# Reservoir Management Decisions Utilizing Markov Decision Processes

**William Schmidt, Andrew Ziskin, Anthony McHugh, Han-Suk Sohn**

**Abstract:** Dam and reservoir managers have a responsibility to find a balance of managing water levels while maximizing power outputs. If the water is too high, they risk damaging the generators. However, if it is too low, they will not be able to meet electrical and water requirements. In this paper, a Markov chain decision process is presented to determine the best management policy of a reservoir, which minimizes the opportunity cost of changing water levels with respect to power generation potential. The presented Markov decision model can determine the optimal release decisions based upon the defined states of a reservoir. It is concluded that managers should decide to release the maximum at all water level states, except when water level is at the medium state, the manager can choose either a minimum or maximum release.

**Index Terms:** Reservoir management, Markov chain, Markov decision process, Linear program, storage level, outflow quantity, risk management.

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## 1 INTRODUCTION

Dams and reservoirs are a means for large metropolitan and rural areas to create energy output, store water surpluses, provide irrigation and drinking water. Dams can prevent flooding during seasons of high rainfall or mountainous snow thaw. Reservoir operations are usually controlled by means of deliberate decisions concerning the release volume during a specific period of time. Reservoir managers and operators often rely on rule curves based on historical data and their own judgement and experience in making reservoir release decisions. Many reservoir systems are still managed based on fixed predefined rules. These rules must satisfy various constraints on the system to minimize overflow and/or maximize energy production while meeting water storage volume targets. A policy optimization is not required to operate reservoirs, but managers use optimization for planning and in practice [1]. As reservoirs become larger and integrate into systems involving multiple reservoirs, the need increases to optimize planning operations [2]. Using optimization techniques to assist in reservoir management can provide significant monetary benefits. The costs and operating expenses of many of these reservoirs are large enough that small enhancements in the system can save considerable amounts of money and resources. Operating rules obtained by simulations and Markov Decision Process (MDP) models have historically been used for the planning and operations of reservoir systems. When MDP solutions are compared to traditional rule curve solutions, Markov solutions have provided similar reliability with rule curve solutions with increased profits [1]. During the last several decades, lots of researches have been done describing optimization models and approaches for reservoir operation, but a consistent conclusive solution has yet to be found. A single method for solving reservoir operation problems has not been decided, however many studies attempt to optimize these decisions using stochastic dynamic programming and MDP [3, 4].


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These methods are preferred because reservoir management decisions are periodic and sequential. Reservoir operations are an industrial application that can be viewed as a MDP. A MDP can be applied to represent and potentially optimize a set of policy decisions to maximize, or minimize, a set of desired outcomes. MDP models can be utilized by reservoir managers for the long-term planning to obtain optimal policies for monthly release and storage targets. In this short paper, we present a simple yet practical optimization approach, which employs a concept of MDP, to determine the release criteria of reservoirs, which are a function of the system state variables; the beginning period storage level and decision of outflow quantity. Due to the number of variables and states within multi-reservoir functions this paper will consider the planning cycle of a single reservoir over a one-year period. The storage and inflow variables will be simplified by discretizing the storage volume and the water inflow quantities by using the probability of transition states from one time period to the next between a finite number of states. Ideally, our model could be applied to other reservoirs using similar characteristics. The optimization of this operation will be determining a set of optimal release decisions for the successive time periods so that long-term operations are maximized.

## 2 PROBLEM STATEMENT

The problem formulated in this study is, in a given year what is the optimal policy mix of water release amounts, given a current water level state, to maximize the potential power output of the dam? To develop this problem formulation we researched several aspects of dam and reservoir operations. Lots of previous studies have explored optimizing reservoir and dam operations through a Markov decision models. There are several ways the team could approach the assumptions, variables and states to develop a model capable of providing a viable policy solution. In the optimization of this reservoir operation, the release decisions are determined at the beginning of the month with the current reservoir water level and transition probability given that particular state. The current water level of the reservoir and the forecast of water intake are estimated in different ways depending on the study. Several studies employed a monthly or seasonal flow forecast of the current state water level variables. Water inflows into reservoirs are highly variable, but it can be modeled using a Markovian stochastic process. To simplify the model formulation, a stochastic variable was not used. We replaced an inflow variable with the transition probabilities of moving from one state to another based on historical water level data. That accounted for both inflow and outflow quantities, as well as secondary factors such as evaporation.In reservoir operations, it is necessary to hold enough water for operations, to produce hydroelectric energy or provide support to other dams when needed. However, too much water held up by a dam could cause a collapse or overflow which could lead to significant infrastructure and environmental damage. Stochastic optimization was the means for our team to reach an optimal policy, and so states and decisions had to be defined. The states of our system are correlated to the water level. It is similar to an inventory problem in that we used the volume of water to be our basis for the decisions. The operations support team must decide, based on the current water level, how much water to release to minimize the opportunity cost of power generation. We define opportunity cost as the difference between the maximum potential of value and the current state value. In other words, any state, or decision, that results in less power being created comes at a cost of lost revenue that would have been earned if the dam were operating at full capacity. These levels are: Minimum Release, 50% Release, and Maximum Release.In order to develop the Markov decision model the team made several assumptions due to the number of variables. The team investigated the stochastic nature of inflows based on historical data and identified the requirement to simplify the water level states and release decisions. It is concluded that a deterministic model would be the best approach given our resources and limited experience with dam and reservoir operations. Given additional time and resources, the team could create more specific states and water release decision amounts and incorporate variable increases in inflow and outflow rates to create a more accurate model. In this problem, many variables were considered but not quantified in our model. We did not have the opportunity to research into every impacting factor. Although we did look at water levels, we did not go into great detail on how sediment can impact reservoirs. Streams and rivers which flow into reservoirs can carry sediment which can impact water level measurements. Additionally, climate change could greatly impact dams through increased or decreased precipitation levels and surface temperature increases [5]. Evaporation is also a significant factor on reservoir water levels and is very difficult to quantify and predict.The goal of our Markov decision model is to represent the basic mechanics of a reservoir. The presented model provides an optimal policy to minimize operational cost. The solution to the problem required the decision policies, which kept operational costs to a minimum.

## 3 MARKOV DECISION MODEL FORMULATION

The data used in this study is from the Lake Powell water database, which includes extensive measurements dating back from 1963 [6]. This data includes daily contents of Lake Powell, water elevation levels, water inflow and outflow measurements as well as water temperatures. Note that Lake Powell reservoir is used for recreation, energy creation and delivering water to California, Arizona, and New Mexico via the Glen Canyon Dam [7]. The year of 2018 is used as the baseline to establish the proposed model and extrapolate the decision process for any given year. However, the model can be modified by taking the averages and inputs from any other year. There are several assumptions needed to make in order to establish this baseline dataset. The first assumption took the average monthly water level variation to establish each water level state. The water content on the first of the month was used as the water content level for that month (*m*). Refer to Table 1 for detailed information, where all of these measurements were in acre feet squared (afs). By taking the difference between the highest monthly water level ($m^{max}$) and the lowest monthly water level ($m^{min}$), the yearly variation in water levels (*b*) are calculated and then divided it by 12 to establish a monthly water level variation (*u*). In order to determine the probability of transition from one state to another, the changes (∆m) from the current month water content (mt) from the previous month water content (mt-1) was measured. It is established that if t>u then the state changed from a higher state to lower state. If -∆m <-u the state changed from a lower state to higher state. Otherwise, the state remained unchanged because this difference (∆m) did not exceed the average monthly water content variation (u) of 296,426 (afs). The water content from 01 January 2018 (mt-1) was 14,055,756 (afs) on 01 February 2018 (mt) the water content level was 13,662,840 (afs) this is a difference (∆m) of 392,916 (afs). Because ∆m exceeded the average monthly variation (u) of 296,426 (afs), in our model, the state changed from a higher state to a lower state. We did this for all 12 months of 2018 to establish the probability of the states changing on any given month (see Table 1).

***TABLE 1***

*WATER CONTENT VARIATION (2018)*

| Date of Measurement | Water Content (Afs) | Changes from previous month (Afs) | Change in state? |
|---|---|---|---|
| 1-Dec-17 | 14,322,262 | n/a | n/a |
| 1-Jan-18 | 14,055,756 | 266,506 | No |
| 1-Feb-18 | 13,662,840 | 392,916 | Yes |
| 1-Mar-18 | 13,335,432 | 327,408 | Yes |
| 1-Apr-18 | 12,948,951 | 386,481 | Yes |
| 1-May-18 | 12,658,966 | 289,985 | No |
| 1-Jun-18 | 12,899,134 | (240,168) | Yes |
| 1-Jul-18 | 12,711,184 | 187,950 | No |
| 1-Aug-18 | 12,095,911 | 615,273 | Yes |
| 1-Sep-18 | 11,458,588 | 637,323 | Yes |
| 1-Oct-18 | 11,016,261 | 442,327 | Yes |
| 1-Nov-18 | 10,860,384 | 155,877 | No |
| 1-Dec-18 | 10,498,645 | 361,739 | Yes |

After these calculations, it is determined that seven months out of the year the state would change from a high-level state to a lower level state (0.58). Four months out of the year the state would remain the same from one month to the next (0.34). One month out of the year the state would increase from a lower state to a higher state (0.08). The month of June did not have a $-\Delta_m$ <-u but because it was the highest water content level increased closest to *-u,* the team included it as a possibility of the state increasing to a higher state from one month to the next. At no point in the year did the state increase or decrease more than one state above or below its current state. To simplify the model into four states, we took each three-month period as one state, i.e., State 0 being the lowest state, State 1 being a medium level, State 2 being a high level and State 3 being an overflow state.

$$P_{ij} = \begin{bmatrix} 0.92 & 0.08 & 0 & 0 \\ 0.58 & 0.33 & 0.08 & 0 \\ 0 & 0.58 & 0.33 & 0.08 \\ 0 & 0 & 0.58 & 0.42 \end{bmatrix} \quad (1)$$

To account for the changes to the states based on our decisions, the team needed to vary the probabilities of transition based on those decisions. We established current state = C higher state = H and lower state = L. The baseline probabilities to transition to a higher state is 0.08, the probability to transition to a lower state is 0.58, and the probability to remain in the same state is 0.34. These transition probabilities were used for the decision to half release. It is assumed the water level will not exceed state 3 (Overflow) or go below State 0 (Low). If the current state is state 3, the H would be added to C, and L = 1 - (H+C). If the current state is in State 0, L would be added to C, and H = 1 - (L+C). This assumption is acceptable because these are the inflows observed for the entire year assuming a variable inflow and a half release will not necessarily increase or decrease the probability to transition between states.

$$P_{ij} = \begin{bmatrix} 0.84 & 0.16 & 0 & 0 \\ 0.16 & 0.68 & 0.16 & 0 \\ 0 & 0.16 & 0.68 & 0.16 \\ 0 & 0 & 0.16 & 0.84 \end{bmatrix} \quad (2)$$

The baseline probabilities H (0.08), L (0.58) and C (0.34) were adjusted for the minimum release decision. With a minimum release decision, the probability to increase state became 2H =0.16. The probability to remain the same became 2C=0.68 and the probability to move to a lower state became L = 1-(2H+2C)=0.16. If the current state is 3, the 2H would be added to 2C, and L = 1- (2H+2C). If the current state is 0, 2L would be added to 2C, and H=1– (2L+2C). Again, this assumption is acceptable because releasing less water implies that the water level may increase and be more likely to transition to a higher state assuming a variable inflow.

$$P_{ij} = \begin{bmatrix} 0.96 & 0.04 & 0 & 0 \\ 0.79 & 0.17 & 0.04 & 0 \\ 0 & 0.79 & 0.17 & 0.04 \\ 0 & 0 & 0.79 & 0.21 \end{bmatrix} \quad (3)$$

The baseline probabilities H (0.08), L (0.58) and C (0.34) were adjusted for the full release decision. With a full release decision, the probability to increase state became ½H =0.04, the probability to remain the same became ½C = 0.17 and the probability to move to a lower state became L = 1-(½H+½C) = 0.79. It is again assumed the water level will not exceed State 3 (Overflow) or go below State 0 (Low). Therefore, if the current state is 3, the ½H would be added to ½C, and L = 1 - (½H+½C), while if the current state is 0, ½L would be added to ½C, and H = 1 - (½L+½C). This assumption is acceptable because if more water is released, the water level is less likely to increase assuming a variable inflow. The water inflow and outflow is measured in cubic feet per second (cfs). The 2018 inflow and outflow rates were used to measure month-to-month variations. Additional factors besides inflow and outflow affect the total water content of the reservoir, to include evaporation. To simplify the model, it is decided to use the previous state as the starting input not an inflow variable. Outflow decision rates were based on the outflow rates grouped into three decision policies: minimum release, half release, and full release. Each of these decisions would affect the probabilities of moving from one state to another given the current state (see Figure 5). Note that the amounts of water (cfs) did not factor into the probabilities of transition.

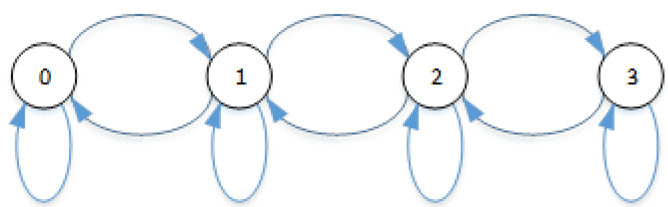


***Fig. 1**. Markov chain for the Reservoir*

***TABLE 2***

*COST ESTIMATION OF EACH STATE AND DECISION*

| State/ Decision | Modifying Coefficient (*yfq*) | Power generated per Month (MW) | Total Value of power generated ($) | Opportunity Cost ($) *G* =max *V* -*V* |
|---|---|---|---|---|
| Low/Min | ⅓(⅓*q*) | 12.21 | 84,249 | 674,751 |
| Low /Half | ½(⅓*q*) | 18.31 | 126,339 | 632,661 |
| Low/Full | ⅓*q* | 36.63 | 252,747 | 506,253 |
| Medium/Min | ⅓(½q) | 18.33 | 126,477 | 632,523 |
| Medium/Half | ½(½q) | 27.5 | 189,750 | 569,250 |
| Medium/Full | ½*q* | 55 | 379,500 | 379,500 |
| High/Min | ⅓*q* | 36.63 | 252,747 | 506,253 |
| High/Half | ½*q* | 55 | 379,500 | 379,500 |
| High/Full | *q* | 110 | 759,000 | 0 |
| Overflow | 0 | 0 | 0 | 759,000 |

Next, we present a linear model to determine the optimal decision policy given the aforementioned states and probabilities. The linear program is based on a transition matrix that showed the transition probability of moving from state (i) to state (j) given a decision (k). The cost of each decision is given a water level state (see Table 2) Note that the value of 1MW per month is assumed to be $6,900 USD. Hence, the linear programming model is to choose the $y_{ik}$ so as to

$$\text{Maximize} \sum_{i \in M} \sum_{k \in K} c_{ik} y_{ik} \quad (4)$$

This objective function is subject to the following constraints:

$$\sum_{i \in M} \sum_{k \in K} y_{ik} = 1 \ , \quad (5)$$

$$\sum_{k \in K} y_{ik} - \sum_{i \in M} \sum_{k \in K} y_{ik} p_{ij}(k) = 0 \quad , \forall j \in M \tag{6}$$

$$y_{ik} \geq 0 \quad , \qquad \forall i \in M, k \in K \tag{7}$$

Readers are refer to [8] for detailed information of this linear programming model formulation.

## 4 RESULTS AND DISCUSSION

The results indicate that managers should decide to release the maximum at all water level states, except when water level is at the medium state, the manager can choose either a minimum or maximum release. The expected monthly cost of $0.49M, and an expected yearly cost of $5.89M. These costs are only to the opportunity costs of power generation attributed to the water level states and the decision variables. We used discrete time Markov Chains to evaluate and determine the optimal solution to the operation of a water reservoir. The results appear to be consistent with a real-world application. The linear program shows, that to minimize cost, the decision maker should always try to maximize output at each state. Interestingly, if the water is at a low state, it would cost more to reduce the flow and get to a higher (more lucrative) state than it would be to stay in a low state and maximize the outflow. This is primarily due to the inflow state probabilities of the Colorado River. Further, this gives insight to the location of future dams, power generation capacity requirements, and risks of floods and droughts.This optimization method may not be applicable to other reservoirs due to the availability of data and/or the length of time data was collected. The water level and inflows for Lake Powell have been documented for over 55 years [9] enabling us to make assumptions to simplify and optimize reservoir operations. Furthermore, the decisions would change based on inflow rates and the probability to change states. In our model, there are very few high inflow months and many low inflow months. We acknowledge that there are many other variables that would add to the cost of operating a reservoir.

## 5 CONCLUSION

In this study, our goal was to demonstrate the applicability of Markov Chain processes to reservoir operations. Through our research and application we demonstrated that Markov Chains be used to improve to reservoir operations. We conclude that, with creative and critical thinking, there are many different ways to apply these processes to account for a myriad of variables that can affect reservoir costs, water levels, and decisions. Optimizing operations with variables that are at the mercy of the environment requires a significant amount of data so analysts can draw conclusions and make assumptions based on historical data. The challenge is determining how much data is needed to accurately extrapolate future predictions. Further research would include more detail in a cost/benefit analysis to include hydroelectric power, supporting drinking water to locals and other areas, possible impact for recreational activities, and any irrigation necessities. Although we held sediment levels constant, any reservoir would likely benefit from some further research into how sediment from the rivers and streams nearby might affect overall volume and inflow month by month or year after year. Additionally, climate changes continuously occur which might have further reaching impact. Climate changes would include precipitation and at some reservoirs how the water temperatures might affect release decisions. Finally, in relation to climate change, operations with excessive amounts of water including flash floods are worth further research [6].This research discussed refining the Markov Chain Decision model and created states. With enough time and money, a more detailed Markov process could be developed to account for the variables we were unable to quantify. Accounting for inflow with a stochastic variable based on the historical data will provide greater accuracy in predicting inflow probabilities and changes to states. Accounting for outflow quantities to include variables like evaporation will assist in more specific quantities of outflow decisions.

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