

# On Rough Interval Multi Criteria Decision Making

Hegazy Zaher , H. A. Khalifa , Soha Mohamed

**Abstract:** Nowadays Multi Criteria becomes a basic tool in Decision Making. Several methods are developed in both certainty and uncertainty domains to facilitate the decision making process. This study presents three different Multi Criteria Decision Making Methods based on rough interval with subjective and objective weights to raise the accuracy of the rating values. The proposed methods are tested against some known methods in the literature. The obtained results show the competence of the proposed techniques.

**Keywords:** Rough Interval, Multiple Criteria Decision Making Methods, subjective weights, objective weights.

## 1 INTRODUCTION

Multi-Criteria Decision Making (MCDM) is the most well-known branch of decision making. It is a branch of a general class of Operations Research (OR) models which deals with decision problems under the presence of a number of conflicting decision criteria. Many researchers contribute to many proposals to solve MCDM which includes several methods such as Simple Additive Weighting (SAW), Weighting Product (WP), Analytic Hierarchy Process (AHP), Multi-Objective Optimization on the basis of Ratio Analysis (MOORA), and the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method [1]. A MCDM problem can be concisely expressed in a matrix format, in which columns indicate criteria (attributes) considered in a given problem; and in which rows list the competing alternatives. Specifically, a MCDM problem with  $m$  alternatives ( $A_1, A_2, \dots, A_m$ ) that are evaluated by  $n$  criteria ( $C_1, C_2, \dots, C_n$ ) can be viewed as a geometric system with  $m$  points in  $n$ -dimensional space[2]. Rough set theory, was first proposed by Pawlak [3]. Generally, a rough interval comprises two parts: an upper approximation interval (UAI) and a lower approximation interval (LAI) [3]. Using arithmetic operations rough interval [4], for the treatment of uncertainty in multi-criteria decision making problems which are based on rough interval. Few papers deal with application of rough numbers with multi-criteria models, the available literature has also shown a lack of studies that deal with application of rough sets and rough numbers in MCDM process, even though they showed considerable advantages. Regarding application of rough numbers in MCDM, there are only few papers dealing with this subject. Xie et al.

[5] combined with variable precision rough set and AHP for group decision-making. Guo and Zhang [6] integrated VIKOR and rough set theory in supplier selection. Li et al. [7] proposed an integrated model by combining with Kano's model, rough set and AHP to rank customer requirements importance. Aydogan [8] presented a hybrid rough-AHP and TOPSIS methods. Wang et al. [9] applied interval rough numbers in dealing with uncertainty when determining weight coefficients of decision attributes. Roy et al. [10] presented a hybrid rough-AHP and MABAC methods. This paper presents a novel three different Multi-Criteria Decision Making (MCDM) methods with rough interval; Rough Interval Operational Competitiveness Rating Analysis (RIO CRA), Rough Interval Multi Objective and Optimization on the basis of Ratio Analysis (RIMOORA) and Rough Interval Additive Ratio Assessment (RIARAS). The three methods are used to choose the best alternative among the various alternatives under the rough interval condition. The Subjective and Objective Weight Integrated Approach (SOWIA) method was presented to determine the weight of each evaluation criterion. Because of disagreement in the ranks obtained by the three different MCDM methods; a final ranking method based on REGIME method is given.

## 2 A BRIEF OVERVIEW OF THE USED METHODS

This section is made for illustrating and analyzing the methods and techniques used in this paper. First the weights allocation method is shown, then after the MCDM techniques used are interpreted.

### 2.1 Criteria Weighting by Subjective and Objective Weight Integrated Approach (SOWIA) Method

SOWIA method was used in this study to determine the weight of each criterion. This method based on a combination of objective and subjective weights that gives its superiority over the subjective and objective standalone techniques well known in literature. In this study, a novel combination of the SOWIA method by employing the Entropy (as the objective part) and Rank Reciprocal (as the subjective part). Both components were never combined before in previous studies.

**The SOWIA method procedure consists of the following steps:**

**Step 1:** The decision matrix  $D = [x_{ij}]$  is prepared

**Step 2:** The decision matrix is normalized using the following equation.

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$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \quad i=1,2,\dots,m; j = 1,\dots,n \quad (1)$$

Where  $p_{ij}$  is the normalized value of  $x_{ij}$  and  $\sum_{i=1}^m x_{ij}$  is the total of the values of  $j^{\text{th}}$  attribute for 'm' number of alternatives [11].

**Step 3:** This step deals with determination of weights of importance of the attributes. The same can be determined by three ways, namely: **(a)** based on objective weight ( $O_j$ ) that depends on the elements in the normalized decision matrix **(b)** based on subjective preference ( $S_j$ ) on attributes and **(c)** based on a combination of objective weights and the subjective preference. The weight, thus determined is known as integrated weight ( $W_j^{\text{int}}$ ) [12].

#### a. Objective weight ( $O_j$ ) of importance of attributes:

Entropy Method is considered to determine the objective weight ( $O_j$ ) of the attributes. Entropy is a term in information theory, also known as the average amount of information (Ding and Shi, [11]). The criteria weights are calculated by the Entropy Method.

The calculation steps are as follows:

The output entropy  $E_j$  of the  $j^{\text{th}}$  factor becomes

$$E_j = - \left( \sum_{i=1}^m p_{ij} \ln(p_{ij}) / \ln(m) \right) \quad j=1,\dots, n \quad (2)$$

Variation coefficient of the  $j^{\text{th}}$  factor:  $G_j$  can be defined by the following equation:

$$G_j = 1 - E_j \quad j= 1,2,\dots,n \quad (3)$$

Calculate the weight of entropy  $w_j$  :

$$O_j = \frac{G_j}{\sum_{j=1}^n G_j} \quad j=1,2,\dots,n. \quad (4)$$

Where  $x_{ij}$  Original measured data,  $E_j$  Information Entropy method and  $O_j$  Entropy method Weight [11].

#### b. Subjective weight ( $S_j$ ) of importance of attributes:

Using Rank Reciprocal weights method (RR) to determine the Subjective weight, the reciprocal of the ranks which are normalized by dividing each term by the sum of the reciprocals. The formula is the following:

$$w_j(RR) = \frac{1/r_j}{\sum_{k=1}^n (1/r_k)} \quad (5)$$

Where  $w_j$  is the normalized weight for dimension  $j$ ,  $N$  is the number of dimensions, and  $r_j$  is the rank of the  $j$ -th criterion,  $j=1, 2,\dots, n$  [13].

#### c. Integrated weight ( $W_j^{\text{int}}$ ) of importance of attributes:

If the decision maker wants to utilize the combined effect of objective and subjective weight as described in earlier steps he/she may use the integrated weight as defined in the following equation:

$$W_j^{\text{int}} = \alpha \cdot O_j + (1 - \alpha) \cdot S_j \quad (6)$$

Where  $\alpha$  is known as objective factor decision weightage. The value of  $\alpha$  lies in the interval  $(0, 1)$ .  $O_j$  and  $S_j$  are objective and subjective weight respectively. As  $\alpha$  can be assigned any value in the interval  $(0, 1)$  [12].

## 2.2 Additive Ratio Assessment (ARAS) Method

The ARAS (Additive Ratio ASsessment) method was firstly introduced by Zavadskas and Turksis [14]. According to the ARAS method, a utility function value determining the complex relative efficiency of a feasible alternative is directly proportional to the relative effect of values and weights of the main criteria considered in a project [14].

## 2.3 Operational Competitiveness Rating Analysis (OCRA) Method

In OCRA method, in the first step, the preference ratings with respect to nonbeneficial or input criteria are determined; in the second step, the preference ratings of the output criteria are determined and in the last step, the overall preference ratings of the available alternatives are evaluated where both the cardinal and ordinal data are used. OCRA uses an intuitive method for incorporating the decision maker's preferences about the relative importance of the criteria [15].

## 2.4 Multi Objective and Optimization on the Basis of Ratio Analysis (MOORA) Method

MOORA (Multi Objective and Optimization on the basis of Ratio Analysis) method was firstly proposed by Brauers and Zavadskas [16]. Considers both beneficial and non-beneficial objectives (criteria) for ranking or selecting one or more alternatives from a set of available options.

## 2.5 Final Ranking Method (REGIME)

The REGIME method was described by Hinloopen et al. [16]. This method is nothing but an ordinal generalization of pairwise comparison methods such as concordance analysis; the final ranking of alternatives is obtained by REGIME method [17].

## 3 PROPOSED TECHNIQUES

All the proposed approaches are using SOWIA method aforementioned in section 2. The three methods proposed are RIARAS, RIMOORA, and RIOCRA for performance evaluation of alternatives.

### 3.1 Rough Interval Additive Ratio Assessment (RIARAS) Method

According to the Interval Additive Ratio Assessment (ARAS) method (Zavadskas and Turskis, [14]). Using operations rough interval and applying in the ARAS method.

**(RIARAS) that includes the following steps:**

**The first stage** is Rough Interval Decision-Making Matrix (RIDMM) forming. In the MCDM is represented by the following RIDMM of preferences for  $m$  reasonable alternatives (rows) rated on  $n$  criteria (columns):

$$X^R = \begin{bmatrix} [x_{01}^{UAI} : x_{01}^{LAI}] & \dots & [x_{0j}^{UAI} : x_{0j}^{LAI}] & \dots & [x_{0n}^{UAI} : x_{0n}^{LAI}] \\ \vdots & & \vdots & & \vdots \\ [x_{i1}^{UAI} : x_{i1}^{LAI}] & \dots & [x_{ij}^{UAI} : x_{ij}^{LAI}] & \dots & [x_{in}^{UAI} : x_{in}^{LAI}] \\ \vdots & & \vdots & & \vdots \\ [x_{m1}^{UAI} : x_{m1}^{LAI}] & \dots & [x_{mj}^{UAI} : x_{mj}^{LAI}] & \dots & [x_{mn}^{UAI} : x_{mn}^{LAI}] \end{bmatrix}; i = \overline{0, m}; j = \overline{1, n}, \quad (7)$$

Where m – number of alternatives, n – number of criteria describing each alternative,  $[x_{ij}^{UAI} : x_{ij}^{LAI}]$  Where  $X^{UAI}$  and  $X^{LAI}$  are upper and lower approximation intervals of  $X^R$ , representing the performance value of the i alternative in terms of the j criterion,

$X^R = [x_{ij}^{UAI} : x_{ij}^{LAI}]$  is defined as an interval lower  $X^{LAI} = [X^{-LAI}, X^{+LAI}]$  and upper bound  $X^{UAI} = [X^{-UAI}, X^{+UAI}]$ .  $[x_{0j}^{UAI} : x_{0j}^{LAI}]$  Optimal value of j criterion.

**If optimal value of j criterion is unknown, then**

$$[x_{0j}^{UAI} : x_{0j}^{LAI}] = \max_i [x_{0j}^{UAI} : x_{0j}^{LAI}], \quad \text{if } \max_i [x_{0j}^{UAI} : x_{0j}^{LAI}] \text{ is preferable, and} \quad (8)$$

$$[x_{0j}^{UAI} : x_{0j}^{LAI}] = \min_i [x_{0j}^{UAI} : x_{0j}^{LAI}], \quad \text{if } \min_i [x_{0j}^{UAI} : x_{0j}^{LAI}] \text{ is preferable.} \quad (9)$$

**In the second stage** the initial values of all the criteria are normalized – defining values  $\bar{x}_{ij}^R$  of normalized decision-making matrix  $\bar{X}^R$ .

**The criteria, whose preferable values are maxima, are normalized as follows:**

$$\bar{x}_{ij}^R = \frac{[x_{ij}^{UAI} : x_{ij}^{LAI}]}{\sum_{i=0}^m [x_{ij}^{UAI} : x_{ij}^{LAI}]} \quad (10)$$

$$\bar{x}_{ij}^R = \frac{[[x_{ij}^{-UAI}, x_{ij}^{+UAI}]: [x_{ij}^{-LAI}, x_{ij}^{-LAI}]]}{[[\sum_{i=0}^m x_{ij}^{-UAI}, \sum_{i=0}^m x_{ij}^{+UAI}]: [\sum_{i=0}^m x_{ij}^{-LAI}, \sum_{i=0}^m x_{ij}^{-LAI}]]} \quad (11)$$

The criteria, whose preferable values are minima, are normalized by applying two-stage procedure:

$$\bar{x}_{ij}^R = \frac{1}{[[x_{ij}^{-UAI}, x_{ij}^{+UAI}]: [x_{ij}^{-LAI}, x_{ij}^{-LAI}]]}, \quad (12)$$

$$\bar{x}_{ij}^R = \frac{[[x_{ij}^{-UAI}, x_{ij}^{+UAI}]: [x_{ij}^{-LAI}, x_{ij}^{-LAI}]]}{[[\sum_{i=0}^m x_{ij}^{-UAI}, \sum_{i=0}^m x_{ij}^{+UAI}]: [\sum_{i=0}^m x_{ij}^{-LAI}, \sum_{i=0}^m x_{ij}^{-LAI}]]}. \quad (13)$$

**The third stage** is Weighted normalized rough interval decision matrix  $\hat{X}^R$  is formed by using  $W^{int}$  criteria significance calculated in SOWIA method. The weighted normalized values  $\hat{x}_{ij}^R$  are calculated as follows:

$$\hat{x}_{ij}^R = \bar{x}_{ij}^R \times w_j; i = \overline{0, m}, \quad (14)$$

$$\hat{x}_{ij}^R = [[\bar{x}_{ij}^{-UAI}, \bar{x}_{ij}^{+UAI}]: [\bar{x}_{ij}^{-LAI}, \bar{x}_{ij}^{+LAI}]] \times w_j; i = \overline{0, m}, \quad (15)$$

Where  $w_j$  is the weight (importance) of the j criterion and  $\bar{x}_{ij}^R$  is the normalized rating of the j criterion.

**The following task is determining values of optimality function:**

$$S_i^R = \sum_{j=1}^n \hat{x}_{ij}^R; i = \overline{0, m}, \quad (16)$$

$$S_i^R = [[\sum_{j=1}^n x_{ij}^{-UAI}, \sum_{j=1}^n x_{ij}^{+UAI}]: [\sum_{j=1}^n x_{ij}^{-LAI}, \sum_{j=1}^n x_{ij}^{+LAI}]] \quad (17)$$

Where  $S_i^R$  is the value of optimality function of i alternative. The biggest value is the best, and the least one is the worst. The result of rough interval decision making for each alternative is grey number  $S_i^R$ .

**The center-of-area is the most practically and simple to apply:**

$$S_i = \frac{1}{4} (x_{ij}^{-UAI} + x_{ij}^{+UAI} + x_{ij}^{-LAI} + x_{ij}^{+LAI}) \quad (18)$$

The degree of the alternative utility is determined by a comparison of the variant, which is analyzed, with the ideally best one  $S_0$ . The equation used for the calculation of the utility degree  $K_i$  of an alternative  $A_i$  is given below:

$$K_i = \frac{S_i}{S_0}; i = \overline{0, m}, \quad (19)$$

Where  $S_i$  and  $S_0$  are the optimality criterion values, that the calculated values  $K_i$  are in the interval [0; 1]. The alternative with the highest utility value is the best choice among the alternatives.

**3.2 Rough Interval Operational Competitiveness Rating Analysis (RIO CRA) Method**

This paper extends the Operational Competitiveness Rating Analysis (OCRA) method [15], for dealing with rough interval and an algorithm for determining the best choice among all possible choices when the data are rough interval is also presented.

**The approach for extending the OCRA method rough interval environment is as follows:**

**Step 1:** construct the Rough interval decision matrix. In the rough interval decision matrix  $X^R$ ,

$$X^R = \begin{bmatrix} [x_{11}^{UAI} : x_{11}^{LAI}] & \dots & [x_{12}^{UAI} : x_{12}^{LAI}] & \dots & [x_{1n}^{UAI} : x_{1n}^{LAI}] \\ \vdots & & \vdots & & \vdots \\ [x_{21}^{UAI} : x_{21}^{LAI}] & \dots & [x_{22}^{UAI} : x_{22}^{LAI}] & \dots & [x_{2n}^{UAI} : x_{2n}^{LAI}] \\ \vdots & & \vdots & & \vdots \\ [x_{m1}^{UAI} : x_{m1}^{LAI}] & \dots & [x_{m2}^{UAI} : x_{m2}^{LAI}] & \dots & [x_{mn}^{UAI} : x_{mn}^{LAI}] \end{bmatrix}; i = \overline{1, m}; j = \overline{1, n}, \quad (20)$$

$$W = [w_1, w_2, \dots, w_n]$$

Where m – number of alternatives, n – number of criteria describing each alternative,  $[x_{ij}^{UAI} : x_{ij}^{LAI}]$  Where  $X^{UAI}$  and  $X^{LAI}$  are upper and lower approximation intervals of  $X^R$ , representing the performance value of the i alternative in terms of the j criterion, and  $w_j$  is the weight of criterion  $C_j$ .

**In this method,** the  $j^{\text{th}}$  criterion of the  $i^{\text{th}}$  alternative is represented by rough interval:

$X^R = [x_{i1}^{UAI} : x_{ij}^{UAI}]$  is defined as an interval lower  $X^{LAI} = [X^{-LAI}, X^{+LAI}]$  and upper bound  $X^{UAI} = [X^{-UAI}, X^{+UAI}]$ .

**Step 2: Compute the Preference Ratings with Respect to the Non-Beneficial Attribute**

The aggregate performance of  $i^{th}$  alternative with respect to all the input attribute is calculated using the following equation:

$$\bar{I}_i^R = \sum_{j=1}^n W_j \frac{\min(x_j^m)^R - (x_i^j)^R}{\min(x_j^m)^R} \quad (i= 1,2,\dots,m; j=1,2,\dots,n; i \neq m) \quad (21)$$

Where  $\bar{I}_i^R$  is the measure of the relative performance of  $i^{th}$  alternative and  $(x_i^j)^R$  is the performance score of  $i^{th}$  alternative with respect to  $j^{th}$  input criterion. The calibration constant  $w_j$  (relative importance of  $j^{th}$  criterion) is used to increase or reduce the impact of this difference on the rating  $\bar{I}_i^R$  with respect to  $j^{th}$  criterion.

**Step 3: Calculate the linear preference rating for the input criteria.**

$$\bar{I}_i^R = \bar{I}_i^R - \min(\bar{I}_i^R) \quad (22)$$

**Step 4: Compute the preference ratings with respect to the beneficial criterion.**

The aggregate performance for  $i^{th}$  alternative on all the beneficial or output criteria is measure using the following expression:

$$\bar{O}_i^R = \sum_{h=1}^H W_h \frac{(x_i^h)^R - \min(x_h^m)^R}{\min(x_h^m)^R} \quad (23)$$

Where  $h=1,2,\dots, H$  indicates the number of beneficial attributes or output attribute and  $w_h$  is calibration constant or weight importance of  $h^{th}$  output criteria. The higher an alternative's score for an output criterion, the higher is the preference for that alternative. It can be mentioned that

$$\sum_{j=1}^n w_j + \sum_{h=1}^H w_h = 1$$

**Step 5: Calculate the linear preference rating for the output criteria:**

$$\bar{O}_i^R = \bar{O}_i^R - \min(\bar{O}_i^R) \quad (24)$$

**Step 6: Compute the overall preference ratings.**

For each alternative ( $P_i^R$ ) is calculated by scaling the sum  $(\bar{I}_i^R + \bar{O}_i^R)$  so that the least preferable alternative receives a rating of zero.

$$(P_i^R) = (\bar{I}_i^R + \bar{O}_i^R) - \min(\bar{I}_i^R + \bar{O}_i^R) \quad (25)$$

**Step 7:** The alternative with the highest overall performance index is the most favorable choice.

**3.2.1 Convert (RIOCR) method into two conventional MCDM interval models:**

Converting (RIOCR) method into two conventional interval number MCDM models that are lower and upper approximation for solving rough interval MCDM problems using the concept of solving the conventional interval models MCDM problems. Therefore, in the present study, we extended the MCDM method for decision making

problem, with lower and upper approximation interval numbers.

A MCDM decision matrix with conventional interval Numbers can be concisely expressed in matrix format as  $X^{IN}$ ,

$$X^{IN} = \begin{bmatrix} [x_{11}^L, x_{11}^U] & \dots [x_{12}^L, x_{12}^U] & \dots [x_{1n}^L, x_{1n}^U] \\ \vdots & \ddots & \vdots \\ [x_{21}^L, x_{21}^U] & \dots [x_{22}^L, x_{22}^U] & \dots [x_{2n}^L, x_{2n}^U] \\ \vdots & \ddots & \vdots \\ [x_{m1}^L, x_{m1}^U] & \dots [x_{m2}^L, x_{m2}^U] & \dots [x_{mn}^L, x_{mn}^U] \end{bmatrix} \quad (26)$$

$W = [w_1, w_2, \dots, w_n]$

Where  $x_{ij}$  is the rating of alternative  $i$  with respect to  $j$ , and is not known exactly and only we know Criterion  $x_{ij} \in [x_{ij}^L, x_{ij}^U]$ . and  $w_j$  is the weight of criterion  $C_j$  [2].

**A. Lower Approximation of the (RIOCR) Method with Interval Numbers:**

Extend the OCRA method to solve MCDM problem with interval numbers by using lower Approximation. Construct the Lower approximation model of (RIOCR), in this method, the  $j^{th}$  criterion of the  $i^{th}$  alternative is represented by Lower approximation interval number:

$$x_{ij}^{LAI} = [x_{ij}^{-LAI}, x_{ij}^{+LAI}] , x_{ij}^{-LAI} = \text{lower lower interval and } x_{ij}^{+LAI} = \text{lower upper interval.}$$

**The approach for extending the OCRA method interval numbers environment is as follows:**

**Step 1:** construct the decision matrix with interval numbers  $X^{IN}$ , shown in (26).

**Step 2: Compute the preference ratings with respect to the non-beneficial attribute**

The aggregate performance of  $i^{th}$  alternative with respect to all the input attribute is calculated using the following equation:

$$\bar{I}_i^{LAI} = \sum_{j=1}^n W_j \frac{\max((x_j^m)^{-LAI}, (x_j^m)^{+LAI}) - ((x_i^j)^{-LAI}, (x_i^j)^{+LAI})}{\min((x_j^m)^{-LAI}, (x_j^m)^{+LAI})} , \quad (i= 1,2,\dots,m; j=1,2,\dots,n; i \neq m) \quad (27)$$

Where  $\bar{I}_i^{LAI}$  is the measure of the relative performance of  $i^{th}$  alternative and  $((x_i^j)^{-LAI}, (x_i^j)^{+LAI})$  are the performance score of  $i^{th}$  alternative with respect to  $j^{th}$  input criterion. The calibration constant  $w_j$  (relative importance of  $j^{th}$  criterion) is used to increase or reduce the impact of this difference on the rating  $\bar{I}_i^{LAI}$  with respect to  $j^{th}$  criterion.

**Step 3: Calculate the linear preference rating for the input criteria.**

$$\bar{I}_i^{LAI} = (\bar{I}_i^{-L}, \bar{I}_i^{+L}) - \min(\bar{I}_i^{-L}, \bar{I}_i^{+L}) \quad (28)$$

**Step 4: Compute the preference ratings with respect to the beneficial criterion.**

The aggregate performance for  $i^{th}$  alternative on all the beneficial or output criteria is measure using the following expression:

$$\bar{O}_i^{LAI} = \sum_{h=1}^H W_j \frac{((x_h^j)^{-LAI}, (x_h^j)^{+LAI}) - \min((x_h^m)^{-LAI}, (x_h^m)^{+LAI})}{\min((x_h^m)^{-LAI}, (x_h^m)^{+LAI})} \quad (29)$$

Where  $h=1,2,\dots, H$  indicates the number of beneficial attributes or output attribute and  $w_h$  is calibration constant or weight importance of  $h^{\text{th}}$  output criteria. The higher an alternative's score for an output criterion, the higher is the preference for that alternative. It can be mentioned that

$$\sum_{j=1}^n w_j + \sum_{h=1}^H w_h = 1$$

**Step 5: Calculate the linear preference rating for the output criteria:**

$$\bar{O}_i^{LAI} = (\bar{O}_i^{-L}, \bar{O}_i^{+L}) - \min(\bar{O}_i^{-L}, \bar{O}_i^{+L}) \quad (30)$$

**Step 6: Compute the overall preference ratings:**

For each alternative ( $P_i^{LAI}$ ) is calculated by scaling the sum ( $\bar{I}_i^{LAI} + \bar{O}_i^{LAI}$ ) so that the least preferable alternative receives a rating of zero.  $P_i^{LAI} = [P_i^{-L}, P_i^{+L}]$

$$P_i^{LAI} = (\bar{I}_i^{LAI} + \bar{O}_i^{LAI}) - \min(\bar{I}_i^{LAI} + \bar{O}_i^{LAI}) \quad (31)$$

$$P_i^{LAI} = ((\bar{I}_i^{-L}, \bar{I}_i^{+L}) + (\bar{O}_i^{-L}, \bar{O}_i^{+L})) - \min((\bar{I}_i^{-L}, \bar{I}_i^{+L}) + (\bar{O}_i^{-L}, \bar{O}_i^{+L})) \quad (32)$$

**B. Upper Approximation of the (RIOCRA) with Interval Numbers:**

Extend the OCRA method to solve MCDM problem with interval numbers by using Upper Approximation. Construct the Upper approximation model of (RIOCRA), **In this method**, the  $j^{\text{th}}$  criterion of the  $i^{\text{th}}$  alternative is represented by Upper approximation interval number:

$$x_{ij}^{UAI} = [x_{ij}^{-UAI}, x_{ij}^{+UAI}], \quad x_{ij}^{-UAI} = \text{upper lower interval and } x_{ij}^{+UAI} = \text{upper upper interval.}$$

**The approach for extending the OCRA method interval numbers environment is as follows:**

**Step 1:** construct the decision matrix with interval numbers  $X^{IN}$ , shown in (26).

**Step 2: Compute the preference ratings with respect to the non-beneficial attribute**

The aggregate performance of  $i^{\text{th}}$  alternative with respect to all the input attribute is calculated using the following equation:

$$\bar{I}_i^{UAI} = \sum_{j=1}^n w_j \frac{\max((X_j^m)^{-UAI}, (X_j^m)^{+UAI}) - (X_i^j)^{-UAI}, (X_i^j)^{+UAI}}{\min((X_j^m)^{-UAI}, (X_j^m)^{+UAI})}, \quad (i=1,2,\dots,m; j=1,2,\dots,n; i \neq m) \quad (33)$$

Where  $\bar{I}_i^{UAI}$  is the measure of the relative performance of  $i^{\text{th}}$  alternative and  $((X_i^j)^{-UAI}, (X_i^j)^{+UAI})$  are the performance score of  $i^{\text{th}}$  alternative with respect to  $j^{\text{th}}$  input criterion. The calibration constant  $w_j$  (relative importance of  $j^{\text{th}}$  criterion) is used to increase or reduce the impact of this difference on the rating  $\bar{I}_i^{UAI}$  with respect to  $j^{\text{th}}$  criterion.

**Step 3: Calculate the linear preference rating for the input criteria.**

$$\bar{I}_i^{UAI} = (\bar{I}_i^{-U}, \bar{I}_i^{+U}) - \min(\bar{I}_i^{-U}, \bar{I}_i^{+U}) \quad (34)$$

**Step 4: Compute the preference ratings with respect to the beneficial criterion.**

The aggregate performance for  $i^{\text{th}}$  alternative on all the beneficial or output criteria is measure using the following expression:

$$\bar{O}_i^{UAI} = \sum_{h=1}^H w_h \frac{((X_h^m)^{-UAI}, (X_h^m)^{+UAI}) - \min((X_h^m)^{-UAI}, (X_h^m)^{+UAI})}{\min((X_h^m)^{-UAI}, (X_h^m)^{+UAI})} \quad (35)$$

Where  $h=1,2,\dots, H$  indicates the number of beneficial attributes or output attribute and  $w_h$  is calibration constant or weight importance of  $h^{\text{th}}$  output criteria. The higher an alternative's score for an output criterion, the higher is the preference for that alternative. It can be mentioned that

$$\sum_{j=1}^n w_j + \sum_{h=1}^H w_h = 1$$

**Step 5: Calculate the linear preference rating for the output criteria:**

$$\bar{O}_i^{UAI} = (\bar{O}_i^{-U}, \bar{O}_i^{+U}) - \min(\bar{O}_i^{-U}, \bar{O}_i^{+U}) \quad (36)$$

**Step 6: Compute the overall preference ratings.**

For each alternative ( $P_i^{UAI}$ ) is calculated by scaling the sum ( $\bar{I}_i^{UAI} + \bar{O}_i^{UAI}$ ) so that the least preferable alternative receives a rating of zero.  $P_i^{UAI} = [P_i^{-U}, P_i^{+U}]$

$$P_i^{UAI} = (\bar{I}_i^{UAI} + \bar{O}_i^{UAI}) - \min(\bar{I}_i^{UAI} + \bar{O}_i^{UAI}) \quad (37)$$

$$P_i^{UAI} = ((\bar{I}_i^{-U}, \bar{I}_i^{+U}) + (\bar{O}_i^{-U}, \bar{O}_i^{+U})) - \min((\bar{I}_i^{-U}, \bar{I}_i^{+U}) + (\bar{O}_i^{-U}, \bar{O}_i^{+U})) \quad (38)$$

**Step 7.** crisp valued overall performance ratings of the Alternatives  $p_i^R$  for each two model, the result of two models for each alternative is rough interval,  $P_i^{LAI} = [P_i^{-L}, P_i^{+L}]$  and  $P_i^{UAI} = [P_i^{-U}, P_i^{+U}]$ .

$$P_i^R = [P_i^{-L}, P_i^{+L}, P_i^{-U}, P_i^{+U}]. \quad (39)$$

$$p_i = \frac{1}{4} [P_i^{-L} + P_i^{+L} + P_i^{-U} + P_i^{+U}] \quad (40)$$

**Step 8.** The alternative with the highest overall performance index is the most favorable choice.

**3.3 Rough Interval Multi Objective and Optimization on the basis of Ratio Analysis (RIMOORA) Method**

In order to take full advantage of Multi Objective and Optimization on the basis of Ratio Analysis (MOORA) method [16], for solving the decision-making problems with rough interval, a new variant of MOORA method, i.e. rough interval MOORA is proposed here which consists of the following procedural steps:

**Step 1:** Based on the valued opinions of the decision makers, develop the decision matrix where each criterion value is measured using rough interval  $X^{RI}$ , shown in (20).

**Step 2:** Calculate the normalized decision-making matrix.

$$x_{ij}^{*R} = \frac{x_{ij}^R}{\sqrt{\sum_{j=1}^m [(x_{ij}^{UAI})^2 + (x_{ij}^{LAI})^2]}} \quad (41)$$

**Step 3:** Calculate the weighted normalized rough interval decision-making matrix by using  $W^{\text{int}}$  criteria significance calculated in SOWIA method, the following formulas are used:

$$v_{ij}^R = x_{ij}^{*R} \times W^{int} \quad (42)$$

Where  $v_{ij}^R$  are lower approximation and upper approximation rough interval of weighted normalized, and  $x_{ij}^{*R}$  is the normalized rough rating of the  $j$  criterion.

**Step 4:** Calculate the overall ratings of beneficial and non-beneficial criteria for each alternative, the overall ratings of an alternative for lower and the upper approximation values of the rough intervals are computed as follows:

$$S_i^{*R} = \sum_{j=1}^g v_{ij}^R - \sum_{j=g+1}^n v_{ij}^R \quad (43)$$

Where  $S_i^{*R}$  represent the overall rating of rough interval. Here  $j = 1, 2, \dots, g$  indicates the criteria to be beneficial and  $j = g+1, g+2, \dots, n$  shows the criteria to be non-beneficial.

**Step 5:** Ranking of the alternatives are obtained by ranking  $S_i^{*R}$  values in descending order, the alternative with the highest overall performance index is the best choice.

### 3.3.1 Convert (RIMOORA) into one model using Lower approximation and Upper approximation rough interval.

Deterministic Model for solving (RIMOORA) using Lower approximation and Upper approximation rough interval: Convert (RIMOORA) model into one model for solving rough MCDM problems.

**Step 1:** construct the decision matrix with rough interval  $X^{RI}$ , shown in (20).

**Step 2:** Calculate the normalized decision-making matrix. To calculate the Lower approximation and the Upper approximation bounds of a rough interval can be determined using the following formula:

$$x_{ij}^{*-L} = \frac{x_{ij}^{-L}}{\sqrt{\frac{1}{2} \sum_{j=1}^m [x_{ij}^{UAI} + x_{ij}^{LAI}]}} \quad (44)$$

$$x_{ij}^{*+L} = \frac{x_{ij}^{+L}}{\sqrt{\frac{1}{2} \sum_{j=1}^m [x_{ij}^{UAI} + x_{ij}^{LAI}]}} \quad (45)$$

$$x_{ij}^{*-U} = \frac{x_{ij}^{-U}}{\sqrt{\frac{1}{2} \sum_{j=1}^m [x_{ij}^{UAI} + x_{ij}^{LAI}]}} \quad (46)$$

$$x_{ij}^{*+U} = \frac{x_{ij}^{+U}}{\sqrt{\frac{1}{2} \sum_{j=1}^m [x_{ij}^{UAI} + x_{ij}^{LAI}]}} \quad (47)$$

Where  $x_{ij}^{*-L}$ ,  $x_{ij}^{*+L}$  are the normalized lower approximation and  $x_{ij}^{*-U}$ ,  $x_{ij}^{*+U}$  are the normalized upper approximation performance ratings.

**Step 3:** Calculate the weighted normalized rough interval decision-making matrix by using  $W^{int}$  criteria significance calculated in SOWIA method, the following formulas are used:

$$v_{ij}^{-L} = x_{ij}^{*-L} \times W^{int} \quad (48)$$

$$v_{ij}^{+L} = x_{ij}^{*+L} \times W^{int} \quad (49)$$

$$v_{ij}^{-U} = x_{ij}^{*-U} \times W^{int} \quad (50)$$

$$v_{ij}^{+U} = x_{ij}^{*+U} \times W^{int} \quad (51)$$

Where  $v_{ij}^{-L}$ ,  $v_{ij}^{+L}$  are lower approximation and  $v_{ij}^{-U}$ ,  $v_{ij}^{+U}$  are the normalized upper approximation of weighted normalized.

**Step 4:** Calculate the overall ratings of beneficial and non-beneficial criteria for each alternative. For beneficial criteria, the overall ratings of an alternative for lower and the upper approximation values of the rough intervals are computed as follows:

$$(S_i^+)^{-LAI} = \sum_{j=1}^n v_{ij}^{-L} \quad (j \in j^{max}), \quad (52)$$

$$(S_i^+)^{+LAI} = \sum_{j=1}^n v_{ij}^{+L} \quad (j \in j^{max}), \quad (53)$$

$$(S_i^+)^{-UAI} = \sum_{j=1}^n v_{ij}^{-U} \quad (j \in j^{max}), \quad (54)$$

$$(S_i^+)^{+UAI} = \sum_{j=1}^n v_{ij}^{+U} \quad (j \in j^{max}). \quad (55)$$

Where  $(S_i^+)^{-LAI}$ ,  $(S_i^+)^{+LAI}$ ,  $(S_i^+)^{-UAI}$  and  $(S_i^+)^{+UAI}$  are the benefit values of lower and upper approximation intervals.

**Similarly**, the overall rating of non-beneficial criteria  $(S_i^-)^{-LAI}$ ,  $(S_i^-)^{+LAI}$ ,  $(S_i^-)^{-UAI}$  and  $(S_i^-)^{+UAI}$ , or more precisely its lower and upper approximation intervals, are calculated using the following formulas:

$$(S_i^-)^{-LAI} = \sum_{j=1}^n v_{ij}^{-L} \quad (j \in j^{min}), \quad (56)$$

$$(S_i^-)^{+LAI} = \sum_{j=1}^n v_{ij}^{+L} \quad (j \in j^{min}), \quad (57)$$

$$(S_i^-)^{-UAI} = \sum_{j=1}^n v_{ij}^{-U} \quad (j \in j^{min}), \quad (58)$$

$$(S_i^-)^{+UAI} = \sum_{j=1}^n v_{ij}^{+U} \quad (j \in j^{min}). \quad (59)$$

Where  $(S_i^-)^{-LAI}$ ,  $(S_i^-)^{+LAI}$ ,  $(S_i^-)^{-UAI}$  and  $(S_i^-)^{+UAI}$  are the cost values of lower and upper approximation intervals.

**Step 5:** Calculate the overall performance index  $S_i$ , for each alternative. As a result of performing the previous steps. For this, the crisp values of the overall performance ratings obtained on the basis of rough intervals beneficial and non-beneficial for each alternative, can be calculated using the following formula:

$$S_i^* (S_i^{UAI}, S_i^{LAI}) = \frac{1}{4} [ ((S_i^+)^{-LAI} - (S_i^-)^{-LAI}) + ((S_i^+)^{+LAI} - (S_i^-)^{+LAI}) + ((S_i^+)^{-UAI} - (S_i^-)^{-UAI}) + ((S_i^+)^{+UAI} - (S_i^-)^{+UAI}) ] \quad (60)$$

Where  $S_i^*$  crisp valued overall performance index for each alternative.

**Step 6:** Ranking of the alternatives are obtained by ranking  $S_i^*$  values in descending order, the alternative with the highest overall performance index is the best choice.

## 4 A NUMERICAL EXAMPLE WITH ROUGH INTERVAL

We consider a numerical example in order to explain the proposed approaches. The numerical example, presented in, By using the set of five criteria :  $c_1$ ,  $c_3$ ,  $c_4$  and  $c_5$  are the beneficial type and  $c_2$  is the cost or non-beneficial type criteria, together with the set of nine alternatives  $A_1$ ,

A<sub>2</sub>, A<sub>3</sub>, A<sub>4</sub>, A<sub>5</sub>, A<sub>6</sub>, A<sub>7</sub>, A<sub>8</sub>, A<sub>9</sub>. The data are tabulated in Table 1.

**TABLE 1. ROUGH INTERVAL DECISION MAKING MATRIX**

Optimization Alternatives	Criteria									
	max		min		max		max		max	
	c <sub>1</sub>		c <sub>2</sub>		c <sub>3</sub>		c <sub>4</sub>		c <sub>5</sub>	
	UAI	LAI	UAI	LAI	UAI	LAI	UAI	LAI	UAI	LAI
A <sub>1</sub>	[[130,350]:[160,250]]		[[150,240]:[180,220]]		[[230, 420]:[270, 380]]		[[30,200]:[70,160]]		[[300,500]:[340,450]]	
A <sub>2</sub>	[[150,290]:[180,250]]		[[160,300]:[200,250]]		[[320, 600]:[430, 570]]		[[200, 460]:[260,400]]		[[700,1000]:[800,900]]	
A <sub>3</sub>	[[130,340]:[160,300]]		[[150, 340]:[180,300]]		[[400, 700]:[450, 680]]		[[150,480]:[200,400]]		[[500,1200]:[590,1000]]	
A <sub>4</sub>	[[190,360]:[240,330]]		[[200,350]:[220,320]]		[[460,750]:[520,700]]		[[250,550]:[350,490]]		[[400,700]:[500,650]]	
A <sub>5</sub>	[[180,340]:[220,320]]		[[180,400]:[230,350]]		[[500,800]:[580,770]]		[[350,620]:[450,580]]		[[600,1400]:[800,1300]]	
A <sub>6</sub>	[[500,680]:[540,650]]		[[140,350]:[200,300]]		[[900,1350]:[960,1300]]		[[450,850]:[550,750]]		[[1300,1780]:[1450,1650]]	
A <sub>7</sub>	[[550,800]:[630,750]]		[[230,500]:[270,360]]		[[1200,1700]:[1400,1650]]		[[700,1200]:[850,1150]]		[[2000,2550]:[2250,2500]]	
A <sub>8</sub>	[[600,900]:[650,830]]		[[200,400]:[250,350]]		[[1000,1400]:[1150,1300]]		[[550,860]:[590,790]]		[[1100,1400]:[1150,1300]]	
A <sub>9</sub>	[[100,280]:[150,250]]		[[130,240]:[160,210]]		[[300,600]:[400,560]]		[[300,520]:[360,480]]		[[500,800]:[570,730]]	

Source: R. Kumar et al. [18]

**4.1 SOWIA Method Computations**

First, applying the SOWIA method to determine the weights of criteria.

The steps of the SOWIA method:

1. To begin with this method, we prepare the normalized decision matrix from the original decision matrix.
2. Determine the objective weights (O<sub>j</sub>) of the attributes by the Entropy method.

**TABLE 2. OBJECTIVE WEIGHTS OF CRITERIA**

Criteria	c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>	c <sub>4</sub>	c <sub>5</sub>
E <sub>j</sub>	0.935552	0.992277	0.947448	0.94547	0.940585
G <sub>j</sub>	0.064448	0.007723	0.052552	0.05453	0.059415
∑ G <sub>j</sub>	0.238669	0.238669	0.238669	0.238669	0.238669
O <sub>j</sub>	0.270032	0.032357	0.22019	0.228476	0.248945

3. Determine the Subjective weights (S<sub>j</sub>) of importance of attributes by reciprocal weights method (RR):

- a. The importance ranking of the criteria.

**TABLE 3. RANK THE IMPORTANCE OF THE CHANGES IN THE CRITERIA FROM THE BEST LEVELS**

Criteria	Importance ranking
c <sub>3</sub>	1
c <sub>4</sub>	2
c <sub>1</sub>	3
c <sub>5</sub>	4
c <sub>2</sub>	5

b. Calculate the RR weight value for each criterion.

**TABLE 4. SUBJECTIVE WEIGHTS OF CRITERIA**

Criteria	c <sub>3</sub>	c <sub>4</sub>	c <sub>1</sub>	c <sub>5</sub>	c <sub>2</sub>
S <sub>j</sub>	0.437956	0.218978	0.145985	0.109489	0.087591

4. Combined the objective and subjective weights W<sub>j</sub><sup>int</sup>

W<sub>j</sub><sup>int</sup> (α = 0.5) As α can be assigned value 0.5

$$W_j^{int} = \alpha \cdot O_j + (1 - \alpha) \cdot S_j$$

**TABLE 5. INTEGRATED WEIGHT ( $W_j^{int}$ ) USING SOWIA METHOD**

Criteria	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$
$O_j$	0.270032	0.032357	0.22019	0.228476	0.248945
$S_j$	0.145985	0.087591	0.437956	0.218978	0.109489
$W_j^{int}(\alpha=0.5)$	0.208009	0.059974	0.329073	0.223727	0.179217

**4.2 RIARAS computations**

Applying the further steps of the RIARAS method. Rough optimality function ( $S_i^R$ ) for each of the alternative is

calculated. Then the corresponding values of the utility degree ( $K_i$ ) are determined for all the alternatives.

**TABLE 6. RANKING USING RIARAS METHOD**

Alternatives	$S_i^R$	$S_i$	$K_i$	Rank
$A_0$	[0.115647, 0.281967]:[0.143297, 0.229027]	0.192485	1	
$A_1$	[0.021584, 0.07888]: [0.028876, 0.055671]	0.046253	0.240294	9
$A_2$	[0.035173, 0.108537]: [0.047652, 0.082415]	0.068444	0.355583	8
$A_3$	[0.032433, 0.122737]: [0.041656, 0.0927]	0.072382	0.376039	6
$A_4$	[0.038637, 0.118142]: [0.051355, 0.09223]	0.075091	0.390114	5
$A_5$	[0.045014, 0.138001]: [0.059892, 0.109702]	0.088152	0.457969	4
$A_6$	[0.083461, 0.212649]: [0.100023, 0.16728]	0.140853	0.731763	3
$A_7$	[0.111567, 0.268951]: [0.140053, 0.220178]	0.185187	0.962089	1
$A_8$	[0.091011, 0.219078]: [0.108068, 0.172202]	0.14759	0.766761	2
$A_9$	[0.033979, 0.110454]: [0.046566, 0.085146]	0.069036	0.358657	7

Then the final ranking of alternatives is obtained with ARAS-RI method as  $A_7 > A_8 > A_6 > A_5 > A_4 > A_3 > A_9 > A_2 > A_1$  and the best alternative is  $A_7$  and the worst alternative is  $A_1$ .

**4.3 RIMOORA Computations**

By applying the procedure of a Deterministic Model for solving (RIMOORA). In Table 7, the beneficial, non-beneficial, and composite scores are listed for all alternatives. The  $A_7$  alternative should be selected because it has the maximum composite score.

**TABLE 7. RANKING USING RIMOORA METHOD**

Materials	$y_i^R$	$y_i$	Rank
$A_1$	[0.057116, 0.141374]:[ 0.071944, 0.114373]	0.096202	9
$A_2$	[0.113377, 0.209747]:[ 0.141938, 0.190451]	0.163878	7
$A_3$	[0.10448, 0.239811]:[ 0.124386, 0.213402]	0.17052	6
$A_4$	[0.126076, 0.232984]:[ 0.158267, 0.213733]	0.182765	5
$A_5$	[0.1543, 0.276573]:[ 0.190199, 0.263067]	0.221035	4
$A_6$	[0.305798, 0.452273]:[ 0.335951, 0.425197]	0.379805	3
$A_7$	[0.412873, 0.588952]:[ 0.48004, 0.576049]	0.514478	1
$A_8$	[0.332978, 0.471699]:[ 0.363819, 0.437557]	0.401513	2
$A_9$	[0.108651, 0.210682]:[ 0.138325, 0.194363]	0.163005	8

Then the final ranking of alternatives is obtained with MOORA - RI method as  $A_7 > A_8 > A_6 > A_5 > A_4 > A_3 > A_2 > A_9 > A_1$  and the best alternative is  $A_7$  and the worst alternative is  $A_1$ .

**4.4 RIOCRRA computations:**

By applying the procedure of two conventional interval number models that is a lower and upper approximation for solving (RIOCRRA) method.

**TABLE 8. COMPUTATIONAL DETAILS AND RANKING USING UPPER APPROXIMATION AND LOWER APPROXIMATION OF THE RIOCRRA METHOD**

Alternatives	$P_i^{UAI}$	$P_i^{LAI}$	$P_i$	Rank
$A_1$	(-5.75857, 5.758568)	(-2.02896, 2.028963)	0	9
$A_2$	(-5.35465, 8.124347)	(-1.43346, 3.257299)	1.148384	8
$A_3$	(-5.44444, 8.644669)	(-1.61459, 3.52091)	1.27663725	6
$A_4$	(-5.27933, 8.981143)	(-1.31922, 3.690048)	1.51816025	5
$A_5$	(-5.08437, 9.937442)	(-1.03316, 4.373024)	2.048234	4
$A_6$	(-4.15798, 13.39229)	(-0.02486, 6.215672)	3.8562805	3
$A_7$	(-3.3927, 17.17128)	(1.152016, 8.481165)	5.85294025	2
$A_8$	(1.780913, 13.74133)	(2.182338, 6.389897)	6.0236195	1
$A_9$	(-5.35229, 8.445362)	(-1.42474, 3.426184)	1.273629	7

Then the final ranking of alternatives is obtained with RI OCRA method as  $A_8 > A_7 > A_6 > A_5 > A_4 > A_3 > A_9 > A_2 > A_1$  and the best alternative is  $A_8$  and the worst alternative is  $A_1$ .

#### 4.5 Results of the proposed Methods

**TABLE 9. THE FINAL RESULTS OF THE PROPOSED METHODS**

Alternatives	RI OCRA	RI MOORA	RI ARAS
$A_1$	9	9	9
$A_2$	8	7	8
$A_3$	6	6	6
$A_4$	5	5	5
$A_5$	4	4	4
$A_6$	3	3	3
$A_7$	2	1	1
$A_8$	1	2	2
$A_9$	7	8	7

The proposed methods such RI OCRA, RI MOORA, and RI ARAS are proposed to solve MCDM problems with rough interval data are very simple to understand and easy to implement; the proposed methods clearly outperform the other MCDM methods. From the above table, it is evident that RIMOORA and RIARAS are providing an almost similar set of rankings for the alternatives. However, the

ranking obtained by RIOCRA varies from the other two methods due to its concept behind. The concept of competitive analysis of OCRA is the main reason of this variation of rankings. Because of disagreement in the ranks obtained by the proposed MCDM methods a final ranking based on REGIME method was used.

#### 4.6 Discussions

**TABLE 10. COMPARISON OF THE PROPOSED METHODS**

MCDM methods	Computational Time	Flexibility	Mathematical calculations required	Simplicity	Problem structure	Concept	Method using
OCRA method	Less	High	Moderate	Simple	Many criteria and alternatives	Aggregating function	- Large scale data -Complex rough interval MCDM problems
MOORA method	Very less	Very high	Minimum	Very simple	Many criteria and alternatives	Aggregating function	-Large scale data -Complex rough interval MCDM
ARAS method	Less	High	Moderate	Simple	Many criteria and alternatives	Aggregating function	-Large scale data -Complex rough interval MCDM problems

Compared with the different traditions MCDM methods, the proposed methods such as OCRA, MOOSRA and ARAS are less computational time, very simple and stable, minimum mathematical calculations involved and easy to implement. The calculation algorithm of the methods are not complicated, being implemented either without the help of a computer or by applying very simple computer programs. The proposed methods are also a newly proposed MCDM method and successfully are applied to rough interval under incomplete data, the calculation processes are always effective when a new alternative is added or removed. The proposed methods are suitable for large-scale data and relatively simple, the proposed methods are applicable when large numbers of alternatives and criteria are involved because the proposed methods are direct and causes no complication in calculation despite the large-scale data. Therefore, calculations using the proposed methods principles are easy to perform and implement, so that decision

making can be more flexible.

#### 4.7 REGIME method

Combines the final ranking results of the three MCDM methods in Table 11. by using REGIME method. The only difference is in place of criteria used in REGIME, we are considering the three MCDM methods as the criteria. The main objective behind doing so is to obtain a composite final rank that combines the ranking results of all the other methods. In this case, the assigned equal weightage to the criteria for the calculation of the concordance index  $C_{ij}$ . Therefore, for each criterion, is taken as  $\frac{1}{3}$ . The paired comparison matrix  $V_{ij}$  for composite rank calculation is shown in Table 11. The final ranking obtained by this method is noted in Table 12.

**TABLE 11. PAIRED COMPARISON MATRIX**

Alternatives	A1	A2	A3	A4	A5	A6	A7	A8	A9
A1	0	-1	-1	-1	-1	-1	-1	-1	-1
A2	1	0	-1	-1	-1	-1	-1	-1	-1
A3	1	1	0	-1	-1	-1	-1	-1	1
A4	1	1	1	0	-1	-1	-1	-1	1
A5	1	1	1	1	0	-1	-1	-1	1
A6	1	1	1	1	1	0	-1	-1	1
A7	1	1	1	1	1	1	0	1	1
A8	1	1	1	1	1	1	-1	0	1
A9	1	1	-1	-1	-1	-1	-1	-1	0

**TABLE 12. FINAL RANK OF ALTERNATIVES**

Alternatives	Total no. of positive elements (+1)	Final rank
A1	0	9
A2	1	8
A3	3	6
A4	4	5
A5	5	4
A6	6	3
A7	8	1
A8	7	2
A9	2	7

Then the final ranking of alternatives is obtained with REGIME method as  $A_7 > A_8 > A_6 > A_5 > A_4 > A_3 > A_9 > A_2 > A_1$  and the best alternative is  $A_7$  and the worst alternative is  $A_1$ .

## 5. CONCLUSION

This paper proposes three different methods based on rough interval technique to determine the best alternative among the various alternatives. The proposed methods based on the SOWIA technique with modification for each proposed method. An illustrative example is given to show the results and to compare the performance of the three methods. Hence the proposed methods are more practical, realistic, comprehensive and applicable for any rough interval multi-criteria decision making problems and very simple to understand and easy to implement in comparison to other methods; that are always complex to comprehend and sometimes difficult to implement and the calculation processes are always ineffective when a new alternative is added or removed. The decision maker can easily apply RIMOORA method because of its several advantages such as highly stable, simple and easy to implement decision making approach with less mathematical calculations over other two MCDM methods such as (RIOCR) and (RIARAS). The final ranking of alternatives is obtained by REGIME method.

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