Designing A Nonlinear Integer Programming Model For A Cross-Dock By A Genetic Algorithm

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Abstract: This paper presents a non-linear integer programming model for a cross-dock problem that considers the total transportation cost of inbound and outbound trucks from an origin to a destination and the total cost of assigning strip and stack doors to trucks based on their number of trips and the distance between doors in cross-dock. In previous studies, these two cost-based problems are modeled separately; however, it is more realistic and practical to use both of them as an integrated cross-docking model. Additionally, this model is solved for a randomly generated numerical example with three suppliers and two customers by the use of a genetic algorithm. By comparing two different parameter levels (i.e., low and high numbers of populations), the optimum solution is obtained considering a high level population size. A number of strip and stack doors are equal to a number of inbound and outbound trucks in the same sequence as 4 and 6, respectively. Finally, the conclusion is presented.

Index Terms: Cross-docking, Genetic algorithm, Non-linear integer programming, Strip and stack doors, Transportation cost.

1 INTRODUCTION

Kinnear provided a definition of a cross-docking system as “receiving product from a supplier or manufacturer for several end destinations and consolidating this product with other suppliers’ product for common final delivery destinations”. He focused on the consolidation of shipments to achieve economies in transportation costs. The Material Handling Industry of America (MHIA) defines cross-docking as “the process of moving merchandise from the receiving dock to shipping [dock] for shipping without placing it first into storage locations” [1]. Cross-docking is a relatively new logistics system used in the retail and trucking industries to rapidly consolidate shipments from disparate sources and realize economies of scale in outbound transportation. Cross-docking essentially eliminates the inventory-holding steps of a warehouse while it allows serving the consolidation and shipping functions. The idea is to transfer incoming shipments directly to outgoing trailers without storing them in between. Shipments typically spend less than 24 hours at the facility, sometimes less than an hour. The improvements that are possible to obtain from cross-docking are important not only from an economic point of view, but also it gives the possibility to produce a reduction of noise pollution, road accident and urban blight. Therefore, it is important to reduce the transportation cost considering a number of trips between dock doors in a cross-dock. In a study the total weighted travel distance is minimized by using a genetic algorithm to assign doors in less-than-truck-load break bulk terminals [2].

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In a study considering a truck scheduling problem during a fixed time window and capacity in a shipment network through a cross-docking system with more trucks than available docks, a model is formulated as an integer programming problem considering integer programming constraints that minimizes the total travel distance [3]. The model is solved by tabu search and genetic algorithms. Although the travel time of the freight between dock doors was ignored, they considered this term in their next studies and used the same methods for solving the problem minimizing the operational costs. It seems that in this case tabu search is better than the genetic algorithm [4]. In a study a comprehensive literature review about mathematical models in cross-docking scheduling was provided. They categorized models in three different levels based on their decision levels and specified each subject to its level [5]. A cross-dock door assignment problem (CDAP) is considered in the case of serving more than one origin (destination) by strip (stack) door [6]. Two heuristics are proposed, in which the first one is a multi-start local search and the second one uses the convex hull relaxation to linearize the quadratic CDAP function. Then, the proposed heuristics are compared with one exact solution method to optimize door assignments in a cross-dock layout [7]. An extensive review is provided over cross-docking that deal with the scheduling of inbound and outbound trucks precisely [8]. In another study a time-indexed formulation for scheduling trucks in cross-docking terminals is proposed. They tested integer programming formulation and used a branch-and-bound method for solving it [9]. A hybrid meta-heuristic algorithm is presented for planning the trucks in cross-docking systems. This algorithm consists of three parts, namely ant colony optimization (ACO) as an initial population generation method, simulated annealing (SA) as an evolutionary algorithm to employ a certain probability avoiding being trapped in local optimum, and variable neighborhood search (VNS) to improve the population [10]. The algorithms are compared with the heuristic algorithm proposed by Yu and Egbelu (2008) [11]. Utilizing mixed-integer programming with two-dimensional loading constraints, a mathematical model is proposed for minimizing the total transportation costs in a cross-docking network [12]. A two-stage mixed-integer programming (MIP) model for cross-docking center location and vehicle routing scheduling problem is considered in a supply chain. They proposed a two-stage hybrid simulated annealing (HSA) embedded by tabu search [13]. A scheduling problem is considered in a multi-product cross-docking system with deliveries and pickups in order to minimize the total transportation and holding costs. An imperialist competitive algorithm is proposed to solve the model [14]. This paper is presented as follows. Section 2 presents the structure of our mathematical model and its advantages. Section 3 describes a genetic algorithm. Section 4 shows the computational results and sensitive analysis. Conclusions and further research recommendations are presented in section 5.

2 Problem description

2.1 Mathematical model

In this study, it is considered M suppliers (origins), N customers (destinations) for a cross-dock. Product flow is done from origins to destinations through cross-dock considering customers demand and strip (stack) doors capacity. Therefore, the objective is to find the best trucks transshipment plan from origins to cross-dock, then from cross-dock to destinations; so that, the total transportation cost be minimized. The cross-dock door assignment problem and costs created regarding (1), the distance between strip doors and stack doors, (2), number of trips required by trucks to move products between origins and destinations through cross-dock, is the second objective of our problem, that we try to cover both of these objectives as one objective, model the problem, then, solve it. A schematic picture of the cross-docking system is depicted as follows.

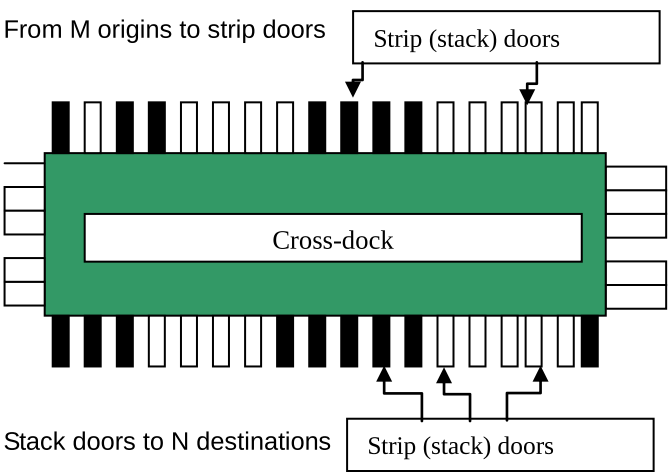

***Fig. 1** Schematic picture of a cross-dock*

Concerning the above assumptions and using the following notations, the problem can be formulated as a non-linear integer programming model.

Notations:

- M Number of suppliers (origins),
- N Number of customers (destinations),
- I Number of strip doors,
- J Number of stack doors,
- P Number of trucks at origin m (inbound trucks), where m ≤ M
- Q Number of trucks at a cross-dock (outbound trucks),
- w_{mn} Number of trips required by trucks to move items from origin m to cross-dock, in which items are destined for destination n,
- d_{ij} Distance between strip door i and stack door j,
- c^*_{pmi} Cost of truck p used for strip door i at origin m,
- c_{qnj} Cost of truck q used for destination n at stack door j,
- s_m Volume of goods from origin m,
- s_i Capacity of strip door i,
- r_n Demand from destination n,
- R_j Capacity of stack door j.

Decision variables:

- $x_{mi}=1$ If origin m is allocated to strip door i,
- $y_{nj}=1$ If destination n is allocated to stack door j,
- $v^*_{pmi}=1$ If truck p is used for strip door i at origin m,
- $v_{qnj}=1$ If truck q is used for destination n at stack door j.

The mathematical model can be formulated as follows:

$$\text{Min} = \sum_{m=1}\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{j=1} d_{ij} w_{mn} x_{mi} y_{nj} + c_{qnj} r_n v_{qnj} + \sum_{i=1}\sum_{m=1}\sum_{p=1} c^*_{pmi} s_m v^*_{pmi}$$

s.t.

$$s_m x_{mi} \le s_i \qquad i=1,\ldots\ldots,I \qquad (2)$$
$$\sum x_{mi} = 1 \qquad m=1,\ldots\ldots,M \qquad (3)$$
$$\sum r_n y_{ni} \le R_i \qquad j=1,\ldots\ldots,J \qquad (4)$$
$$\sum y_{ni} = 1 \qquad n=1,\ldots\ldots,N \qquad (5)$$
$$\sum v_{pmi} \le 1 \qquad i=1,\ldots\ldots,I \;\&\; m=1,\ldots\ldots,M \qquad (6)$$
$$\sum v_{qni} \le 1 \qquad j=1,\ldots\ldots,J \;\&\; n=1,\ldots\ldots,N \qquad (7)$$
$$x_{mi}=0 \text{ or } 1 \qquad (8)$$
$$y_{nj}=0 \text{ or } 1 \qquad (9)$$
$$v_{pmi}=0 \text{ or } 1 \qquad (10)$$
$$v_{qnj}=0 \text{ or } 1 \qquad (11)$$

Objective function (1) is to minimize the total transportation cost of inbound and outbound trucks from an origin to a destination, considering the total cost of assigning strip and stack doors to trucks based on their number of trips and the distance between doors. The constraints make sure that conditions be true. Constraints (2) show that the volume of goods from each origin is not exceeded s_i. Constraints (3) ensure that each origin is allocated to only one strip door. Constraints (4) show that a demand from each destination is not exceeded R_j. Constraints (5) show each destination is allocated to only one stack door. Constraints (6) guarantee that each inbound truck is allocated to the maximum one strip door. Constraints (7) make sure that each outbound truck is allocated to the maximum one stack door.

2.2 Advantages of the model

The proposed model is a modified model based on the cross-dock door assignment problem ([6]) and the mathematical model for a two-dimensional loading problem in a cross-docking network ([12]). The purpose of the presented model is to benefit the objectives of both of them. It is to minimize the total transportation cost of inbound and outbound trucks from an origin to a destination and minimize the total cost of assigning strip and stack doors to trucks based on their number of trips and the distance between doors. The previous studies suggested each of these objectives in separate from each other. The model presented by Zhu et al. (2009) ([6]) was a cross-dock door assignment and the model presented by Küçükoğlu et al. (2013) [12] considered the total transportation costs in a cross-docking network. Therefore, it is the first time that the problem is designed in this paper in order to present the model more realistic and practical in real life.

3 Genetic Algorithm

A genetic algorithm (GA) was introduced for the first time by John Holland in 1975 [15]. It is a heuristic search technique that mimics the process of natural evolution and has been successfully used to generate solutions to optimization and search problems. The GA population consists of a number of chromosomes (i.e., solutions). At first, an initial population of possible solutions is randomly generated, in which the best candidates are selected to be parents to create offspring via a crossover operator. Based on the objective function, each chromosome is assessed and given a fitness score. Crossover and mutation operators are used to produce better offspring, whose fitness scores are better than the fitness scores of their parents. Then, the offspring are placed in the current population to develop a new population for the next generation. The process is iterated until the stopping criterion is met [16].

4 Computational Results

To express the performance of the proposed model, we test the proposed GA on a set of random instances used in the previous studies. The GA procedure is shown in Figure 2.

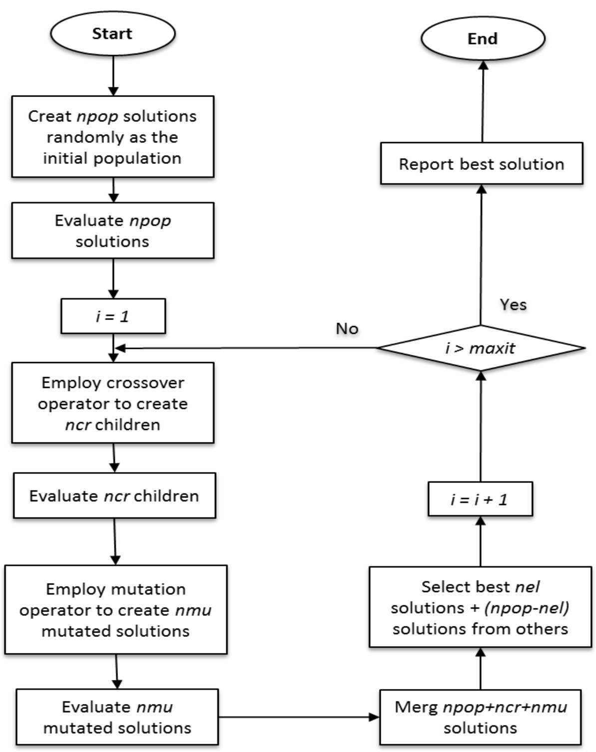

Fig. 2 *Genetic algorithm flowchart*

Parents' selection is done based on binary tournament selection. In this variant, two individuals are chosen at random and the better of the two individuals is selected with fixed probability p, (where, 0.5 < p ~ 1). The procedure of selection is stated as follows. Firstly, select two solutions from the population, secondly compare their objective function values, then choose the best one as a parent, and finally the process is finished if ncr parents have been selected; otherwise, the procedure is repeated. The initial amounts of the parameters used in our model are shown in Table 1. Based on the previous studies, our test problems are presented in Table 2.

TABLE 1
Assumed Parameters

Parameter	Pattern
S_m	$\sim DU\ (0,1000)$
S_i	$\sim DU\ (0,10000)$
r_n	$\sim DU\ (0,1000)$
R_j	$\sim DU\ (0,10000)$
d_{ii}	$\sim DU\ (8,12)$
w_{mn}	$\sim DU\ (100,1000)$
$C\ *_{vmi}$	$\sim DU\ (50,150)$
C_{qni}	$\sim DU\ (50,150)$

TABLE 2
Test Problems Used in This Study

Problems	*M*	*I*	*N*	*J*	*P*	*Q*
1	1	1	5	5	2	5
2	2	2	6	5	2	5
3	3	2	4	4	4	4
4	3	4	2	6	4	6
5	5	5	3	5	6	7
6	6	2	3	3	5	3
7	7	2	2	3	2	3
8	9	2	4	2	7	4

Necessary assumptions and parameters are considered, such as the distance matrix, transportation costs of inbound and outbound trucks used at origins and each destination, strip and stack doors capacity, destinations demands, origins volume of goods, flow matrix shown the number of trips between origins and destinations, and the like. Finally, regarding Table 3, we use the proposed to solve the given problem. All the computations are obtained by the Matlab software. To find the best solutions by the proposed GA for our test problems considering levels shown in Table 3, we run the program by 100 times and the best solutions are included in our analysis. Based on our computations, the results for Problem 1, as one of our test problems, are shown in Table 4.

TABLE 3
Selecting 2 class levels in GA regarding to our test problems

	Level 1(low level)	Level 2(high level)
$npop$	25	200
$maxit$	25	200
pcr	0.25	0.75
pmu	0.1	0.5
pel	0.1	0.9

TABLE 4
BEST GA RESULTS FOR PROBLEM 1

			$npop = 25$		$npop = 200$	
			$maxit =$	$maxit$	$maxit$:	$maxit$:
pcr	pmu	pel	OFV	OFV	OFV	OFV
		0.1	40908	43500	31216	32498
	0.1	0.5	36132	35202	32196	32576
		0.9	34730	50506	31422	33878
		0.1	38452	36052	31932	34428
0.25	0.3	0.5	35710	37414	31932	31950
		0.9	31950	45055	33294	33294
		0.1	45606	55541	31462	33390
	0.5	0.5	50748	35222	29818	31746
		0.9	33674	36356	33614	35710
		0.1	42193	33126	31764	32274
	0.1	0.5	44276	39036	31726	35180
		0.9	35258	36788	34202	34752
		0.1	59370	38510	32234	36318
0.5	0.3	0.5	35318	48275	34770	33576
		0.9	35318	35524	33878	35672
		0.1	40146	36602	32010	32156
	0.5	0.5	33576	40150	32010	35426
		0.9	36698	36944	34938	31686
		0.1	41356	58448	32196	33312
	0.1	0.5	44148	33350	33312	36904
		0.9	35426	31950	33048	30384
		0.1	34124	57114	33692	31500
0.75	0.3	0.5	33576	52782	34674	32576
		0.9	46686	47108	33390	33350
		0.1	33144	35486	32010	31950
	0.5	0.5	40805	33106	30100	31892
		0.9	42024	53234	32234	36582

All the test problems are calculated and the best solution of each problem is summarized in Table 5. As we can observe from the results, Problem 4 has the optimum solution. It is clear that in higher levels we have better solutions; however, longer time is taken to find the answer and the solutions are not optimal.

TABLE 5
BEST GA RESULTS FOR THE TEST PROBLEMS

Test problems	Best OFV	Time (S)	$npop$
1	28456	1.611056	200
2	55029	1.769794	200
3	58768	9.231942	200
4	26216	2.630019	200
5	79377	20.35295	200
6	96908	2.239543	200
7	43848	0.26737	25
8	165708	12.59292	200

5 CONCLUSION

This study considered a cross-dock problem for the first time that minimizes the total transportation cost of inbound and outbound trucks from origin to destination and minimizes the total cost of assigning strip and stack doors to trucks based on their number of trips and the distance between doors simultaneously. Therefore, the problem became more realistic and practical in real life and the presented model could be used to find applicable solutions in cross-docking operations. As discussed before, in the model presented by Zhu et al. (2009) ([6]), minimizing the number of trips required by trucks to move items from origin to destination through cross-dock was considered, while in our model minimizing the total transportation cost considering the volume of goods from origin and demand of them from destination is perceived in addition to their model. In the model presented by Küçükoğlu et al. (2013) ([12]), the objective is only minimizing the total transportation cost, whereas in our model minimizing the total transportation cost regarding the volume of goods from origin and demand of them from destination is modeled and minimizing the number of trips required by trucks to move items from origin to destination through cross-dock is another objective in our model. As it is taken from the results, the minimum transportation cost under the given assumptions was obtained by the proposed genetic algorithm (GA). The main parameters were set as $maxit$ =50, pcr =0.5, pmu =0.3 and pel =0.9. This study can be extended for future studies in different ways. It is possible to use other meta-heuristics in order to find the good solutions in a cross-dock system and analyze the results in large-scale problems. Additionally, the problem can be modeled as a multi-objective one in cross-docking operations and scheduling.

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