A Designed Paradigm for Contestant Quality Evaluation using Analytic Hierarchy Process

Alade O. A., Ganiyu R. A., Oladipupo E. T.

Abstract: Election of contestants into positions in a civilized society is a product of choice among few or many alternatives. In order to make a good choice among the available alternatives, a number of criteria must be considered. Several methods had been adopted in the past at local, national and international scene but in most cases with prejudice and biasness. These had subsequently produced contentious results which eventually led to political violence and insecurity. This paper proposes a multi-criteria decision making algorithm which is based on Analytic Hierarchy Process (AHP) for quality leadership selection, free of strive and violence. Alternative election contestants were considered; the required qualities for a post were defined. Their evaluations were translated into reciprocal matrix in order to determine the priority vector while the validity of the designed paradigm was assessed using secondary data. Out of the three alternative election contestants under consideration, the results of the designed paradigm show that the political aspirant Y is the best choice, followed by aspirant X and aspirant Z. That is, it can be inferred that aspirant Y is 3.87 times more preferable than choice Z, and choice Y is 1.3 times more preferable than choice X. Also, the obtained overall composite weight of 0.092 further proves that the result of the analysis is consistent. Thus, the practical application of the designed paradigm would eliminate an atmosphere of rancor, which may arise from unfair selection of contestants and decision making processes.

Index Terms: Analytic hierarchy process, consistency ratio, priority vector, pair-wise comparison matrix, random index

1 INTRODUCTION

In computing, solving a problem requires many preliminary works. On conceiving the idea of a solution to a problem, the solution is presented in algorithmic paradigm of the flow in a logical view of computer. In this paper, an algorithmic flow of a mathematical solution to electing quality leaders is presented to solve political crisis and insecurity. In election contests, it is the expectation of the electorates to have a leader that will care for the welfare at the grass root. This leader must possess some qualities of life that can offer these demands of the electorates. These qualities are not hidden from the electorates since the political office seekers are also members of the public. Since leaderships of groups, unions or and organizations greatly determine the growth and image of such associations and governments are very critical in electing leaders. In most cases, election is based on a number of criteria. These criteria must be evaluated among various alternative candidates who are aspiring to take the responsibility of such leadership. Election of a candidate among many alternative contestants is a multi-criteria decision-making (MCDM) process [1]. MCDM process has been widely used in various fields such as location selection, information project selection, material selection, management decisions, strategy selection, and problems relating to decision-making [2].

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Multi-criteria decision making methods are classified into two: discrete and continuous methods based on the nature of alternatives to be considered in a decision making process [3]. Continuous method deals with quantities which vary continuously in the decision problem, such method includes linear /goal programming and aspiration based model. On the other hand, discrete method has a finite number of alternatives, a set objective, criteria for evaluating alternatives and a method for ranking the alternatives [4]. Discrete technique is further divided into two: weighing technique, such as simple additive weighting (SAW) [5]; ranking techniques, such as Preference Ranking Organization METHod for Enrichment Evaluation (PROMETHEE)[6], Technique for Ordered Preference by Similarity to Ideal Solution (TOPSIS) [5] and Ordered Weighted Average (OWA); and mixed techniques such as the ELECTRE [7], the Analytic Hierarchy Process (AHP) [8], the Multi-Attribute Value Theory [9], and Value Focused Thinking (VFT) [10]. Discrete multi-criteria decision making analysis methods are most suitable for election of candidates into an office because the criteria for selection are discrete in nature with varying degree of intensity.

2 RESEARCH METHODOLOGY

2.1 Analytic Hierarchy Process (AHP)

Saaty Analytic Hierarchy Process (AHP) attempts to support multi-criteria decision analysis of decision variables. Thomas Satty created AHP in the late 1960's in order to determine the relative importance of each variable in the decision making matrix on a pair-wise basis [11]. The AHP deals with independences among variables or cluster of decision structure to combine the statistic and judgmental information [12]. The analytic hierarchy process is a popular and classical method of evaluation where properties are derived from eigenvalue of the pair-wise comparison matrix (PGM) of a set of elements expressed onto ratio scales. It is based on the well-defined mathematical structure of consistent matrices and their associated right eigenvector's ability to generate true or approximate weights. The AHP, among others uses the same approach as VFT; however the AHP adopts different approach in evaluating the weight and in assigning scores to alternatives. It means two elements at a same time can be compared. The values of the pair-wise comparison are the member of the set: {9,8,7,6,5,4,3,2,1, 1/2,1/3,1/4,1/5,1/6,1/7,1/8,1/9} – 9 represents absolute importance and 1/9 the absolute triviality. The simplicity and power of the AHP has led to its widespread use across multiple domains in every part of the world [13]. In general, this technique was developed to select the best from a number of alternatives with respect to several criteria [14]. In [15], the superiority of AHP to other MCDA methods was stressed. The Analytic Hierarchy Process (AHP) is based on the following axiom:

- **(i)** Decomposition of problem solution into criteria and other the associated alternatives
- **(ii)** Comparison matrix
- **(iii)** Reciprocal judgment
- **(iv)** Homogenous elements
- **(v)** Feedback dependent structure and
- **(vi)** Rank order expectations.

The AHP algorithm and pseudocode are as stated hereunder.

2.2 The AHP Algorithm

1. Read the decision variables
2. Produce the reciprocal matrix for the pair-wise comparison matrix (pwc)
3. Computer the eigen value
4. Compute the eigen vector
5. Compute the weight
6. Evaluate the consistency of the weight
7. Repeat steps 1 to 6 above for other nodes in the hierarchy
8. Compute the overall composite weight of the hierarchy
9. Deduce the best candidate among the alternatives

2.3 The AHP pseudocode

```
Loop i = 1 to n
    Read decision variables a1, a2,…, an
        Produce pwc
        Loop k = 1 to n-1
            If imp(n) > imp(n-1) then
                pwc(n →n-1) =r (r=2, …, 9)
            else
            if imp(n) = imp(n-1) then
                pwc(n → n-1) = 1
            else
                pwc(n → n-1) = 1
        endloop
endloop
‘ …. Compute eigenvalue and eigenvector
Loop i=1 to n
    Loop j = 1 to n
        Sum(j) = sum(a1j,a2j,a3j,…,anj)
        E(ij) = aij/sum(j)   (Eij = eigenvalue)
    endloop
    λmax = avg(ai1,ai2,ai3,…,αin)   (λmax = eigenvector)
endloop
loop d = 1 to p   (p is the number of alternative aspirants)
    Read decision variable rating, ri  (r = 1,2,3,4,5)
    compute wi = ∑riR(i)
endloop
    rank r = relative size of wi
return
```

2.4 Decomposition

As shown in Fig.1, Level 0 is the goal of the analysis. Level 1 represents the attributes of contestants. The last level 2 represents the alternative leaders. The lines between levels indicate relationships between the goal, the attributes, and the alternatives.

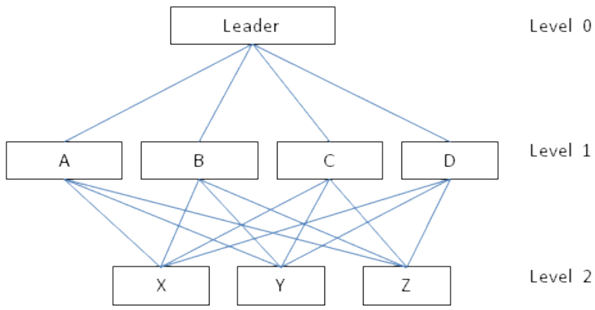

Fig.1: Hierarchical decomposition of leadership election problem

2.5 Pair-wise Comparison Matrix

Saaty's pair-wise comparison matrix was used to measure the preferences of the criteria which are the strength of the importance of the variable over one another. This is presented on an absolute scale as shown in Table 1. The comparison was done from top level of the hierarchy down in order to establish the priority index by transforming top diagonal matrix into reciprocal matrix to obtain the absolute values of the lower diagonal matrix.

Table 1: Pair-wise comparison matrix for level 1 with respect to the goal

Criteria	A	B	C	D
A	1.00	3.00	7.00	9.00
B		1.00	5.00	7.00
C			1.00	3.00
D				1.00

2.6 Reciprocal Matrix

Assuming we have a matrix M,

M(i,j) is the PCM with elements i and j (i=1,2,3,…,n; j=1,2,3,…,n)

$\{i, j\} \in n_k$ (n_k = node k of the AHP tree)

The larger the value of M(i,j) the more the element i is preferred to element j in the rating. In generating the reciprocal matrix in Table 2, the statements depicted by equations 1.1 and 1.2 were used:

$$M(i,j) = M(j,i)^{-1} \quad (1.1)$$

$$\text{If } M(i,j) = M(j,i), \text{ then } M(i,j) = M(j,i) = 1 \quad (1.2)$$

Based on our own paired comparison, we make several comparison matrices. The diagonal is always 1 and the lower triangular matrix is filled using the reciprocal equation 1.3.

$$m_{j,i} = \frac{1}{m_{i,j}}. \qquad (1.3)$$

Table 2: Reciprocal PCM for level 1

Criteria	A	B	C
A	1.00	3.00	7.00
B	0.33	1.00	5.00
C	0.14	0.20	1.00
D	0.11	0.14	0.33

The priority vector is obtained from normalized eigenvector of the matrix.

Table 3: The priority vector of the matrix

	A	B	C	D	Priority Vector
A	1.00	3.00	7.00	9.00	**57.39%**
B	0.33	1.00	5.00	7.00	**29.13%**
C	0.14	0.20	1.00	3.00	**9.03%**
D	0.11	0.14	0.33	1.00	**4.45%**
Sum	1.59	4.34	13.33	20.00	100.00%

$$\lambda max = (05739)(1.59) + (0.2913)(4.34) + (0.0903)(13.33) + (0.0445)(20) = 4.2692$$

2.7 Consistency

In AHP, priorities are derived from consistent or near consistent matrices. The consistency is calculated using the consistency index CI in equation 1.4.

$$CI = \frac{\lambda_{max} - n}{n-1} \qquad (1.4)$$

Therefore

$$CI = \frac{4.2692-4}{3} = 0.0897$$

The consistency ration is calculated using equation (1.5) which is the ratio of consistency index and consistency random index (a standard scale shown in Table **4**)

$$CR = \frac{CI}{RI} \qquad (1.5)$$

$$CR = \frac{0.0897}{0.90} = 9097\% < 10\%$$

λ_{max} =4.2692, CI = 0.0897, CR = 9.97% < 10% (acceptable)

Table 4: Random consistency index

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Table 5: Reciprocal matrix and priority vector for level 2 with respect to attribute A

A	X	Y	Z	Priority Vector
X	1.00	1.00	7.00	**51.05%**
Y	1.00	1.00	3.00	**38.93%**
Z	0.14	0.33	1.00	**10.01%**
Sum	2.14	2.33	11.00	100.00%

λ_{max} =3.104, CI = 0.05, CR = 8.97% < 10% (acceptable)

Table 6: Reciprocal matrix and priority vector for level 2 with respect to attribute B

B	X	Y	Z	Priority Vector
X	1.00	0.20	0.50	**11.49%**
Y	5.00	1.00	5.00	**70.28%**
Z	2.00	0.20	1.00	**18.22%**
Sum	8.00	1.40	6.50	100.00%

λ_{max} =3.088, CI = 0.04, CR = 7.58% < 10% (acceptable)

We can do the same for paired comparison with respect to criteria C and D. However, the weight of C and D are very small (See Table 3; they are approximately 9% and 5% respectively). Therefore the effect of leaving them out from further consideration is negligible. We ignore these two weights and set them as zero. So we do not use the paired comparison matrix level 2 with respect to alternatives C and D. In that case, the weight of alternatives A and B in Table 3 was adjusted so that the sum still maintains 100%.

Adjusted weight for attribute A = $\frac{57.39}{57.39+29.13} = 0.663$

Adjusted weight for attribute B = $\frac{29.13}{57.39+29.13} = 0.337$

Then, the overall composite weight of each of the alternatives based on the weight of levels 1 and 2 was computed. The overall weight is the normalization of linear combination of multiplications between weight and priority vectors. This is shown in Table 7.

X = 0.663(51.05) + 0.337(11.49) = 37.72%

Y = 0.663(38.93) + 0.337(70.28) = 49.49%

Z = 0.663(10.01)+0.337(18.22) = 12.78%

Table 7: Overall composite weight of the alternatives

	Alternative A	Alternative B	Composite Weight
(Adjusted) **Weight**	*0.663*	*0.337*	
Choice X	51.05%	11.49%	**37.72%**
Choice Y	38.93%	70.28%	**49.49%**
Choice Z	10.01%	18.22%	**12.78%**

The overall consistency of hierarchy can also be checked by summing up all levels, with weighted consistency index (CI) in the nominator and weighted random consistency index (RI) in the denominator. Overall consistency of the hierarchy above is given by equation 1.6.

$$\overline{CR} = \frac{\sum_i w_i CI_i}{\sum_i w_i RI_i} \qquad (1.6)$$

Where $\overline{CR}$ is overall consistency index of the hierarchy w_i is the adjusted weight of alternatives i (i=1,2,3,…,n); CI_i is the consistency index of level of the hierarchy RI_i is the random consistency index.

$$\overline{CR} = \frac{0.0897(1)+0.05(0.663)+0.04(0.337)}{0.90(1)+0.58(0.663)+0.58(0.337)} = 0.092 < 0.1$$

(Acceptable)

3 RESULTS AND DISCUSSION

In level 1, one comparison matrix corresponds to pair-wise comparisons between 4 criteria with respect to the goal. Thus, the comparison matrix of level 1 has size of 4 by 4 as depicted in Table 1 and Table 2. Because each alternative is connected to each criterion and we have 4 criteria and 3 alternatives, then in general we have 4 comparison matrices at level 2, each of these matrices has size 3 by 3. However, it is observed that some weight of level 2 matrices are too small to contribute to overall decision, thus we can ignore them. The results obtained in Table 7 showed that the political aspirant Y is the best choice, followed by X as the second choice and the worst choice is Z. The composite weights are ratio scale. It can be inferred that aspirant Y is 3.87 times more preferable than choice Z, and choice Y is 1.3 times more preferable than choice X. The overall composite weight of 0.092 also proved that the result of the analysis is consistent since it is less than 0.1. Therefore it is acceptable.

4 CONCLUSION

Most of the election violence experienced in developing countries before, during and after local and national elections can be avoided if pragmatic scientific approaches were employed with appropriate mathematical models. Loss of lives and material possessions and other forms of insecurities could have been averted. This paper proposed an AHP algorithmic paradigm for electing quality leaders into political offices. The algorithm was used to demonstrate the choice of most preferred aspirants based on some expected leadership qualities among three contesting aspirants. The consistency of the selected candidate was tested using some mathematical tools. It was observed that the selected candidate falls within the acceptable limit of the composite weight. Precisely, we can say that the requirement of consistency is the most critical issue in the practical application of AHP. The use of the balanced scale improves consistency, but it would be most helpful to have well defined, theoretically founded cut-off limits, independent from scales and priority derivation methods.

5 REFERENCES

[1]. Y. Chena, H. Lienb,G. Tzengc and L.Yange, "Fuzzy MCDM approach for selecting the best environment-watershed plan", pp. 1-13, 2009.

[2]. H. K. Chiou, G. H. Tzeng and D. C. Cheng, "Evaluating sustainable fishing development strategies using fuzzy MCDM approach". Elsevier , 33 (3), pp. 223–234, 2005.

[3]. S. A. Hajkowcz, G. T. McDonald and P. N. Smith, " An evaluation of multiple objective support weighting technique in natural resource management", Journal of Environmental Planning and Management , 43, pp.505-518.

[4]. J. Anada and G. Herath, " A critical review of multi-criteria decision making methods with special reference to forest management and planning", Ecological Economics , 68, pp. 2535-2548, 2009.

[5]. C. J. Hwang and P. Vincke, "Multi-attribute decision making methos and applications", Spingers , 86,1981.

[6]. J. P.Brans and P. Vinche, "A preference ranking organization method: the PROMETHEE method for MCDM", Management Sciences , 31 (6), pp. 647-656, 1985.

[7]. O. Roux and J. Elloy, "ELECTRE: a language using control structure expression to specify synchronization", Range of Computing Method: Mid 80s Perspectives, pp. 240-245, CO, USA: ACM, 1985.

[8]. T. L. Saaty, "Priorities in systems with feedback", International Journal of System Measurement and Decision, 1, pp. 24-38, 1981.

[9]. D.Von Winterfeldt and W. Edwaards, "Decision Analysis and Behavioural Research", Cambridge: Cambridge University Press, 1986.

[10]. R. I. Keeney, "Creativity in MS/OR: Value focussed thinking - creativity diredted towards dedision making", Interfaces , 23 (3), pp. 62-67, 1993.

[11]. T. L.Saaty, "A scaling method for priorities in hierarchical structure", Journal of mathematical psychology , 15, pp. 19-43, 1977.

[12]. F. E. Uzoka, O. Okure, K. Barker and J. Osuji, " An experimental comparison of fuzzy logic and analytic hierarchy process for medical decision support systems. Computer Methods and Programs in Biomedicine , 103, pp. 10-27, 2011.

[13]. Kristof, "Planning improvement using analytical hierarchy process and design structure matrix, Montana: Montana State Univeristy, 2005.

[14]. R. Mosadeghi, R. Tomlinson, H. Mirfenderesk and J. Warnken, "Coastal management issues in Queensland and application of the Multi- Criteria Decision Making techniques", Journal of Coastal Research , Special Issue , (56), pp. 1252 - 1256, 2009.

[15]. V. A. Salmonon and J. A. Montevechi, " A compilation of comparisons on the analytic hierarchy process and othe multi-criteria decision making methods: Some cases developed in Brazil", 6th ISAHP Conference, pp. 413-420, 2001, Berne, Switzerland.