A New Formulae Method For Solving The Simultaneous Equations
Avinash A. Musale, Eknath S. Ugale

ABSTRACT: Actually there are too many Methods which are used for solving the various types of equations by various methods. The equations may be like linear, quadratic, simultaneous, radial, and exponential as well as the equations with the number of variables. And for the Simultaneous Equation the methods are like Addition, Substitution, Elimination as well as Graphical Method also. We know there are too many methods like Gauss Elimination, Gauss Seidal, and Jacobi used for solving the Simultaneous Equations. But here we are going to introduce a new Method for solving the Simultaneous Equations as well as the comparison between the Methods above told and the new one. The paper will show you a new easy method for solving the Simultaneous Equation and the difference between the methods that are being used in the colleges which are Gauss Elimination, Gauss Seidal etc.

Keywords: Simultaneous, Equations, Formulae, Method, Gauss, Elimination, Derivation

1. INTRODUCTION:
The paper represents a method for solving the simultaneous equations in a different way. We use the substitution, addition, matrices Methods for solving the simultaneous equations but in this paper the method is related to directly the formulas for solving the simultaneous equations.

2. METHOD (FORMULAE):
For Two Unknowns – X & Y
\[ a_{11}X + a_{12}Y = b_1; \]
\[ a_{21}X + a_{22}Y = b_2; \]
Solution:
\[ u = \frac{(a_{11}b_2 - a_{21}b_1)}{(a_{11}a_{22} - a_{12}a_{21})}; \]
\[ v = \frac{(a_{11}b_1 - a_{21}b_2)}{(a_{11}a_{22} - a_{12}a_{21})}; \]
\[ X = \frac{b_1 - a_{12}Y}{a_{11}}; \]
\[ Y = \frac{b_2 - a_{11}X}{a_{12}}; \]
\[ Z = \frac{b_3 - a_{13}Y}{a_{13}}; \]
For Four Unknowns – X, Y, Z & W
\[ a_{11}X + a_{12}Y + a_{13}Z + a_{14}W = b_1; \]
\[ a_{21}X + a_{22}Y + a_{23}Z + a_{24}W = b_2; \]
\[ a_{31}X + a_{32}Y + a_{33}Z + a_{34}W = b_3; \]
\[ a_{41}X + a_{42}Y + a_{43}Z + a_{44}W = b_4; \]
Solution:
\[ r = \frac{(a_{11}b_4 - a_{14}b_1)}{(a_{11}a_{44} - a_{14}a_{41})}; \]
\[ l = \frac{(a_{11}b_3 - a_{13}b_1)}{(a_{11}a_{33} - a_{13}a_{31})}; \]
\[ m = \frac{(a_{11}b_2 - a_{12}b_1)}{(a_{11}a_{22} - a_{12}a_{21})}; \]
\[ n = \frac{(a_{11}b_1 - a_{11}b_1)}{(a_{11}a_{11} - a_{11}a_{11})}; \]
\[ p = \frac{(a_{11}b_3 - a_{13}b_1)}{(a_{13}b_1 - a_{13}b_1)}; \]
\[ q = \frac{(a_{11}b_2 - a_{12}b_1)}{(a_{12}b_1 - a_{12}b_1)}; \]
\[ t = \frac{(a_{11}b_1 - a_{11}b_1)}{(a_{11}b_1 - a_{11}b_1)}; \]
\[ g = \frac{(a_{11}b_4 - a_{14}b_1)}{(a_{11}b_4 - a_{14}b_1)}; \]
\[ h = \frac{(a_{11}b_3 - a_{13}b_1)}{(a_{11}b_3 - a_{13}b_1)}; \]
\[ i = \frac{(a_{11}b_2 - a_{12}b_1)}{(a_{11}b_2 - a_{12}b_1)}; \]
\[ j = \frac{(a_{11}b_1 - a_{11}b_1)}{(a_{11}b_1 - a_{11}b_1)}; \]
\[ e = \frac{(a_{11}b_4 - a_{14}b_1)}{(a_{11}b_4 - a_{14}b_1)}; \]
\[ f = \frac{(a_{11}b_3 - a_{13}b_1)}{(a_{11}b_3 - a_{13}b_1)}; \]

\[ W = \frac{(a_{11}b_4 - a_{14}b_1)}{(a_{11}b_4 - a_{14}b_1)}; \]
\[ Z = \frac{(a_{11}b_3 - a_{13}b_1)}{(a_{11}b_3 - a_{13}b_1)}; \]
\[ Y = \frac{(a_{11}b_2 - a_{12}b_1)}{(a_{11}b_2 - a_{12}b_1)}; \]
\[ X = \frac{(a_{11}b_1 - a_{11}b_1)}{(a_{11}b_1 - a_{11}b_1)}; \]

• Avinash A. Musale is currently pursuing bachelor degree in Mechanical Engineering in MIT AOE Pune in Savitribai Phule Pune University, India, Mobile No. +918484972706. E-mail: musaleavinash24@gmail.com
• Mr. Eknath S. Ugale is an Assistant Professor in the Department of Mechanical Engineering in MIT AOE Pune, India, E-mail: esugale@mech.mitaoe.ac.in
3. Derivation: For Two Unknowns – X & Y

\[ a_{11}X + a_{12}Y = b_1; \]
\[ a_{21}X + a_{22}Y = b_2; \]

Using Gauss Elimination Method
Convert the given equations in matrix AX=B form

\[ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \]

Make the Matrix A as Upper Triangular Matrix

Make row 1 as following Instruction

\[ R_1 = R_1/a_{11} \]

\[ \begin{bmatrix} 1 \\ a_{21} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} b_1/a_{11} \\ b_2 \end{bmatrix} \]

\[ a_{12}' = a_{12}/a_{11} \]
\[ b_1' = b_1/a_{11} \]

………..(1)

………..(2)

Make row 2 as following Instruction

\[ R_2 = R_2 - a_{21}'R_1 \]

\[ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1' \\ b_2'' \end{bmatrix} \]

\[ a_{22}' = a_{22} - a_{21}'a_{12}' \]
\[ b_2'' = b_2 - a_{21}'b_1' \]

………..(3)

………..(4)

Make row 2 as following Instruction

\[ R_2 = R_2/a_{22}' \]

\[ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} b_1'' \\ b_2'' \end{bmatrix} \]

\[ b_2'' = b_2/a_{22}' \]

………..(5)

We Get…..

\[ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} b_1'' \\ b_2'' \end{bmatrix} \]

From the above solved Matrix
By the Backward Substitution we will get
Here Y = b_2'' ……..(a)

\[ X + a_{12}'Y = b_1' \] ……..(b)

And from equation (a)

\[ Y = b_2'' \]

From eq(5)

\[ Y = \begin{bmatrix} b_2'' \\ a_{22}' \end{bmatrix} \]

From equation (3),(4)

\[ Y = \begin{bmatrix} b_2'' - a_{21}'b_1' \\ a_{22}' - a_{21}'a_{12}' \end{bmatrix} \]

From equation (1),(2)

\[ Y = \begin{bmatrix} a_{11} - b_1/b_1 \] 
\[ a_{22}' - a_{21}'a_{12}' \end{bmatrix} \]

a_{11} gets cancelled…..

\[ Y = \begin{bmatrix} a_{11} - b_1/b_1 \\ a_{12}' - a_{21}'b_1' \end{bmatrix} \]

and from given equation

or from equation (b)

We get

\[ X = \begin{bmatrix} b_1 - a_{12}'Y \\ a_{11} \end{bmatrix} \]

or \[ X = \begin{bmatrix} b_2 - a_{22}'Y \\ a_{21} \end{bmatrix} \]

If

\[ u = [(a_{11}*b_2 - a_{21}*b_1)] \]

\[ v = [(a_{11}*a_{22} - a_{12}'*a_{21})] \]

\[ Y = \frac{u}{v} \]

\[ X = \begin{bmatrix} b_1 - a_{12}'Y \\ a_{11} \end{bmatrix} \]

For Three Unknowns – X, Y & Z

\[ a_{11}X + a_{12}Y + a_{13}Z = b_1; \]
\[ a_{21}X + a_{22}Y + a_{23}Z = b_2; \]
\[ a_{31}X + a_{32}Y + a_{33}Z = b_3; \]

By Gauss Elimination Method
Convert the given equations in matrix AX=B form

\[ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \]

Make the Matrix A as Upper Triangular Matrix therefore

Make row 1 as following Instruction

\[ R_1 = R_1/a_{11} \]

\[ \begin{bmatrix} 1 \\ a_{21} \\ a_{31} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \]

\[ a_{12}' = a_{12}/a_{11} \]
\[ a_{13}' = a_{13}/a_{11} \]
\[ b_1' = b_1/a_{11} \]

………..(1)

………..(2)

………..(3)

Make row 2 and row 3 as following Instruction

\[ R_2 = R_2 - a_{21}'R_1 \]

\[ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} b_1'' \\ b_2'' \\ b_3'' \end{bmatrix} \]

\[ a_{22}' = a_{22}' - a_{21}'a_{12}' \]
\[ b_2'' = b_2 - a_{21}'b_1' \]

………..(4)

………..(5)

………..(6)

\[ a_{32}' = a_{32}' - a_{31}'a_{12}' \]
\[ b_3' = b_3 - a_{31}'b_1' \]

………..(7)

………..(8)

………..(9)

Make row 2 as following Instruction

\[ R_2 = R_2/a_{22}' \]

\[ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} b_1'' \\ b_2'' \\ b_3'' \end{bmatrix} \]

\[ a_{23}' = a_{23}'/a_{22}' \]
\[ b_2'' = b_2''/a_{22}' \]

………..(10)

………..(11)

Make row 3 as following Instruction

\[ R_3 = R_3 - a_{31}'R_1 \]

\[ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} b_1'' \\ b_2'' \\ b_3'' \end{bmatrix} \]

\[ a_{32}' = a_{32}' - a_{31}'a_{12}' \]
\[ b_3' = b_3 - a_{31}'b_1' \]

………..(12)

………..(13)

………..(14)

………..(15)

………..(16)

………..(17)

………..(18)

………..(19)
\[
R3 = R3 - a32' * R2
\]
\[
\begin{bmatrix}
1 & a12' & a13' \\
0 & 1 & a23' \\
0 & 0 & a33'
\end{bmatrix}
\times
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
= \begin{bmatrix}
b1' \\
b2'' \\
b3''
\end{bmatrix}
\]
\[
a33'' = a33' - a32' * a23''
\]
\[
b3'' = b3' - a32' * b2''
\]

Make row 3 as following Instruction
\[
R3 = R3/a33''
\]
\[
\begin{bmatrix}
1 & a12' & a13' \\
0 & 1 & a23' \\
0 & 0 & 1
\end{bmatrix}
\times
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
= \begin{bmatrix}
b1' \\
b2'' \\
b3''
\end{bmatrix}
\]
\[
b3'' = b3''/a33''
\]

From the above Matrix,

By the Backward Substitution we get
\[
\begin{align*}
Z &= b3'' \quad \text{……(a)} \\
Y + a23''Z &= b2'' \quad \text{……(b)}
\end{align*}
\]

From equation (a)

We get
\[
Z = b3'' \\
Y = b2'' - a23''Z
\]

\[
Z = b3'' - a32'' * b2'' \\
a33'' - a32'' * a23''
\]

\[
\begin{bmatrix}
(b3' - a31*b1') - (a32' - a31*a12') + (b2' / a22') \\
(a33' - a31*a13') - (a32' - a31*a12') + (2a23' / a22')
\end{bmatrix}
\]

…… From eq(1,2,3,4,5,6)

\[
Z = \begin{bmatrix}
\left(\frac{(a11+b3'-a31*b1')}{a11}\right) - \left(\frac{(a32-a31*a12')}{a11}\right) + \left(\frac{(b2' - a23')}{a11}\right) \\
\left(\frac{(a11+a33-a31*a31)}{a11}\right) - \left(\frac{(a11+a32-a31*a12)}{a11}\right) + \left(\frac{(a23' - a22')}{a11}\right)
\end{bmatrix}
\]

……From eq(1,2,3)

\[
Z = \begin{bmatrix}
\left(\frac{1}{a11}\right) \left(\frac{(a11+b3' - a31*b1)}{1}\right) - \left(\frac{(a11+a32-a31*a12)}{1}\right) + \left(\frac{(a11+b2 - a21*b1)}{a11}\right) \\
\left(\frac{1}{a11}\right) \left(\frac{(a11+a33-a31*a31)}{1}\right) - \left(\frac{(a11+a32-a31*a12)}{1}\right) + \left(\frac{(a12' - a32' * a23' / a22')}{a11}\right)
\end{bmatrix}
\]

\[
\begin{align*}
\frac{1}{a11} \text{ gets cancelled} \\
as well as a11 gets cancelled
\end{align*}
\]

\[
Z = \begin{bmatrix}
\left(\frac{1}{a11}\right) \left(\frac{(a11+b3' - a31*b1)}{1}\right) - \left(\frac{(a11+a32-a31*a12)}{1}\right) + \left(\frac{(a11+b2 - a21*b1)}{a11}\right) \\
\left(\frac{1}{a11}\right) \left(\frac{(a11+a33-a31*a31)}{1}\right) - \left(\frac{(a11+a32-a31*a12)}{1}\right) + \left(\frac{(a12' - a32' * a23' / a22')}{a11}\right)
\end{bmatrix}
\]

\[
\begin{align*}
\frac{1}{a11} \text{ will get cancelled...}
\end{align*}
\]
\[
Y = \left[ \frac{1}{(a_{11} + a_{21} + b_1)} \right] \left( \frac{1}{(a_{11} + a_{23} - a_{23} + a_{13}) - 2} \right)
\]

\[
Y = \left[ \frac{(a_{11}b_2 - a_{21}b_1)}{(a_{11} + a_{22} - a_{22} + a_{12})} \right]
\]

\[
s = (a_{11}b_2 - a_{21}b_1)
\]

\[
t = [(a_{11}a_{22}) - (a_{12}a_{21})]
\]

\[
v = [(a_{11}a_{23}) - (a_{13}a_{21})]
\]

\[
Y = \frac{t - s}{t}
\]

And from given equations we get

\[
X = \frac{Y}{a_{11}}
\]

And here Simultaneously we can derive the formulas for the four unknowns in the same way above derived. As well as we can also derive the formulas for the Simultaneous equations with the number of derivatives.

4. COMPARISION

A. Gauss Elimination Method

\[
x - 3y + z = 4
\]

\[
2x - 8y + 8z = -2
\]

\[
-6x + 3y - 15z = 9
\]

Convert into AX=B Matrix form

Make the Matrix A as Upper Triangular Matrix

\[
\begin{bmatrix}
1 & -3 & 1 \\
2 & -8 & 8 \\
-6 & 3 & -15 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} =
\begin{bmatrix}
4 \\
-2 \\
9 \\
\end{bmatrix}
\]

Make row 2 and row 3 as following Instruction

\[
R_2 = R_2 - 2R_1
\]

\[
R_3 = R_3 - (-6)R_1
\]

\[
\begin{bmatrix}
1 & -3 & 1 \\
0 & -2 & 6 \\
0 & -15 & -9 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} =
\begin{bmatrix}
4 \\
-10 \\
33 \\
\end{bmatrix}
\]

Make row 2 as following Instruction

\[
R_2 = \frac{R_2}{-2}
\]

\[
\begin{bmatrix}
1 & -3 & 1 \\
0 & 1 & -3 \\
0 & -15 & -9 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} =
\begin{bmatrix}
4 \\
5 \\
33 \\
\end{bmatrix}
\]

Make row 3 as following Instruction

\[
R_3 = R_3 - (-15)R_2
\]

\[
\begin{bmatrix}
1 & -3 & 1 \\
0 & 1 & -3 \\
0 & 0 & -54 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} =
\begin{bmatrix}
4 \\
5 \\
108 \\
\end{bmatrix}
\]

And then by Backward Substitution Method

We will get

\[
-54Z = 108
\]

\[
Z = -2
\]

\[
Y - 3Z = 5
\]

\[
Y = -1
\]

And finally

\[
X = 3
\]

Hence we can conclude

\[
X = 3
\]

\[
Y = -1
\]

\[
Z = -2
\]

B. New Formula Method

\[
x - 3y + z = 4
\]

\[
2x - 8y + 8z = -2
\]

\[
-6x + 3y - 15z = 9
\]

Compare it with

\[
a_{11}X + a_{12}Y + a_{13}Z = b_1
\]

\[
a_{21}X + a_{22}Y + a_{23}Z = b_2
\]

\[
a_{31}X + a_{32}Y + a_{33}Z = b_3
\]

Then here for three unknowns Substitute the values in the given formulae Hence we will get

\[
r = (a_{11}b_3 - a_{31}b_1)
\]

\[
r = 33
\]

\[
s = (a_{11}b_2 - a_{21}b_1)
\]

\[
s = -10
\]

\[
t = (a_{11}a_{22} - a_{12}a_{21})
\]

\[
t = -2
\]

\[
u = (a_{11}a_{33} - a_{13}a_{31})
\]

\[
u = 6
\]

\[
w = (a_{11}a_{32} - a_{12}a_{31})
\]

\[
w = -15
\]

\[
Z = \frac{r - w}{u - w}
\]

\[
Z = -2
\]

\[
Y = \frac{t - w}{t}
\]

\[
Y = -1
\]

\[
X = \frac{b_1 - a_{12}Y - a_{13}Z}{a_{11}}
\]

\[
X = 3
\]

Hence

\[
X = 3
\]

\[
Y = -1
\]

\[
Z = -2
\]

Hence here is the solution by the formulae method
5. CONCLUSION:
Here we can conclude that there is no confusion in solving the simultaneous equation by the Formulae Method. As well as it is easier and takes less time than the other methods like Gauss Elimination Method, Gauss Seidal Method, etc. And in the Engineering Calculators like CASIO fx-991ES PLUS there is no solution for the simultaneous equation with four unknowns so we can use these formulas for solving the Simultaneous equations easily as well as fast.

6. REFERENCES:


