Source Equation Analysis Of Cavitation In Fluid Flow Over A Surface

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ABSTRACT: The main objective of this paper is to form source equation for cavitation.Mathematical analysis of physical phenomenon gives an ease for the study of parameters. Cavitation phenomenon critically affects several applications of hydronautics and hydrodynamics. cavitation shows losses in several turbines as well as pumps. Inception of cavitation over a surface caused due to fall in pressure below atmospheric. Pressure difference in flow gives rise to spherical bubbles. For physical interpretation one can think for coefficient of pressure as threshold value for inception of cavitation. Deciding parameters for range of operation of rotodynamic machines and performance of same can be decided with mathematical equation. Simulation of Mathematical equation on computer will give cost efficient experimental base for designing machines

Keywords: Cavitation, Coefficient of pressure, Separated flow, Unseparated flow

I. INTRODUCTION

Cavitation is phenomenon which can be understood for detailed flow. It helps to analyze parameters in flow losses over surface. Cavitation for unseparated flow as well as separated flow have different physical reasons. There are Microscopic and macroscopic approach of cavitation.

II. EQUATIONS AND ANALYSIS

III Inception of Cavitation (macroscopic approach):

III.1 Unseparated Flow:

Surfaces experience continuous flow. Difference in pressure over surface results in inception of cavitation. One can clearly define origin of cavitation as ratio of difference in pressure to total dynamic pressure. Flow without separation has certain assumptions, given equation for minimum coefficient pressure varies accordingly, in fluid flow particular size of nucleus has minimum pressure that is responsible for instability of that nucleus. Let us consider minimum pressure as critical pressure (p^*) for size of nucleus, it may be less than vapor pressure. Based on this commencement of cavitation can be

$$\frac{p^* - p_o}{\frac{1}{2}\rho v^2} = c_{p,\min}$$

Seems to be modified form of

$$\frac{p^* - p_v + p_v - p_o}{\frac{1}{2}\rho v^2} = \frac{p^* - p_v}{\frac{1}{2}\rho v^2} + \frac{p_v - p_o}{\frac{1}{2}\rho v^2} = c_{p,\min}$$

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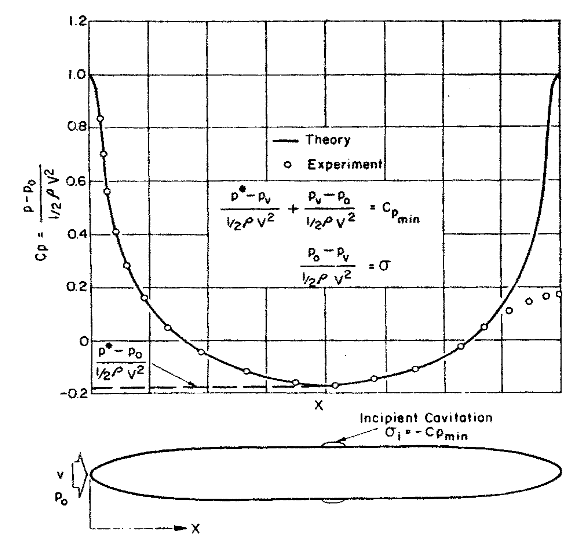
Above equation explains relationship between vapour pressure (p_v) and cavitation inception, It makes a threshold for cavitation in unseparated flow. Density (ρ) and free stream velocity (v) of flowing fluid makes Cavitation inception independent for particular fluid. Free stream pressure makes a difference for cavitation as its value increses inception will increase. C_{pmin} varies according to different flow conditions, It's value over surface is not desirable to avoid it

Figure.1
Above figure shows cavitation inception corresponding to C_{pmin} over surface.



III.2 Separated Flow

Deceleration of flow over surface indicates boundary layer separation and makes it difficult to observe minimum pressure, as a result minimum pressure coefficient is merely indication of cavitation inception. Therefore it is essential to define different coefficient of pressure based on minimum pressure coefficient which will consider stream velocity for cavitation inception. For Separated boundary layer, velocity of flow will be given by the following equation

$$u_s = v(1 - c_{p,\min})$$

$$\bar{c}_{p,\min} \cong 2c_{p,\min} - 1$$

Or to control it several measures can be practicized which will be further explained in this paper. Graphically variation of coefficient of pressure can be visualized as Here u_s is velocity of separated flow and v is velocity of free stream, Therefore modified value of minimum coefficient of pressure in separated flow for inception of cavitation becomes, Position of Minimum coefficient of pressure and minimum pressure are as shown in figure.2 Tip vortex are based on surface roughness, core of tip vortex gives rise to cavitation Hand finishing and filing of leading edge results in surface roughness. It results in earlier inception of boundary layer.

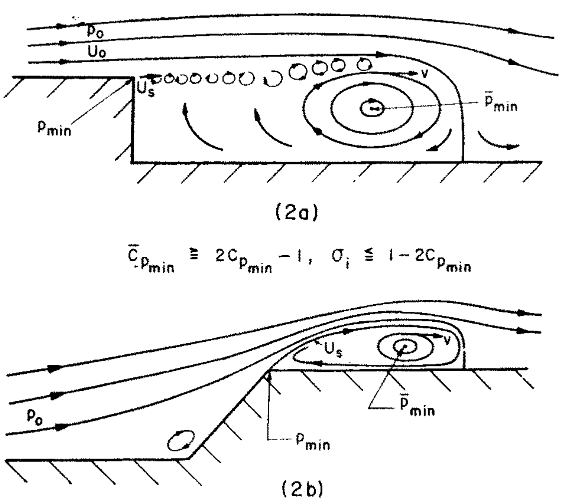


FIGURE.2

It explains position of minimum pressure in case of separated flow,It is in vortex formed. $C_{pmin}$for unseparated flow and separated Flow are related with each other. Eddies formation inside the separated flow gives rise to minimum pressure,In order to reduce eddies formation,control over

IV Body conditions affecting cavitation:

Surface of an object is also reason for inception of cavitation. For positive angle of incidence, roughness on suction side of blade will not affect cavitation number. For negative angle of attack, roughness on pressure side will reduce cavitation number. Where represents surface tension force per unit length and r represents radius of the bibble, second term on right hand side represents surface tension pressure and may be responsible for collapsing bubble. Pressure P_g for a given weight of gas will vary inversely with volume of the gas bubbles, therefore it can be shown as $P_g = A/r^3$, in which A is proportional to number of molecules or weight of the gas and r refers to radius of sphere Above equation becomes,

V Inception of Cavitation (microscopic approach):

V.1 Critical pressure affecting Cavitation:

Critical pressure is a crucial phenomenon of cavitation as it originates bubble of different sizes. Parameters affecting critical pressures are described in following session.

V.2 Nuclei Theory (Study of static stability of cavitation):

Static stability of bubble formed as result of cavitation based on equilibrium of internal pressure and external pressure. Let us consider pressure inside bubble is partially gas pressure (p_g) and partially vapor pressure (p_v), these two pressures balanced by ambient pressure (p), and surface tension force. Mathematical equation of equilibrium can be expressed as boundary layer is required.

$$p_g + p_v = p + 2\gamma/r$$

$$p - p_v = \frac{A}{r^3} - \frac{2\gamma}{r} \quad - \qquad (6)$$

Now since $r = r_o$ at the free stream pressure p_0, value of constant A in terms of ambient nuclei size and pressure is

$$A = (P_O - P_v + \frac{2\gamma}{r_o})r_o^3 \quad - \qquad (7)$$

In terms of diameter Equation (6) becomes,

$$\left(\frac{p^* - p_v}{p_o - p_v}\right)_{critical} = -\frac{2\left(\frac{8}{\sigma_v W}\right)^{\frac{3}{2}}}{3\sqrt{3}\left(1+\frac{8}{\sigma_v W}\right)^{\frac{1}{2}}} \quad - \qquad (8)$$

Defining weber number as

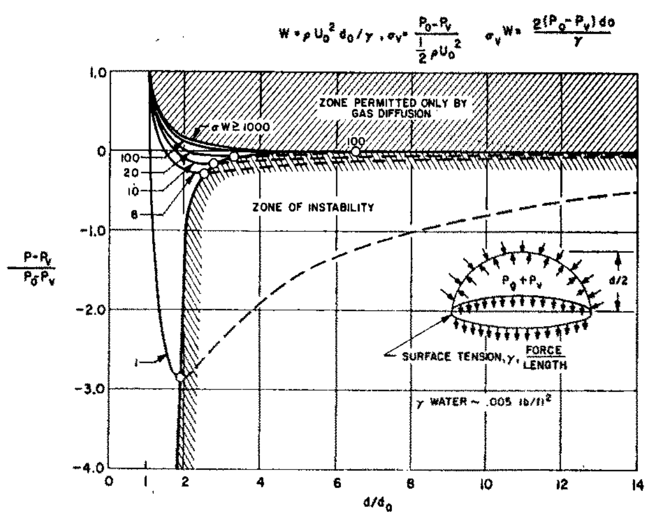
$W = \rho U_O^2 d_o/\gamma$

And vapor cavitation number as

$\sigma_v = (p_o - p_v)/0.5\rho U^2$

Equation number (8) may also be written as

$$\frac{p - p_v}{p_o - p_v} = \left(\frac{d_o}{d}\right)^3\left[1 + \frac{8}{\sigma_v}\frac{1}{W}\left(1 - \frac{d^2}{d_o^2}\right)\right] \quad - \qquad (9)$$



The quasi steady growth of spherical gas bubbles in liquid For the values of $\sigma_v W$,curve have a minimum and thus the well-known instability or cavitation inception occurs. The critical point for inception in terms of the parameters $\sigma_v W$ may be determined as From above equation cavitation inception and its instability can be observed,

VI Results:

Source equation above results in inception of cavitation. For macroscopic approach source of cavitation inception based on free stream velocity. Higher free stream velocity gives weaker cavitation. For microscopic approach it is based on several parameters like volume, pressure difference, density as well as surface tension force.

VII Conclusion:

A detailed cavitation inception observed with mathematical equation are useful for Stable cavitation in the flow occurring at high velocities. Unstable cavitation are due to pressure difference as well as Separation of flow with growing vapor pressure over ambient fluid pressure.

VIII Discussion:

It is very essential to trace the path of cavitation bubbles, Source equations are useful for tracing cavitation. An analysis can be done for trajectory of cavitation bubbles using source terms.

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