates combine light weight and form efficiency with high load-carrying capacity, economy, and technological effectiveness. So far, most of the work by several authors. as mentioned in Guan, W. “Modal Analysis of a Thick-Walled Cylinder. MSc. Thesis, University of Saskatchewan, Saskatoon, Canada, 1993. was dedicated to discovering a theoretical or numerical way of determining the frequencies of cylinders accurately and concisely. Some of the work considered a solid bar of infinite length only, others were suitable for finite length circular hollow cylinders with either thin walls or thick walls. Few authors gave a description of the mode shapes of the cylinders, and none of them provided a unique way for describing all of the modes shapes. McMahon gave some mode charts showing sand patterns on the plane surface of a solid cylinder and the approximate form of the vibration at a diametrical cross section. The mode shapes of thin-walled cylinders were presented. Singal and Williams gave a description for the mode shapes of thick-walled hollow cylinders. However, all of the descriptions mentioned are based on two parameters, the number of circumferential nodes, and number of axial nodes. Such a description is insufficient for describing the mode shapes of a three-dimensional structure uniquely. as we know from McMahon G.W. Expermental Study of the Vibrations of Solid, Isotmprc Elastic Cylinders", Journal of the Acoustical Society of America. Vol. 36, 1964, pp 85-92. Mode shape information is a very important vibrational characteristic of any structure. Without mode shape information, it is difficult to find a way to control and eliminate the vibration and noise of the structure itself. The energy analysis has been shown to be ideally awaited to the determination of the natural frequencies of both thick-walled hollow cylinders and rings, however, it does not provide much information with regards to the eigenvector problem. The technique provides only the magnitude of n and whether m is an even or odd integer. The meaning of n (where 2n = the number of circumferential cross points in the radial displacement shape) and m (the number of cross points in the radial displacement shape along any axial generatrix) are illustrated in Figures 1a and 1b. The actual magnitudes of the integer m were obtained experimentally in the journal. It is impossible for several frequencies to have the exact same mode shape, therefore it must be concluded that the descriptors n and m alone are not adequate for describing such a three-dimensional problem. Natural frequencies and mode shapes of finite length thick cylinders are of
considerable engineering importance because of strong potential applications and more demanding requirements imposed on cylindrical structures. In general thick cylindrical structures have radial, axial, circumferential and extensional modes as shown in fig 1.

**FIG 1.1:** Different types of Mode Shapes of a Cylinder.

**FIG 1.2:** Circumferential Nodes pattern.
When some of these free vibrational modes are coincide with the machine operating frequency range can lead to potential resonance and causes huge damages to structures. In order to avoid the resonance we can shift the system natural frequencies away from the machine operating range by using topology optimization technique. One can shift the system natural frequency by either increasing or decreasing the Eigen values. In case of turbo-generators the forcing function, which is combination of various deforming stresses that deforms the structure into an oval shape. In this paper we prepared a ring stiffener using conventional dimensions of a missile ring stiffener provided by a research organisation and on this stiffener we applied the Topology optimization of ring stiffener thereby trying to reduce its weight and unwanted material and also shifting of Oval mode Shapes.

Topology optimisation is a mathematical approach that optimises material layout within a given design space, for a given set of loads and boundary conditions such that the resulting layout meets a prescribed set of performance targets. Using topology optimisation, engineers can find the best concept design that meets the design requirements. Topology optimisation has been implemented through the use of finite element methods for the analysis, and optimisation techniques based on the method of moving asymptotes, genetic algorithms, optimality criteria method, level sets and topological derivatives. Topology optimisation is used at the concept level of the design process to arrive at a conceptual design proposal that is then fine tuned for performance and manufacturability. This replaces time consuming and costly design iterations and hence reduces design development time and overall cost while improving design performance. In some cases, proposals from a topology optimisation, although optimal, may be expensive or infeasible to manufacture. These challenges can be overcome through the use of manufacturing constraints in the topology optimisation problem formulation. Using manufacturing constraints, the optimisation yields engineering designs that would satisfy practical manufacturing requirements. In some cases Additive manufacturing technologies are used to manufacture complex optimized shapes that would otherwise need manufacturing constraints.

1.1 Ring stiffener of a turbo generator

The turbo-generator structure consists of number of cylindrical rings that are arranged in parallel and are covered by the envelope plate circumferentially and thus forms a ring stiffened cylindrical shell. There has been an extensive work done by the researchers to predict free vibrational behavior of cylindrical shells, and it was found that in ring stiffened cylindrical shell increasing the stiffeners stiffness increases the cylinders natural frequency. This Ring stiffened cylindrical shell forms a cage or support to the stator part of turbo-generators. Typically the turbo-generator structures consist number of ring-stiffened cylindrical segments arranged in parallel to each other. A typical ring-stiffened cylindrical shell is as shown.
1.2 Objective and Goal:

The project deals with the optimization of ring stiffeners of missile primary goals of preventing resonance between engine and ring stiffener, retain structural stability of missile body in its highest operating frequency range appropriate location for cutouts in ring stiffeners. All this is done by simple concept of determining the highest possible oval mode of vibration of ring stiffener which is a cylinder itself, under the highest possible operating frequency and the shifting this oval mode by making changes in the design. Natural frequencies and mode shapes of finite length thick cylinders are of considerable engineering importance because of strong potential applications and more demanding requirements imposed on cylindrical structures. Normally this cylindrical have various free vibration modes and some of these modes may be in the range of machines operating frequency. The main objective of the project is to shift the system natural frequency away from machines operating range. Our project particularly concentrates on a turbo generator in which the forcing function tends to deform the structure into oval shape. Our project main aim is how to shift oval mode frequencies using topology optimization scheme in the context of a finite element (FE) approach. In detail, this includes:

a) A theoretical modal analysis using the finite element method to obtain the natural frequencies and mode shapes.

b) An experimental modal analysis to verify the results of the theoretical analysis. Using the post processors. In both the finite element program and the experimental test software package, to examine the mode shape results and understanding of the mode shapes in general.

c) Shifting Oval mode frequencies using topology optimization scheme in the context of a finite element (FE) approach.

d) During a topological optimization scheme, there will be a progressive change in the geometry and material, which may cause removal/shifting of the mode of interest.

2 LITERATURE REVIEW

The approach of R.K singhal and W.C Guan to find true descriptors for the mode shapes of cylinders in their journal titled Modal Analysis of Thick Cylinder. Where the vibrational behavior of a thick-walled circular cylinder is of considerable engineering importance us such elements have numerous applications in electrical machines, stiffened cylinders. and gears, to name a few. Besides the natural frequencies of a structure, mode shapes are also an important aspect and are wry important sources of information for understanding and controlling the vibration of a structure. A theoretical modal analysis and an experimental modal analysis were carried out using a thick-walled cylinder model in order to obtain its natural frequencies and mode shapes. The theoretical modal analysis was done using the finite element method. The results for the frequency range from 20 Hz to 20 kHz were verified using experimental modal analysis. The correlation between the analytical and the experimental results is very good The largest error for all frequencies is 4.05 %; less than 2 % for most frequencies. The intent of the paper Aircraft Engine Attachment and Vibrational Control provided the data with a fundamental background to the engine vibration/noise problem in modern aircraft and present the available solutions that can be used to treat the engine vibration problem. Additionally, a design approach that provides technology options to the aircraft OEM throughout the design and flight test phases of the program is outlined. All mounting systems need to accomplish two basic functions:

1) constrain motion,
2) provide vibration isolation and noise reduction.

“Constraining Motion” refers to limiting the relative motion between two structures created by thrust, ‘g’ loads, weight, and torque. “Providing isolation” and “reducing noise” involves minimizing the transmission of vibration from one structure to another so as to reduce the transmitted noise into the cabin area. To provide the first basic function, the mounting system must be stiff to minimize relative motions. In order to minimize transmitted vibration (or noise), the mounting system must be dynamically soft. This inherent problem sets up competing objectives that require compromise and flexibility in the engine attachment design. This basic issue, along with the need for longer service lives and reduced costs, is the reason for new technology development. In rotordynamics, nonlinear coupling forces between the rotating and surrounding stationary parts can result in unexpected significant displacements and subsequent high stresses leading to structural failure. More specifically in aircraft engines, several mechanisms can contribute to such rotor-to-stator interactions and are usually classified in three main categories:

1) interacting forces due to variations of fluid pressure without structural contact.
2) interacting forces reduced to a unique contacting point along the circumference of both structures.
3) interacting forces induced by multiple simultaneous contacting points at different locations along the circumference.

The first category comprises phenomena such as stall flutter, forced response due to aerodynamic surroundings or acoustic resonance]. This category is out of the scope of the present paper. The second category is more or less well understood. The related works usually analyze the vibrations of a rotating shaft with a non-uniform cross-section supported by journal bearings where different levels of nonlinearity are considered: oil-film pressure field implicating nonlinear hydrodynamic equations, direct rub and friction forces, viscous damping forces, nonconstant angular velocity to name a few. These studies mainly necessitate small models with a few coupled nonlinear second-order differential equations suitable for the investigation of a real shaft behavior. Nonlinear and chaotic behaviors as different as dry whip, oil whip or whirling motion are highlighted. On the other hand, the third category is an emerging field of research and is more specific to aircraft engines like the one depicted. By virtue of the need of high machine efficiency, it became apparent that more realistic descriptions of fully flexible structures, principally bladed disks and outer casings, within a contact mechanic framework, was required. This efficiency, simply defined as the ratio of energy output to energy input, strongly depends on the clearance between the rotating and stationary components: the wider the clearance, the less efficient the machine. Higher efficiency is achieved by reducing this tip clearance in order to avoid aerodynamic losses. Unfortunately, an obvious consequence is a significant increase in the possibility of rub between the two components with origins such as: gyroscopic effect under
certain operating conditions, maneuvering loads during take-off and landing of the aircraft, apparition of a rotor imbalance due to design uncertainties, bird strikes or blade-off, vibrations due to aerodynamic excitations, outer casing distortion caused by a temperature gradient. Depending on the nature of the induced contact, theses interactions can give rise to either very short and transient dynamic responses encountered in crash analysis for example or long lived phenomena characterized by initially intermittent soft contacts that can lead to the excitation of the mode shapes of the structures, undesirable large amplitudes and very high stress levels into the structures. In this paper, our emphasis will be in a better way. This approach gives the shape of initial and total deflection of plate. From this analysis, the large deflection behaviour of plate under transverse load can be expressed as a function of the pre to post buckling in-plane stiffness of plate. Liew et al. developed the differential quadrature method and harmonic differential quadrature method for static analysis of three dimensional rectangular plates. This methodology can be used to find the bending and buckling of plates, which are simply supported and clamped boundary conditions only. It is a certification requirement of aero-engines to demonstrate blade containment and rotor dynamic integrity in the unlikely event of loss of a fan blade during operation. In such an event, the engine would be shut down, but the shaft would continue to rotate due to the incoming airflow. Out-of-balance forces caused by a missing fan blade provide a source of excitation for the whole engine-wing-aircraft structural assembly. This condition is referred to as ‘windmilling imbalance’ (FAA, 1999). Rotor clearances which pertain to normal operation could now be overcome by vibrating components, thus leading to the rotor rubbing against the stator which, in turn, can potentially cause a rich mixture of ejects associated with contact/impact-related phenomena. These ejects manifest themselves in the occurrence of multiple solutions for steady-state response scenarios, including amplitude jumps during rotor acceleration and deceleration, and vibration responses at different/multiple frequencies of the exciting unbalance force.

The studies presented here form part to complete understanding of the contributing mechanisms of the post-fan-blade-off windmill dynamics and to anciently perform windmilling analysis for large-scale engine models. The FAA, in collaboration with industry, is developing a certification procedure requiring engine and airframe manufacturers to prove by analysis or test that an airplane under windmilling conditions is still safe to fly. It is part of the understanding required for the analysis to and out what loads are involved and to what level low-frequency vibration modes of the airframe can be excited. Computer aided topology optimization of structures is a relatively new but rapidly expanding field of structural mechanics. Topology optimization is used in an increasing rate by for example the car, machine and aerospace industries as well as in materials, mechanism and Micro Electro Mechanical Systems (MEMS) design. The reason for this is that it often achieves greater savings and design improvements than shape optimization. The topology optimization problem solves the basic engineering problem of distributing a limited amount of material in a design space, where a certain objective function has to be optimized. In the case where the design domain is subjected to any static loads, we speak of topology optimization of static problems. A common objective in static problems is to minimize the compliance (maximum global stiffness). In dynamic topology optimization problems, the objective is related to Eigen value optimization. These problems are relevant for the design of machines and structures which are subjected to a dynamic load. A possible motivation for this type of problems is, for example, to keep the Eigen frequency of a structure away from the driving frequency of an attached vibrating machine with a given frequency of vibration. A common objective in dynamic topology optimization is to maximize the fundamental Eigen frequency, for example to shift the fundamental Eigen frequency away from certain disturbing frequencies. Moreover, structures with a high fundamental frequency tend to be reasonable stiff for static loads. The goal of this project is to implement two different optimization algorithms in dynamic topology optimization problems and to compare the results. The implemented algorithms are the optimality criteria method and the method of moving asymptotes (MMA). In literature survey embedded topics are collected from different books and journals. Firstly clear description of the survey is given for the geometric detail. In “Jane’s Strategic weapon systems” Which is edited by Duncan Lennox in Forty-seven issue of their book (Can also found in jsws.janes.com) it is clearly described about the detail explanation of missile cross section. It gives dimension aspect of explanation for the complete report in a proper manner. In this particular journal it has been clearly explained about the previous flight vehicle cross sections which are practically proved as air worthy. With this type of study from this particular book it made easier to consider the dimensional values which are acceptable and also the further applications of this type of cross section in different fields. Therefore it can be concluded by stating that with the help of the existing matter in the given book helped out in the analysis application which made work much easier and also gave a practical example of existence.

3 THEORETICAL BACKGROUNDS

3.1 Definition of Problem

In any structural problem, response of the structure depends on the load applied i.e., the structure parameters, which affects the structure adversely and some of the definitions are to be mentioned clearly. Some of the general result or responses are mentioned

- Where the maximum deflection occurs?
- What is allowable stress?
- Failure of structure due to external load applied?

The unambiguous definition of the problem makes the structural analyst easy. Probably the most critical step in the structural analysis is the definition of the problem. The true problem is not always what is seems to be at first glance because this first step requires such a small part of the total time to analyse the structure, its important is often overlooked. This project deals with the FEA Analysis of the ring stiffener, which are mostly used in many areas like aerospace, automotive etc in turbo-generators or electric machine stators the in-plane vibrational modes are of pure radial mode, which persists even if the transverse vibrational modes are eliminated [4]. The turbo-generator structure consists of number of cylindrical rings that are arranged in parallel and are covered by the envelope plate circumferentially and thus forms a ring stiffened cylindrical shell. There has been an extensive work done by the researchers [9-17] to predict free vibrational behavior of cylindrical shells, and it was found that
in ring stiffened cylindrical shell increasing the stiffeners stiffness increases the cylinders natural frequency [5]. This Ring stiffened cylindrical shell forms a cage or support to the stator part of turbo-generators. The turbo-generators mainly falls into 2 categories; 2 pole and 4-pole. This paper discusses specifically about 2-pole machines. For a two-pole turbo-generator the electromagnetic force acting on the stator core is as shown in Figure 2 and it deforms the stator core into oval shape which exactly matches with the mode shape of \( i=2 \) i.e. is a pure radial mode. The occurrence and pattern of radial modes in a normal turbo generator is shown in the figure.

In this project the segment is modeled with shell 63 elements (Figure 5(a)) and a symmetric boundary condition is applied on free edges of envelope plate. Using the Ansys TM finite element code, a natural frequency analysis was performed using Block Lanczos method between 0-200Hz. As the project deals with the FEA Analysis of the structural plate, the previous work is reviewed and the results are taken as a reference for the present project work. The project deals with the thickness optimization rather than shape optimization. The main problem, is that the rectangular structural plate "t" thickness should be optimize for its minimum weight objective function and it should also be taken care that stress should not exceed allowable which are given for the materials.

3.2. METHODOLOGY

3.2.1 Finite Element Analysis

Finite element analysis (FEA) is a powerful computational technique used for solving engineering problems having complex geometries that are subjected to general boundary conditions. While the analysis is being carried out, the field variables are varied from point to point, thus, possessing an infinite number of solutions in the domain. So, the problem is quite complex. To overcome this difficulty FEA is used; the system is discretized into a finite number of parts known as elements by expressing the unknown field variable in terms of the assumed approximating functions within each element. For each element, systematic approximate solution is constructed by applying the variation or weighted residual methods. These functions (also called interpolation functions) are, included in terms of field variables at specific points referred to as nodes. Nodes are usually located along the element boundaries, and they connect adjacent elements. Because of its flexibility in ability to discretize the irregular domains with finite elements, this method has been used as a practical analysis tool for solving problems in various engineering disciplines. FEA is used in new product design, and existing product refinement. Because of its characteristics, researchers are able to verify a proposed design to the user’s specifications before manufacturing or construction. In case of structural failure, FEA may be used to determine the design modifications to meet the required conditions. Structural analysis consists of linear and non-linear models. Linear models consider simple parameters and assume that the material is not plastically deformed. Non-linear models consider that the structure is pre-stressed and is plastically deformed. FEM is a numerical method used as an effective analysis tool in various field of engineering to provide numerical solutions to engineering problems. Basic idea in FEM is to find the solution of complicated problem by replacing simpler one by finding solution we would able to find the approximate solution. The existing mathematical tools will not be sufficient to find the exact solution and sometimes even an approximate solution of most of the practical problems. Thus, in the absence of any other convenient method to find even the approximate solution we have to prefer FEM. Moreover in FEM it will often be possible to improve or refining the approximate solution by spending more in computational method

3.2.1 Need for FEM:

To predict the behavior of the structure, the designer adopts three tools/methods. They are:

1. Analytical Method,
2. Experimental Method &
(1) Analytical Method:

Advantages:
- Used for regular section shapes of known geometries or primitives where component geometry is expressed mathematically.
- Mathematical expression used for finding unknowns using knowns.
- Solution is exact.
- It takes less time.

Disadvantages:
- Imposing boundary conditions are difficult.
- Cannot write generalized code.
- Cannot model dissimilar materials.

(2) Experimental Method:
- Testing Equipment, Instruments Test setup is required.
- Preparation of specimen according to requirement.
- High initial cost.
- More time.
- Exact result.

(3) Numerical Method:
- There are many numerical schemes such as FEA, FDM, FVM, BEM, and CVM & Hybrid methods are used to estimate the approximate solution of acceptable tolerances.
- Predicts behavior at desired accuracy of any complex and irregular geometry.
- Least cost.

The specific need of FEM in most of the engineering problems the field variables (like displacement, stress, temperature & pressure) are often represented as linear differential equation or partial differential equation. Solving these differential equation using classical analytical method is tedious processes and at certain conditions becomes near to impossible in such cases the FEM helps to convert the differential equations in to set of algebraic equations that can be solved easily by using various matrix manipulation techniques.

3.2.3 CATIA:
CATIA version 5 is a process-centric computer-aided design/computer-assisted manufacturing/computer-aided engineering (CAD/CAM/CAE) system that fully uses next generation object technologies and leading edge industry standards. Seamlessly integrated with Dassault Systemes Product Lifecycle Management (PLM) solutions, it enables users to simulate the entire range of industrial design processes from initial concept to product design, analysis, assembly, and maintenance. The CATIA V5 product line covers mechanical and shape design, styling, product synthesis, equipment and systems engineering, NC manufacturing, analysis and simulation, and industrial plant design. In addition, CATIA KnowledgeWare enables broad communities of users to easily capture and share know-how, rules, and other intellectual property (IP) assets. CATIA V5 builds on powerful smart modeling and morphing concepts to enable the capture and reuse of process specifications and intelligence. The result is an easily scaleable, Web-enabled system that covers all user requirements within the digital extended enterprise, from the simplest design to the most complex processes. This capability allows optimization of the entire product development process while controlling change propagation. CATIA V5 moves beyond traditional parametric or variational approaches, accelerating the design process and helping designers, engineers, and manufacturers increase their speed and productivity. CATIA V5 has an innovative and intuitive user interface that unleashes the designer's creativity. Context-sensitive integrated workbenches provide engineers with the tools they need for the task at hand, and they are beneficial for multi-discipline integration. The workbenches have powerful keyboard-free direct object manipulators that maximize user productivity. CATIA V5 applications are based on a hybrid modeling technology. These applications provide expanded digital product definitions, process definitions, and review functions capable of operating on projects with any degree of design complexity. CATIA V5 has produced domain-specific applications that have addressed global digital enterprise requirements that span the areas of mock-up, manufacturing, plant, and operations.

3.2.4 HYPERMESH:
Altair Engineering is a product design and development, engineering software and cloud computing software company. Altair was founded by Jim Scapa, George Christ, and Mark Kistner in 1985. Over its history, it has had various locations near Detroit, Michigan, USA. It is currently headquartered in Troy, Michigan with regional offices throughout America, Europe and Asia. Altair Engineering is the creator of the HyperWorks suite of CAE software products. Altair was established in 1985. In 1990, Hyper Mesh was released. In 1994, Altair receives Industry Week’s “Technology of the Year” award for OptiStruct. During the 2008 economy crisis, Altair started a program to offer free training on its product for unemployed persons in Michigan. In September 2010, Altair purchased a 136,000-square-foot (12,600 m²) Annex Facility in Troy, to initially house Altair’s subsidiary Ilumisys, Inc. Altair also acquired SimLab in October 2011.2011 began with another acquisition, AcuSim, with their CFD Solver, AcuSolve. In September 2011, Altair Product Design unveiled, a hybrid hydraulic bus.

3.2.5 ANSYS RELATED WORK:
ANSYS is a general purpose finite element modeling package for numerically solving a wide variety of mechanical problems. To solve the problem considered initially section is modeled with respect to the considered dimensions. And then the section must be meshed with in the tool. Proper meshing has to be done to get the result with out any errors. We need to get the structural and thermal analysis values for the designed model. As per the requirement load must be applied on the left most end of the section end ring and right most end of the section end ring must be constrained. This is how loads are applied and constraining is done in ANSYS. The above given procedures are done in preprocessor phase. Now entering in to the solving phase of the model procedure to be followed is as follows. Firstly mode must be changed to static such that after solving we get the value of stress and deformation of the structure when load is applied. After the first run post processor phase has to be opened and in this phase required
values and analysis plots are taken. Similar solving phase must to followed for the second cycle but this run is to get the of BLF and therefore mode must be changed to eigen buckling. After the run it has to be saved and post processing phase must be opened. In this phase values must be noted down and plots must be taken. This is how steps are followed to get the results in ANSYS.

3.3 MATERIAL PROPERTIES:
The main material considered in this project for the ring stiffener is steel since its all mechanical physical and thermal properties are well knowned.

3.3.1 Steel properties:
Steel is an alloy of iron and other elements, including carbon. When carbon is the primary alloying element, its content in the steel is between 0.002% and 2.1% by weight. The following elements are always present in steel carbon, manganese, phosphorus, sulfur, silicon, and traces of oxygen, nitrogen and aluminum. Alloying elements intentionally added to modify the characteristics of steel include: manganese, nickel, chromium, molybdenum, boron, titanium, vanadium and niobium. Varying the amount of alloying elements and the form of their presence in the steel (solute elements, precipitated phase) controls qualities such as the hardness, ductility, and tensile strength of the resulting steel. Steel with increased carbon content can be made harder and stronger than iron, but such steel is also less ductile than iron. Alloys with a higher than 2.1% carbon (depending on other element content and possibly on processing) are known as iron. Today, steel is one of the most common materials in the world, with more than 1.3 billion tons produced annually.

### Table 3.3.1: Steel material properties

<table>
<thead>
<tr>
<th>SL-NO</th>
<th>MATERIAL</th>
<th>E(Gpa)</th>
<th>UTIMATE-TENSILE STRENGTH</th>
<th>DENSITY (Kg/m³)</th>
<th>YIELDSTRESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Steel</td>
<td>200</td>
<td>830</td>
<td>7800</td>
<td>700</td>
</tr>
</tbody>
</table>

#### 3.3.2 SHELL63 Element Description
SHELL63 has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Stress stiffening and large deflection capabilities are included. A consistent tangent stiffness matrix option is available for use in large deflection (finite rotation) analyses. See SHELL63 in the ANSYS, Inc. Theory Reference for more details about this element. Similar elements are SHELL43 and SHELL181 (plastic capability), and SHELL93 (midside node capability). The ETCHG command converts SHELL57 and SHELL157 elements to SHELL63.

3.4 LOADS & BOUNDRY CONDITIONS:

#### 3.4.1 DESIGN CONFIGURATION OF STIFFENER:
The variables of cross section of a Ring Stiffener are shown in the figure, these variables define the Natural Frequency of any ring stiffeners and are the main part in designing the ring stiffener in Catia.

![FIG 3.1: Ring Stiffener variables.](image)

Here,
- \( b_2 \) = length of Envelope Plate.
- \( d_2 \) = thickness of Envelope Plate.
- \( d_1 \) = length of Stiffening Ring.
- \( b_1 \) = thickness of Stiffening Ring.
- \( r_1 \) = Inner Radius.
- \( r_2 \) = Outer Radius.
- \( y \) = distance from Datum line.

Typically the Turbo generator structure consists of Ring Stiffened Cylindrical segments arranged in parallel to each other. A typical Ring Stiffened Cylindrical Shell is as shown in the figure 6 above.
3.4.2 LOADS ON THE RING STIFFENER IN A MISSILE:

For any Missile, loading is divided into two major categories: limit loads and ultimate loads. Limit loads are often just flight loads and are further divided into maneuvering loads and gust loads. Ultimate loads are crash loads. Maneuvering loads are determined based on the performance limits of the aircraft whether imposed by the flight manual or by the actual aerodynamic performance of aircraft. Gust loads are determined statistically are taken from guidelines or requirements given by the applicable regulatory agency. Crash loads are loosely bounded by the ability of humans to survive extreme accelerations and are also typically taken from regulations. Cylindrical or spherical pressure vessels (e.g., hydraulic cylinders, gun barrels, pipes, boilers and tanks) are commonly used in industry to carry both liquid s and gases under pressure. When the pressure vessel is exposed to this pressure, the material comprising the vessel is subjected to pressure loading, and hence stresses, from all directions. The normal stresses resulting from this pressure are functions of the radius of the element under consideration, the shape of the pressure vessel (i.e., open ended cylinder, closed end cylinder, or sphere) as well as the applied pressure. Two types of analysis are commonly applied to pressure vessels. The most common method is based on a simple mechanics approach and is applicable to “thin wall” pressure vessels which by definition have a ratio of inner radius, r, to wall thickness, t, of r/t≥10. The second method is based on elasticity solution and is always applicable regardless of the r/t ratio and can be referred to as the solution for “thick wall” pressure vessels. Both types of analysis are discussed here, although for most engineering applications, the thin wall pressure vessel can be used. Other loads that may be critical are pressure loads (for pressurized, high-altitude aircraft) and ground loads. Loads on the ground can be from adverse braking or maneuvering during taxi. Finally, you cannot discuss aircraft loading without hearing about fatigue and damage tolerance. Aircraft are constantly subjected to cyclic loading. These cyclic loads initiate cracks and cause them to grow. Thermal loading is rarely considered for the analysis of the primary structure of aircraft but it can become critical under extreme operating conditions and should be examined where materials of disparate coefficients of thermal expansion are joined. Hence the loads on a ring stiffener are

- Pressure Loads.
- Thermal Loads.
- Bending Loads.
- Buckling loads.
- Torsional Loads.

Since these types of loads will induce different stresses in the ring stiffener, some of the practically knowable stresses that can be known are

- Hoop stresses.
- Von Mises stresses.

**Hoop stresses** For the thin-walled assumption to be valid the vessel must have a wall thickness of no more than about one-tenth (often cited as one twentieth) of its radius. This allows for treating the wall as a surface, and subsequently using the Young–Laplace equation for estimating the hoop stress created by an internal pressure on a thin wall cylindrical pressure vessel:

\[
\sigma_\theta = \frac{P r}{t} \quad \text{(for a cylinder)} \\
\sigma_\theta = \frac{P r}{2t} \quad \text{(for a sphere)}
\]

where

- \( P \) is the internal pressure
- \( t \) is the wall thickness
- \( r \) is the inside radius of the cylinder.
- \( \sigma_\theta \) is the hoop stress.

The hoop stress equation for thin shells is also approximately valid for spherical vessels, including plant cells and bacteria in which the internal turgor pressure may reach several atmospheres. When the vessel has closed ends the internal pressure acts on them to develop a force along the axis of the cylinder. This is known as the axial stress and is usually less than the hoop stress.

\[
\sigma_z = \frac{F}{A} = \frac{P d^2}{(d + 2t)^2 - d^2}
\]

Though this may be approximated to

\[
\sigma_z = \frac{P r}{2t}
\]

Also in this situation a radial stress \( \sigma_r \) is developed and may be estimated in thin walled cylinders as:

\[
\sigma_r = \frac{-P}{2}
\]

**Von misses stress**: The von Mises yield criterion suggests that the yielding of materials begins when the second deviatoric stress invariant \( J_2 \) reaches a critical value. For this reason, it is sometimes called the \( J_2 \)-plasticity or \( J_2 \) flow theory. It is part of a plasticity theory that applies best to ductile materials, such as metals. Prior to yield, material response is assumed to be elastic. In materials science and engineering the von Mises yield criterion can be also formulated in terms of the von Mises stress or equivalent tensile stress, \( \sigma_y \), a scalar stress value that can be computed from the stress tensor. In this case, a material is said to start yielding when its von Mises stress reaches a critical value known as the yield strength, \( \sigma_y \). The von Mises stress is used to predict yielding of materials under any loading condition from results of simple uniaxial tensile tests. The von Mises stress satisfies the property that the two stress states with equal distortion energy have equal von Mises stress. Because the von Mises yield criterion is independent of the first stress invariant, \( J_1 \), it is applicable for the analysis of plastic deformation for ductile materials such as metals, as the onset of yield for these materials does not depend on the hydrostatic component of the stress tensor.
Although formulated by Maxwell in 1865, it is generally attributed to Richard Edler von Mises (1913). Tytus Maksymilian Huber (1904), in a paper in Polish, anticipated to some extent this criterion. This criterion is also referred to as the Maxwell–Huber–Hencky–von Mises theory.

4 DEFINING THE GEOMETRY OF STIFFENER
The geometry of the Missile stiffener is taken by considering various missile dimensions. As the data related to the design of any missile is a very confidential data and a very strict privacy should be maintained, so we were asked to take conventional design data related to the original one. From a source of one of the important Research organization the following pictures of the conceptual design is taken into consideration.

FIG 4.1: Different configurations of Agni missiles.
FIG 4.2: Comparisons of Prithvi II and other Agni Missiles.
FIG 4.3 : Typical Ring Stiffener Dimensions.
Studying these the conventional dimensions for a Ring Stiffener are calculated using proportional limits. And using the phenomenon of Theory of Similarity which states that for undistorted models in which the geometry of the undistorted model is similar to the original model then the scale effects in models will not have a deviating effect on the results, moreover the results may vary with respect to proportionality accordingly. By applying the Dynamic similarity to Specific Model concept to these Ring Stiffener and predicting the geometry of the Ring Stiffener as follows

4.1 CONVENTIONAL GEOMETRY DETAILS:
- Stiffening Ring Thickness = 2.0 inches.
- Envelope Plate Thickness = 0.75 inches.
- Stiffening Ring Inner Diameter = 110 inches.
- Stiffening Ring Outer Diameter = 150 inches.
- Envelope Plate length = 15 inches.

5 MATHEMATICAL BACKGROUND AND FORMULATION
5.1 MATHEMATICAL MODELING
The use of mathematics is one of the many approaches to solving real-world problems. Others include experimentation either with scaled physical models or with the real world directly. Mathematical modelling is the process by which a problem as it appears in the real world is interpreted and represented in terms of abstract symbols. This makes mathematical modelling challenging and at the same time demanding since the use of mathematics and computers for solving real-world problems is very widespread and has an impact in all branches of learning. Structures which are subjected to dynamic loading, particularly aircraft, vibrate or oscillate in a frequently complex manner. An aircraft, for example, possesses an infinite number of natural or normal modes of vibration. Simplifying assumptions, such as breaking down the structure into a number of concentrated masses connected by weightless beams (lumped mass concept), are made but whatever method is employed the natural modes and frequencies of vibration of a structure must be known before flutter speeds and frequencies can be found. We shall discuss flutter and other dynamic aeroelastic phenomena in T.H.G MEGSON but for the moment we shall concentrate on the calculation of the normal modes and frequencies of vibration of a variety of beam and mass systems. The determination of natural frequencies and normal mode shapes for beams of nonuniform section involves the solution of and fulfillment of the appropriate boundary conditions. However, with the exception of a few special cases, such solutions do not exist and the natural frequencies are obtained by approximate methods such as the Rayleigh and Rayleigh–Ritz methods.
which are presented here. Rayleigh’s method is discussed first. A beam vibrating in a normal or combination of normal modes possesses kinetic energy by virtue of its motion and strain energy as a result of its displacement from an initial unstrained condition. From the principle of conservation of energy the sum of the kinetic and strain energies is constant with time. In computing the strain energy \( U \) of the beam we assume that displacements are due to bending strains only so that at the ends of the beam then a good approximation to the true natural frequency will be obtained. We have noted previously that if the assumed normal mode differs only slightly from the actual mode then the stationary property of the normal modes ensures that the approximate natural frequency is only very slightly different to the true value. Furthermore, the approximate frequency will be higher than the actual one since the assumption of an approximate mode implies the presence of some constraints which force the beam to vibrate in a particular fashion; this has the effect of increasing the frequency. The Rayleigh–Ritz method extends and improves the accuracy of the Rayleigh method by assuming a finite series for \( V(z) \), namely where each assumed function \( V_i(z) \) satisfies the slope and displacement conditions at the ends of the beam and the parameters \( B_i \) are arbitrary. Substitution of \( V(z) \) then gives approximate values for the natural frequencies. The parameters \( B_i \) are chosen to make these frequencies a minimum, thereby reducing the effects of the implied constraints. Having chosen suitable series, the method of solution is to form a set of equations Eliminating the parameter \( B_i \) leads to an \( n \)-th order determinant in \( \omega^2 \) whose roots give approximate values for the first \( n \) natural frequencies of the beam. The above study is with reference to T.H.G MEGSON, pg no : 341 to 343.

### 5.2 Governing Equation for Natural Frequency:
Excessive vibration due to resonance occurs when the frequency of a dynamic excitation is close to one of the natural frequencies of a structure. Therefore, it is necessary to restrict the fundamental or higher natural frequencies of a structure to a prescribed range in order to avoid severe vibration.

#### 5.2.1 Equivalent Section Stiffness:
The moment of inertia of each section can be calculated by using the following formula,

\[
I = \frac{1}{12} \sum b_i d_i^3 + \sum b_i d_i (y_i - \bar{y})^2 \quad \cdots \quad \text{eq. (1)}
\]

In the above equation \( \bar{y} \) the position of neutral axis relative to inner diameter of the stiffening ring and can be written as

\[
\bar{y} = \frac{\sum b_i D_i y_i}{\sum b_i d_i} \quad \cdots \quad \text{eq.(2)}
\]

From above, using thin ring theory the deflection at the unit force for the diametrically opposed forces can be written as,

\[
U_b = 0.074 \frac{R^3}{EI} \quad \cdots \quad \text{eq.(3)}
\]

Where

\[
R = r_i + \bar{y}
\]

\[
U = \frac{1}{2} u^T Ku \quad \cdots \quad \text{eq.(4)}
\]

\[
U_b = -0.068 \frac{R^3}{EI}
\]

In Turbo-Generator the forces are rotating with respective to the ring and thus the average deflection of any point on the ring can be written as

\[
U_b = 0.071 \frac{R^3}{EI} \quad \cdots \quad \text{eq (5)}
\]

From Eqn. (6), one can write the expression radial stiffness as

\[
S_i = \frac{1}{U_b} = \frac{EI}{71R^3} \quad \cdots \quad \text{eq.(6)}
\]

For thick rings used in generator frame, the effect of shear must also be included. Therefore, the stiffness (Eqn. (7)) must be modified as,
\[ S_0 = S_1 \left( \frac{U_b}{U_b + U_s} \right) = S_1 \left( \frac{1}{1 + \left( \frac{U_s}{U_b} \right)^2} \right) \quad \ldots \quad eq(7) \]

In the above, \( U_s \) is the deflection due to shear and the ratio of deflections can be written as

\[ \frac{U_s}{U_b} = \frac{4EI}{gA_1R^2} \quad \ldots \quad eq \ (8) \]

Where \( A_1 \) is stiffening ring cross-sectional area \( E \) is the young’s modulus \( R \) is the radius to neutral axis Substitute the above equation in (6) Then we have

\[ S_0 = S_1 \left( \frac{1}{1 + \frac{10I}{A_1R^2}} \right) \quad \ldots \quad eq \ (9) \]

5.2.2 Equivalent Section weight calculation:
Individual segment weight can be broken down as follows,

Stiffening ring

\[ W_1 = \pi(r_2^2 - r_1^2)b_1\gamma \]

Envelope circular plate

\[ W_2 = 2\pi r_2 d_2 b_2\gamma \]

In the above, \( \gamma \) is weight density and the total segment weight,

\[ W = \sum W_i \]

5.2.3 Section oval mode frequency :
The frequency of any mode of vibration for a circular ring can be written as

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{EgI}{\gamma AR^4} \left( i^2(1 - i^2) \right)} \quad \ldots \quad \ldots \quad \ldots \quad eq \ (10) \]

Where

\( g \) is the acceleration due to gravity.
\( E \) is the young’s modulus.
\( I \) is the moment of inertia.
\( A \) is stiffening ring cross-sectional area.

For 2-pole turbo-generators the excitation conforms to \( i=2 \) and the ring performs fundamental mode of flexural vibration. For pure radial mode \( (i=2) \) the Eqn. (10) can be simplified as

\[ F_2 = 177.5 \sqrt{\frac{S_0 \gamma W}{W_m}} \text{ Hz} \]

Where,

\( W_m \) is weight of miscellaneous parts

5.3 Numerical Solution for Natural Frequency of Ring Stiffener:
Overall inertial and stiffness properties are computed by using the above Eqs, as given below,

Moment of Inertia \( I = 2278.99 \text{ in}^4 \)
Stiffness \( S_1 = 3162.2 \text{ lb} / \text{mil} \)
Corrected stiffness \( S_0 = 2808.7 \text{ lb} / \text{mil} \)
Weight of segment \( W = 6123.46 \text{ lb} \)
Oval mode frequency \( F = 177.5 \sqrt{\frac{S_0 \gamma W}{W_m}} = 120.2 \text{ Hz} \).

The stiffness of full structure is then the sum of the stiffness’s of all the individual segments. The total structure weight is the sum of the weights of all individual segments plus miscellaneous weights such as pipes and hence the natural frequency of complete structure can be written as,
6 FREQUENCY PREDICTIONS THROUGH FE APPROACH

6.1. Designing The Model

**TABLE 6.1 :** Design Configuration of the Stiffener.

<table>
<thead>
<tr>
<th>Geometry details of the Ring Stiffener (inches)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness Ring Thickness</td>
<td>2.0</td>
</tr>
<tr>
<td>Envelope plate thickness</td>
<td>0.75</td>
</tr>
<tr>
<td>Stiffening Ring inner diameter</td>
<td>110</td>
</tr>
<tr>
<td>Stiffening Ring outer diameter</td>
<td>150</td>
</tr>
<tr>
<td>Envelope plate Length</td>
<td>15</td>
</tr>
<tr>
<td>Material</td>
<td>steel</td>
</tr>
</tbody>
</table>

Using these configurations the Ring Stiffener is modeled in Catia. The following figure show the various Projections of the Stiffener in Catia.

**FIG 6.1 :** Catia image 1.
Hence the optimized structure of the Ring Stiffener is created in Catia thereby ending the modeling stage of Ring Stiffener. Then comes the stage of Modal Analysis, this is done using the Ansys tool.

6.2 WORK ON ANSYS :

6.2.1 Initial work : Ansys Main Menu

1. Preferences> Structural Preferences> Discipline Option> h.Method and then Click “ok”
2. Pre-processor > Element Type > add/edit/delete > Shell63 (shell → Elastic 4 Node63) click “ok”
6.2.2 Importing the Catia IGES file

FIG 6.6: Procedure for Importing the IGES file

FIG 6.7: Imported Ring Stiffener in Ansys.
6.2.3 MESHING PART IN ANSYS:

3. Pre-processor > Material Properties > material models > Structural > linear > elastic > isotropic > give density & Young’s modulus and then Click “ok”
4. Preferences > Meshing > Size Control > Mesh Size > Areas > Pick All Areas and then Preferences > Meshing > Mesh > Areas > Mapped > 3 or sided click “ok”

5. Solution > Analysis type > Modal > “ok”
6. Solution > Define Loads > Apply > Structural > Displacement > Onlines (select 2side) > select Ux, Uy, Uz and then “ok”
7. Solution Type > Solve click “ok”

Fig 6.10 : Meshed model of Ring Stiffener.
6.2.4 Results of Modal analysis:
The Oval mode occurs at 120.9 Hz.
7 COMPARISON OF ANALYTICAL AND FEA RESULTS

The analysis has been carried out as follows and the Natural Frequencies of the Ring Stiffener is calculated analytically and by using the FEA tools. The comparison of the natural frequency is done as shown in the table.

Table 7.1: comparisons of analytical and FEA results for frequencies

<table>
<thead>
<tr>
<th>Analytical Formulae</th>
<th>Finite Element Results</th>
<th>% of difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>120.2 Hz</td>
<td>120.9 Hz</td>
<td>0.5</td>
</tr>
</tbody>
</table>

This Results shows that FEA is in poor agreement with closed form of solutions..

8 OPTIMIZATION OF RING STIFFENER

8.1 Introduction to Topology Optimization

Computer aided topology optimization of structures is a relatively new but rapidly expanding field of structural mechanics. Topology optimization is used in an increasing rate by for example the car, machine and aerospace industries as well as in materials, mechanism and Micro Electro Mechanical Systems (MEMS) design. The reason for this is that it often achieves greater savings and design improvements than shape optimization. The goal of structural optimization is to achieve the best performance from a structure while satisfying various constraints, such as a certain amount of material and specific mechanical conditions. For the past several decades, topology optimization techniques such as the homogenization method [1], the solid isotropic material with penalization (SIMP) method [2], the evolutionary structural optimization (ESO/BESO) method [3], and the level set technique [4] have been developed. These methods have been successfully adopted in topology optimization for static, dynamic, and practical engineering problems.

3.1.1 Governing Equation for Dynamic Problems

Governing equation for free vibration systems considered in this study can be written as

\[ M \ddot{u} + K \dot{u} = 0 \quad \ldots \ldots \text{eq}(8.1.1) \]

By using Laplace transformation Equation can be rewritten as

\[ MU(\omega)^2 + KU(\omega) = 0 \quad \ldots \ldots \text{eq} (8.1.2) \]

By substituting \( \omega^2 \) for \( \omega \) into Eq. (12), the final Eigen value problem is defined as

\[ [K - \omega_i^2 M]U_{i=0} = \ldots \ldots \text{eq}(8.1.3) \]

Where

\( K \) and \( M \) are the global stiffness and mass matrix, respectively. \( \omega_i \) is the \( i \)-th eigen frequency and \( u \) denotes the corresponding eigenvector depending on \( \omega_i \). In order to numerically solve Eq.
(21), $K$ and $M$ have to be the symmetric and positive definite (Lehoucq et al, 1998) stiffness and mass matrices of the finite element-based, generalized structural eigen value.

### 8.2 Topology Optimization Formulation for Dynamic Problems

Eigen value optimization designs are profitable for mechanical structural systems subjected to dynamic loading conditions like earthquakes and wind loads. The dynamic behaviors of structural systems can be estimated by Eigen frequency which describes structural stiffness. In general maximizing first-order eigen frequency can be an objective for dynamic topology optimization problems since stiffness of structures also increases when eigen frequency increases. Problems of topology optimization for maximizing natural Eigen frequencies of vibrating elasto static structures have been considered in the studies (Diaz et.al, 1992; Krog et. al, 1999; Pedersen, 2000) Assuming that damping can be neglected, such a dynamic design problem can be formulated as follows.

$$\omega_i^2 = \frac{K_i}{M_i} = \frac{U_i^T K U_i}{U_i^T M U_i} \ldots \ldots eq(8.1.4)$$

Where $K_i$ and $M_i$ are the model stiffness and mass, respectively. The optimization problem for maximizing the first Eigen frequency can be written as,

$$\max_x \omega_1^2(x) = \frac{U_1^T K U_1}{U_1^T M U_1} \ldots \ldots eq (8.1.5)$$

Subjected to \ldots \ldots \frac{v_i}{V_i} \geq f \frac{[K - \omega_1^2 M]U_1}{0 < x_{\min} \leq x \leq x_{\max}}$

The optimization problem for minimizing the first Eigen frequency can be written as,

$$\min_x \omega_1^2 = \frac{U_1^T K U_1}{U_1^T M U_1} \ldots \ldots eq (8.1.6)$$

Subjected to \ldots \ldots \frac{v_i}{V_i} \leq f \frac{[K - \omega_1^2 M]U_1}{0 < x_{\min} \leq x \leq x_{\max}}$

### 8.3 Numerical consistency of an artificial boundary condition w.r.t modal analysis

Detailed steps involved in the topology optimization scheme adopted for the present study are given in this section. The modal analysis explained in section 3.2 is reevaluated using Hyper OptiStruct and resulted are tabulated in Table and pure radial modes which are of current interest are shown in
The modal analysis explained in the above section and it is reevaluated using Hyper OptiStruct and resulted are tabulated in Table 4 and pure radial modes which are of current interest are around 9th and 10th mode.

Table 8.1: Mode numbers and frequencies

<table>
<thead>
<tr>
<th>Model no</th>
<th>Frequency (Hz)</th>
<th>Type of Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>Global-rigid body</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>Global-rigid body</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>Global-rigid body</td>
</tr>
<tr>
<td>4</td>
<td>67.9</td>
<td>Axial bending</td>
</tr>
<tr>
<td>5</td>
<td>78.9</td>
<td>Axial bending</td>
</tr>
<tr>
<td>6</td>
<td>79</td>
<td>Axial bending</td>
</tr>
<tr>
<td>7</td>
<td>106.7</td>
<td>Axial bending</td>
</tr>
<tr>
<td>8</td>
<td>106.8</td>
<td>Axial bending</td>
</tr>
<tr>
<td>9</td>
<td>120.9</td>
<td>Pure radial (i=2)</td>
</tr>
<tr>
<td>10</td>
<td>120.9</td>
<td>Pure radial (i=2)</td>
</tr>
<tr>
<td>11</td>
<td>144.7</td>
<td>Axial bending</td>
</tr>
<tr>
<td>12</td>
<td>144.7</td>
<td>Axial bending</td>
</tr>
<tr>
<td>13</td>
<td>190.6</td>
<td>Axial bending</td>
</tr>
<tr>
<td>14</td>
<td>190.6</td>
<td>Axial bending</td>
</tr>
</tbody>
</table>
The free mode shapes of the FE model of the segment it is clear that the Lowest \( i=2 \) mode at 120.9 Hz occurs at a 9th mode and the highest \( i=2 \) mode of 120.9 Hz occurs at 10th mode. During the topology optimization process the modes of the structure under consideration are going to change, as the geometry and mass distribution of the parent structure is changing. In order to ensure that the particular mode under consideration is retained during the optimization process, one has to provide an artificial boundary conditions in consistent with the structure under consideration. In this study, the mode under consideration is an oval mode which is a pure radial mode and it involves purely in plane displacements. Arresting all out of plane degrees of freedom will help us to avoid all the out of plane modes, which are having out of plane displacements. The coordinate system of the structure is as shown in Figure 7, in which X and Y directions are in plane and Z is out of plane. For the structure under consideration in plane displacements are due to UX, UY and ROTZ and out of plane displacements are UZ, ROTX and ROTY. In order to avoid out of plane modes, a displacement constraint applied on stiffening ring as UZ=0, ROTX=0 and ROTY=0. Application of this artificial boundary condition is not going to add any in plane stiffness and hence the Oval mode frequency is not altered. The natural frequency simulation results with this artificial boundary condition are shown in

![Image]

**FIG 8.2 : Applying Boundary Conditions on free ends of Envelope.**

After application of boundary condition which means fixing the free sides of envelope plate, since the Envelope plate is fixed by means of different Fastening systems like riveted joints, screws and nuts, by welding or even adhesives. The stress distribution in each of this is different in different fastening systems however here we assume a permanent strong fix for the stiffener in the Missile Structure and hence it is fixed as shown in the above Figure 8.2. Again by doing Modal Anlysys on the Ring Stiffener with the ends fixed, i.e applications of arbitrary Boundary Conditions on the free Edges of the Envelope plate and thus solving on Ansys solver using Block Lanczos Method. The Mode shapes of the Ring Stiffener are depicted as shown in the table in the other page.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Frequency (Hz)</th>
<th>Type of mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>Global-rigid body</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>Global-rigid body</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>Global-rigid body</td>
</tr>
<tr>
<td>4</td>
<td>120.9</td>
<td>Pure radial ((i=2))</td>
</tr>
<tr>
<td>5</td>
<td>120.9</td>
<td>Pure radial ((i=2))</td>
</tr>
</tbody>
</table>

Table 8.2 : List of modes and frequencies after applying artificial boundary condition.

In this section the radial modes occurs at 4th and 5th mode and this will be the main concern in the process of topology optimization of this stiffener the counter plots of these two modes are shown in the figure as shown
FIG 8.3 : Contour plots of 4th mode.

FIG 8.4 : Contour plot of 5th mode.
From these data of the Mode shapes of Ring Stiffener with arbitrary Boundary condition it is apparent that 4th and 5th modes are oval modes for the structure under consideration and these oval mode frequencies remain unchanged (Table 8.2). This ensures the consistency of the artificial boundary condition introduced in the FE model.

8.4 Minimizing cylinder oval modes using topology optimization:

8.4.1 Optimization problem and results:
For a 2-pole 60Hz turbo-generator (used conventionally), the pole passing frequency occurs at 120Hz. In order to avoid the resonance, the structure oval mode should be away from 120Hz. For the structure under consideration, the oval mode frequency is 120 Hz, which needs to be shifted away. This section discusses minimization of oval mode frequencies. Topology optimization is performed using Finite element Analysis code Hyper Optistuct. The approach that is taken is reduction of frequencies by removing material. The stiffening ring is considered as design region and envelope cover plate is as non-design region. There are two optimization responses defined. One is Frequency-5, which corresponds to oval mode. Second is Volume fraction. And the design constraint is specified as Oval mode (mode 5) frequency to be in between 110Hz and 115Hz. Objective is defined as minimization of volume. There are two oval mode frequencies (4th and 5th), 5th mode frequency would be on higher side. As we would like to have the highest oval mode frequency between 110-115Hz it is necessary to optimize with respective to 5th mode. In topology optimization scheme the thickness of stiffening ring is defined as a design variable and started the optimization with a base value of 0.0in. oval mode frequency is 120 Hz, which needs to be shifted away. This section discusses minimization of oval mode frequencies. Figure 9. The contour plot shown is element density in which red indicates highly significant material and blue is insignificant material. For all practical reasons it is not possible to remove blue regions completely.

The counter plot shows the region with more stressed and with no stress region. The optimization process is performed by using the trial and error method. a step by step counter plots are read for the 4th and 5th oval modes by varying the thickness of the design region which is the stiffener. at a random instance the above figure appears in which we can see clearly the regions with high significance and the region with low significance. The optimization is performed by removing the unwanted material from the structure as far as possible. As in this case we sought to decrease the frequency, the removal of the material is will surely help to get a low frequency. By removing the insignificant material that is less than the elemental density of 0.0105 a new approach to design is depicted. Hence the optimized design is shown I the figure below.
Basing on the redesigned Structure of Stiffener the Optimized design is drawn to have a clear cut idea of the new design in MS paints or any other soft tool to have future work proceedings.

8.5 Validation using Finite Element:
The new design thus is needed to be validated. this validation can be done in two ways either experimentally or by using soft tools of Finite Element. Due to lack of time and Financial assistance we opted the Finite Element Techniques to validate the new design. To validate the design here, it is enough to prove that there is a shift of the frequency from 120 Hz to a relatively lower value of it also the stiffness and other bending moments remains constant or the change is negligible. For validation in Finite Element again the same steps have to be followed from the startin of Catia Design Stage.

8.5.1 Design of the Optimized Stiffener 1:
FIG 8.8 : a) Design of Optimized Stiffener 1


Since the direct importing and meshing of the design in Ansys is difficult as the import was not regular and it was difficult to mesh using the shell63 element for a complicated figure like this in Ansys, so preferably Hypermesh was used to mesh the figure the steps involved in the hypermesh is shown in a pictorial form as follows.

FIG 8.11 : Extracting Mid Surface of the Geometry.
FIG 8.12: Mid Surface of the Geometry.

FIG 8.13: Half mid surface of the geometry
FIG 8.14: Half mesh surface of the geometry

FIG 8.15: Full mesh surface of the geometry
FIG 8.16: Assigning the material properties of the geometry.

FIG 8.17: Mesh Model in Ansys.
FIG 8.18: Boundary Conditions of the Model.

FIG 8.19: Lowest \((i=2)\) with a frequency 57.616 Hz at 4\(^{th}\) Mode.
The first oval mode is at 57Hz and the second Oval mode is at 116Hz. The max oval mode (i=2) frequency is at 116Hz that is less than 120Hz and hence the shifting down or lowering oval mode frequencies achieved. The highest oval mode (i=2) frequency is reduced to 116Hz which is slightly less than the desired limit of 115Hz. The reason being for all practical purposes, it is not possible to remove all insignificant material and in this case the material is removed only at inner diameter. This validates that the new Design formed gives better results of frequency by shifting it to down values, moreover the stiffness and other structural properties remains nearly constant.

8.6 Maximizing Cylinder Oval Modes using Topology Optimization

8.6.1 Optimization problem and results:
Oval mode frequency is 120 Hz, which needs to be shifted away. This section discusses minimization of oval mode frequencies. Figure 9. The contour plot shown is element density in which red indicates highly significant material and blue is insignificant material. For all practical reasons it is not possible to remove blue regions completely. We have already discussed that in Topology optimization with the proposed artificial boundary condition 4th and 5th modes are Oval modes and 4th mode is on lower side and 5th mode is on higher side. In order to shift frequencies to upper side it is necessary to optimize with respective to 4th mode. There are two optimization responses defined. One is Frequency-4, which corresponds to oval mode. Second is Volume fraction. The design constraint is specified as Oval mode (mode 4) frequency to be in between 125Hz and 130Hz. Objective is defined as minimization of volume. In this case for the topology optimization scheme, a base thickness of 1.25in is adopted for stiffening ring as a design variable. The topology optimization results are as shown in the figures below.
FIG 8.21 : Element Density Plot 1 for Optimized Stiffener 2.

FIG 8.22 : Element Density Plot 2 for Optimized stiffener 2.
The Figure 14 shows element density that has significance value more than 0.120. Since we have defined base plate thickness for stiffening ring as 1.25 in, the plate thickness at blue regions can be 1.25 in or even less than that but at red regions it should be more. The optimization is performed by removing the unwanted material from the structure as far as possible. As in this case we sought to increase the frequency, the removal of the material is will surely help to get a low frequency. By removing the insignificant material that is more than the elemental density of 0.120 a new approach to design is depicted. Hence the optimized design is shown in the figure below. Figure 8.23 and at these islands the stiffening ring thickness is 1.25 in. Natural frequency analysis results are shown in Figure.

Since the regions with unwanted insignificant material is identified hence grooves are made at the locations as shown in the figure on both sides of the stiffener, removing the material. Since the thickness was increased initially to know the effect it is now neutralized and the overall weight remains low. Now a new Optimized stiffener with new design is brought into picture as shown in the below figure. The conceptual sketch is prepared in MS paint as shown in figure --- Figure 15 and at these islands the stiffening ring thickness is 1.25 in. Natural frequency analysis results are shown in Figure 16.

FIG 8.26 : b) Design of Optimized Stiffener 2
8.6.3 Validation using Finite Element

The new design thus is needed to be validated. This validation can be done in two ways either experimentally or by using soft tools of Finite Element. Due to lack of time and financial assistance, we opted for the Finite Element Techniques to validate the new design. To validate the design here, it is enough to prove that there is a shift of the frequency from 120 Hz to a relatively lower value of it, also the stiffness and other bending moments remain constant or the change is negligible. For validation in Finite Element again the same steps have to be followed starting of Catia Design Stage.

**FIG 8.27:** c) Design of optimized Stiffener 3.

**FIG 8.28:** Half mid surface of the geometry.
FIG 8.29 : Extracting Mid Surface of the Geometry.

FIG 8.30: Mid Surface of the Geometry.
Fig 8.31: Half mid Surface of the Geometry.

Fig 8.32: Half mesh Surface of the Geometry.
FIG 8.33: Full mesh surface of the geometry.

FIG 8.34: Assigning the material properties of the geometry.
FIG 8.35: Mesh Model in Ansys.

FIG 8.36: Boundary Conditions of the Model.

FIG 8.38: Fifth Mode shape of the Geometry.
The lowest oval mode occurs at 123.70Hz and the highest oval mode occurs at 123.71Hz. There is frequency shift by almost 3Hz compared with original case. The lowest Oval mode (i=2) frequency is at 123.7Hz which is slightly less than the desired limit of 125Hz. The reason is that the island shape cannot be same as topology output due to manufacturability. This validates that the new Design formed gives better results of frequency by shifting it to down values, moreover the stiffness and other structural properties remains nearly constant.

9 RESULTS AND DISCUSSIONS
The analyses have been carried out as follows:

- A theoretical method to know the Natural Frequency of the Ring Stiffeners (cylinders) is done and the Frequency of the Ring Stiffener with conventional Dimensions is known Theoretically.
- The Design of the Ring Stiffener with Conventional measurements provided as per the Defence Research Organisation has been designed successfully in Catia V5.
- The Catia design of the Ring Stiffener is imported in Ansys and Modal analysis is done to know the oval mode shapes and the Oval Mode Frequency of the Stiffener is known.
- The Mode shapes of the stiffener is known by applying the artificial boundary condition on the free sections of the Envelope Plate. In this stage we learnt that there is no change in the oval mode frequencies even after the application of the boundary condition.
- Percentage of error is calculated by comparing the theoretical and experimental values of natural frequency.
- Different cases of optimization is done by varying the thickness of the stiffener and various optimized cases with merits are studied. From these cases two cases are concentrated depending on the density plots.
- One relates to the shifting the oval mode frequency to lowest possible by eliminating the insignificant material from the stiffener. The other relates with increasing the thickness and making the grooves which made the oval mode frequency shift on higher than the one with normal condition.
- These results give us the stiffeners with low weight and high stiffness and also these study gives the knowledge of shifting the oval mode frequency to low or high value without changing the stiffness and help in avoiding resonance.

10 CONCLUSIONS AND FUTURE SCOPE
This project mainly focused on the finite element model for finding different Optimized stiffeners using density plots and removing the unwanted and insignificant material. The analysis has been performed by considering a cylinder with stiffener as geometric shape. During the analysis, plate thickness varied from 0.01 to 1.25 in, under different thickness conditions and concentrated on the removal of insignificant unwanted material and removing it and the new structure is validated by using Finite Element. Finally, the results obtained from FEA and ANSYS MECHANICAL ADPL have been compared with exact solutions of normal mode and the shifting of frequencies is done using various Optimization concept. In turbo-generator specifically 2-pole machines, the oval mode (i=2) of structure has high importance. Shifting Oval mode (i=2) frequencies is presented in this paper based on Topology optimization using Hyper optistruct. An artificial boundary condition is used to ensure that the oval mode number is consistent as the design iterations go on. The artificial boundary condition constraints the out of plane deformations. By using the output of topology optimization of Eigen value the stiffening ring was modified and achieved shift in frequencies. Verified the shape proposed by topology optimization by performing FE analysis. Similar approach can be developed to optimize other modes of interest. In future the FEM code developers can customize their tools to optimize different types of cylinder modes. We can further do the analysis by taking different level of optimization and also by extending the scope of Eigen values. By using this concept, different types of stiffeners can be made depending on the requirements. The concept of making the stiffener by making four parts of it and assembling it can also be done using this method. This type of stiffener can greatly reduce the torsion effect while the missile is in curved trajectories and even the maintenance would be easy as we can do the maintenance only on the section under stress or damage.

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