PREFERENCE SOLUTION FOR LINEAR FRACTIONAL ASSIGNMENT PROBLEM

S. V. Gomathi, M Jayalakshmi

Abstract—Linear assignment problems (AP) arise in different situations to find an optimality to assign k-objects to m-other objects in a one to one manner. It is a combinatorial optimization. The assignment problems find a tremendous application of production planning, telecommunication, economics and so on. In view of this paper, a newly approach this is to say fractional-restriction method is proposed for linear fractional assignment problems (LFAP). Then it is transformed to two linear objective functions of the assignment problem so as to get the optimal solution. The solution procedure is illustrated with the numerical examples.

Index Terms—assignment problem, fractional-restriction method, linear fractional assignment problem, simplex method.

1 INTRODUCTION

An assignment problem is a particular type of a linear programming problem which consists of jobs to machines, drivers to motor vehicles, building construction to workers etc., to depute k number of persons into m number of jobs at a minimum cost (time). The elementary assumption of the AP is that one person can perform one job at a time. Kuhn [1] developed the Hungarian method to solve the AP. Dubois and Fortemps [2] proposed a flexible AP which combines with multiple criteria decision making and constraint-direct methodology. The area based on linear fractional programming problem (LFPP) was established by the Hungarian mathematician Matros [3]. McGinnis [4] implemented and testing a primal algorithm for AP. A competitive (dual) simplex method was proposed by Balinski [5] to solve assignment problem. Barr [6] et al., developed an alternating basis algorithm for finding assignment problem. The LFPP problem is an essential tool for the good old days. It is utilized for disparate discipline in engineering, business, economics and finance. LFPP is generally used to solve simulation of real time application problems with more than single objective function as the ratio of profit to the cost, ratio of inventory to the sales, ratio of actual cost to the standard cost and so forth. Later on several authors such as Bajalinov [7], Charnes and Cooper [8], Odior [9], Panday and Punnen [10] proposed different approaches for solving LFP problem. Sharma [11] et al. proposed the extension of simplex technique for solving LFP problem. Stancu-Minasian [12] discovered the theory, methods and applications of LFP problems. Pandian and Jayalakshmi [13] developed an algorithm for LFP by linear programming techniques.

Doke [14] proposed the algorithm to solve the multi objective fractional programming by Taylor’s series. Further he developed an algorithm to solve multi objective linear fractional transportation problem in which each objective function is expressed about optimal solution by Taylor’s series method.

Guzel and Sivri [15] proposed a solution to multi objective linear fractional programming problem by extending the objective function as the first order Taylor series at the optimal points. In this paper, we propose a new method namely, fractional restriction method for finding an optimal solution to the linear fractional assignment problem (LFAP). In this proposed method, we assume the two linear assignment problem from the given LFAP, one is of maximization type and the other one is of minimisation type. Then we obtained the optimal solution of the given LFAP from the solution of two assumed AP. The proposed method is established from the simplex method which differs completely from transformation method introduced by Charnes and Cooper [8] and the fractional method was introduced by Swarup [17]. In the fractional-restriction method, the assignment technique are not used, the optimal solution is obtained by using classical simplex method. Illustrated examples are given for better understand and the solution procedure of the proposed method.

2 PRELIMINARIES

2.1 Mathematical model of Assignment Problem

An Assignment problem is a special case of linear programming problem that deals with the allocation of the various resources to the various activities in one to one manner, hence it minimizes the time or maximizes the profit or sale. Here, \( c_{ij} \) represents the cost associated with machine \( i \) (\( i=1,2,\ldots,n \)) who has performed job \( j \) (\( j=1,2,\ldots,n \)).

The mathematical form of the assignment problem is

\[
\text{Maximise (or) Minimise } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \\
\text{Subject to } \\
\sum_{i=1}^{n} x_{ij} = 1, \forall j = 1, 2, 3 \ldots n \\
\sum_{j=1}^{n} x_{ij} = 1, \forall i = 1, 2, 3 \ldots n \\
& x_{ij} = 0 \text{ or } 1
\]

S V Gomathi Research Scholar, Department of Mathematics, School of Advanced Sciences, VIT University, Vellore-14, India.gomathi.sv@vit.ac.in

M Jayalakshmi Assistant Professor (Sr.), Department of Mathematics, School of Advanced Sciences, VIT University, Vellore-14, India. Corresponding Author: m.jayalakshmi@vit.ac.in

Corresponding Author: m.jayalakshmi@vit.ac.in

www.ijcst.org
Here $x_{ij}$ denote the assignment of $i$th resource to $j$th activity, such that

$$x_{ij} = \begin{cases} 1; & \text{if resources } i \text{ is assigned to activity } j \\ 0; & \text{otherwise} \end{cases}$$

### Table 1-Assignment Model

<table>
<thead>
<tr>
<th>Resources</th>
<th>$c_{ij}$</th>
<th>$c_{i1}$</th>
<th>$c_{i2}$</th>
<th>$c_{i3}$</th>
<th>$c_{i4}$</th>
<th>$c_{i5}$</th>
<th>$c_{i6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$...$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Availability</th>
<th>$c_{11}$</th>
<th>$c_{12}$</th>
<th>$c_{13}$</th>
<th>$c_{14}$</th>
<th>$c_{15}$</th>
<th>$c_{16}$</th>
<th>$c_{17}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$...$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_n$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2-Fractional Assignment Model

#### 2.2 Mathematical model of Fractional Assignment Problem

Fractional assignment problem is also an assignment problem where the objective function is a ratio of two assignment problem, numerator is of the same objective and its denominator is the reciprocal of the given objective function. The objective function is to minimize the total time in return the maximum product (or) maximise the profit in relate to minimize the job. Each fractional assignment problem is associated with the matrix with it and the number in the table indicates the time related to the product outcomes. Assume the problem of fractional assignment 'n' resources to 'n' activities so as to minimize the overall time in such a way that maximise the product. The general formation of fractional assignment problem is given as under:

**Maximise(or Minimise)** $(Z) = \frac{\sum_{j=1}^{n} c_{ij} y_{ij}}{\sum_{j=1}^{n} d_{ij} y_{ij}}$

**Subject to**

$$\sum_{j=1}^{n} x_{ij} = 1, \forall j = 1, 2, 3, ..., n$$  (1)

$$\sum_{i=1}^{m} x_{ij} = 1, \forall i = 1, 2, 3, ..., n$$  (2)

$x_{ij} = 0 \text{ or } 1$

The problem is to find the assignment $x_{ij}$ (which $i$th machine should be assigned to $j$th job) so that to minimize the total time to acquire the maximum product to get the optimality, where $c_{ij}$ the minimum is time of one unit from $i$th origin to $j$th destination and $d_{ij}$ is the maximum product of one unit from $i$th origin to $j$th destination.

Now, the above mathematical formulation of LFAP can be shown in the table as below:

### 3 FRACTIONAL-RESTRICTION METHOD

**Step 1:** Consider the Linear Fractional Assignment Problem $(Z)$

**Step 2:** Construct the two single objective linear assignment problem, the problem $(A)$ as well as the problem $(B)$ from the given problem $(Z)$ as:

**Step 3:** Obtain the optimality of the problem $(A)$ using simplex method. Let its optimal solution be $x_{ij}^{(A)}$ and substitutes the optimal values $(A)$ in $(Z)$

**Step 4:** To find an efficient solution for $(B)$, assume the optimal iteration $(A)$ as the initial iteration for $(B)$.

**Step 5:** Solve step 4 and check the optimality $(B)$

(i) If $P_{ij} - C_{ij} \geq 0$, stop the computation. Let its optimal solution be $x_{ij}^{(B)}$.

(ii) If $P_{ij} - C_{ij} \leq 0$, stop the computation. Let its optimal
solution be \( x_{ij}^P \).

Step 6: Substitute \( x_{ij}^P \) in the objective function \( Z \) in order to get the optimality of LFAP.

4 ILLUSTRATIONS

Illustration 4.1:
Let us consider that manufacturers of four kinds of products A1, A2, A3, A4. The products are manufactured from machines M1, M2 corresponding to the jobs J1, J2 with a time of 6, 7, 6, 8 per unit respectively. However the cost of the product of per unit of the above time is 0, 2, 1, 2. If the objective of this company is to minimise the time in return to the total product, provided that the company has a raw material for manufacturing. Assign the jobs for different machines so as to minimise the time with respect to maximum profit.

Solution:

The Mathematical Formulation of the above problem is:

Now, Min \( Z = \frac{x_{11} + x_{12} + x_{21} + x_{22}}{x_{11} + x_{12} + x_{21} + x_{22}} \)

Subject to:

\[ x_{ij} = 0 \text{ or } 1, \text{ for all } i, j \]

Min \( T = 13 \) and the value of \( Z \) is 13/3.

Now, by step 4 the initial iteration to the problem (B) is:

Since all \( P_{ij} - C_{ij} \geq 0 \) and by step 5(i):

The optimal solution to the linear fractional assignment problem is \( x_{12} = 1 \), \( x_{21} = 1 \) and Min \( \left( Z \right) = 13/3 \).

Illustration 4.2:

Consider the problem of manufacturers of four kinds of products A1, A2, A3, A4. The products are manufactured from machines M1, M2 corresponding to the jobs J1, J2 with a time of 0, 1, 1, 3 per unit respectively. However the cost of the product of per unit of the above time is 4, 6, 7, 9. If the objective of this company is to minimise the time in return to the total product, provided that the company has a raw material for manufacturing. Assign the jobs for different machines so as to minimise the time.

Solution:

The Mathematical Formulation of the above problem is:

Now, the step 2 of the proposed is given as:

Since all \( T_{ij} - C_{ij} \geq 0 \), the optimal solution (A) \( x_{ij} \) is

\( x_{12} = 1 \), \( x_{21} = 1 \).