Implementation Of An Active RC Band-Pass Filter At Varying Quality Factors Using Matlab

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ABSTRACT: In this paper, an active second order RC band-pass filter is designed and simulated at different values of quality factor Q. The filter is designed from given specifications of the filter, center frequency of 15 kHz and roll-off rate of -20dB/decade. The architecture that will be used is the Sallen-Key. The filter parameters and values for the passive components were calculated and the gain magnitude and phase response is then simulated with MATLAB. The simulation shows that at high Quality factors, the bandwidth of the filter response reduces considerably while its frequency selectivity increases without a shift in its center frequency.

Keywords: Band-pass filter, Quality factor, center frequency, MATLAB.

1 INTRODUCTION
A filter is defined as a network, which passes a certain portion of a frequency spectrum and blocks the remaining portion of the spectrum [10]. In modern electronic circuits today, unwanted signals are a major challenge to contend with. This is due to interferences, in the form of noise and harmonics which these unwanted signals pose to certain specified wanted signal frequencies in a band in electronic systems. Filter circuits are used in a wide variety of applications. In the field of telecommunication, band-pass filters are used in the audio frequency range (0 kHz-20 kHz) for modems and speech processing [1]. In state-of-the-art RF receivers, high performance filters are required to remove undesired signals at different stages of the receiving process, such as noise from incoming signals the antenna receives, undesired signals at the image frequency, and the effects of harmonics after the mixing operation which attenuates the desired signal [3]. The most common filter responses are the Butterworth, Chebyshev, and Bessel types. Among these responses, Butterworth type is used to get a maximally-flat response. Also, it exhibits a nearly flat pass band with no ripple. The roll-off is smooth and monotonic, with a low-pass or high-pass roll-off of 20dB/dec for every pole [3],[4],[6]. In this paper, a second order active band pass filter is designed to meet the specifications of different quality factors using MATLAB. In the second part of this paper, the implemented band-pass filter at different quality factors will be compared to ascertain the effect of change of quality factor on the band-pass filter frequency response.

2.1 Design Specification
The architecture that has been used to implement the second order band-pass filter is the Sallen-Key Topology. This topology is chosen due to its simplicity compared to other known architectures such as multiple feedback and state variable. [3] The circuit diagram below shows a second order Sallen-Key band pass filter.

![Fig 1: Second Order Sallen-Key Band-pass filter with Gain (A) >1.](image)

Table 1 illustrates the specifications for the desired band-pass filter. By using the following filter parameters, the required filter characteristic is simulated with MATLAB.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center frequency</td>
<td>15kHz</td>
</tr>
<tr>
<td>Stop band attenuation</td>
<td>-20dB</td>
</tr>
<tr>
<td>Pass band ripple</td>
<td>0.1dB</td>
</tr>
<tr>
<td>VPP</td>
<td>10mV</td>
</tr>
</tbody>
</table>

2.2 Design Implementation
The general transfer function of a second order band pass filter is given as:

\[ H(s) = \frac{a_1 s}{s^2 + b_1 s + b_0} \]  \hspace{1cm} (4)

Where \(a_1\), \(b_1\) and \(b_0\) are constants.
The transfer function of the second order Sallen-Key band pass filter in Fig. 1 is:

$$H(s) = \frac{sC_4R_4R_5G}{s^2C_4C_5R_4R_5 + s[C_4R_4R_5(1 - G) + C_5R_5 + C_4R_2 + C_1R_1] + (R_1 + R_2)}$$

The above equation can be simplified by setting

$$R_1 = R_2 = R, R_5 = 2R, C_1 = C_2 = C$$

this is known as the equal component Sallen-Key band-pass filter [3]:

$$H(s) = \frac{sCRG}{s^2C^2R^2 + sCR(3 - G) + 1}$$

(5)

Resonant frequency: $$f_o = 1/2\pi CR; \text{or} \omega_o = 1/CR$$

Bandwidth: $$\beta = f_o/Q;$$

Op-amp gain: $$G = 1 + R_4/R_5;$$

Gain at $$\omega_o:$$ $$A = G/(3-G);$$

Quality factor: $$Q = 1/(3-G);$$

The poles of the transfer function are:

$$S_{1,2} = [(G - 3)/RC \pm \sqrt{((3 - G)/RC)^2 + 4(1/RC)^2})] / 2$$

From equation (4) it can be seen that the quality factor is governed by the term G (op-amp gain). Hence, the quality factor (Q) can be varied via the inner gain (G) without modifying the center frequency. A drawback is, however, that Q and A (gain at center frequency) cannot be adjusted independently. When G approaches the value of 3, A and Q tend to infinitely increase and cause the circuit to oscillate. As G grows greater than 3, Q becomes negative too, thus from equation (4), the s – coefficient is equal to zero indicating that H(s) has right half-plane poles [7]. This problem can be solved by adding an input buffer that gives a gain adjustment so as to just compensate for a Q adjustment with a change in the input gain. This will increase the number of components used for design and construction thereby raising the cost of the circuit. Substituting, the above transfer function can be written in standard form as:

$$H(s) = \frac{sA(\omega_o/Q)}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

(6)

Let the design components be

$$C = 10nF, R_5 = 10k, R_4 = 10k, f_o = 15kHz$$

$$R = 1/(2\pi^215000 * 10 \times 10^{-9}) = 1.06k$$

$$G = 1 + (10/10) = 2$$

$$A = 2/(3 - 2) = 2$$

$$Q = 1/(3 - 2) = 1$$

Bandwidth = $$f_o/Q = 15 kHz/ 1 = 15 kHz$$

The transfer function:

$$H(s) = \frac{188520 s}{s^2 + 94260s + 8.88 E09}$$

The Quality factor can be varied by varying the gain G through the resistor R_4 or R_5. In this paper, R_3 is chosen to be constant at 10k, while R_4 is chosen as the resistor to be varied. Thus, the different values of Q will be obtained for $$R_4 = 10k, 16k, 17.5k, 18.75k, 19k, 19.9k$$ from the table below:

<table>
<thead>
<tr>
<th>R_4 (kΩ)</th>
<th>10</th>
<th>16</th>
<th>17.5</th>
<th>18.75</th>
<th>19</th>
<th>19.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>1.0</td>
<td>2.5</td>
<td>4</td>
<td>8</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>

By using the transfer function, the frequency response of the filter at varying Q’s can be plotted using MATLAB to verify the design.

3 RESULTS

The results of MATLAB simulation for the second order Active-RC band-pass filter is shown in Fig. 2 below

![Fig 2: Gain response of a second order band-pass filter for different Q (quality factor).](image)

From Fig 2 above,

$$\omega_o = 94.3 rad/s \quad \omega_0 = 2\pi f_o$$

$$f_o = 94.3/ 2*3.142 = 15kHz$$

The filter response in Fig. 2 has pass-band frequencies 9.28 kHz and 24.27 kHz at Q = 1 and roll-off rates of -20dB/dec.
TABLE 3: BANDWIDTH OF 2ND ORDER BANDPASS FILTER AT Fc = 15K WITH VARYING Q

<table>
<thead>
<tr>
<th>Quality Factor (Q)</th>
<th>FL at -3dB (Hz)</th>
<th>FH at -3dB (Hz)</th>
<th>Bandwidth (Hz)</th>
<th>Center Freq. (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.28k</td>
<td>24.27k</td>
<td>14.99k</td>
<td>15.00k</td>
</tr>
<tr>
<td>2.5</td>
<td>12.30k</td>
<td>18.32k</td>
<td>6.02k</td>
<td>15.00k</td>
</tr>
<tr>
<td>4</td>
<td>13.25k</td>
<td>16.99k</td>
<td>3.74k</td>
<td>15.00k</td>
</tr>
<tr>
<td>8</td>
<td>14.12k</td>
<td>15.99k</td>
<td>1.87k</td>
<td>15.00k</td>
</tr>
<tr>
<td>10</td>
<td>14.28k</td>
<td>15.77k</td>
<td>1.49k</td>
<td>15.00k</td>
</tr>
<tr>
<td>100</td>
<td>14.93k</td>
<td>15.09k</td>
<td>160</td>
<td>15.00k</td>
</tr>
</tbody>
</table>

The graph in Fig. 2 and Table 3 shows that the frequency response of second-order band-pass filters gets steeper with rising Q, thus making the filter more selective. At Q = 1, the bandwidth of the filter is noted to be 14.99 kHz, at Q = 10, bandwidth is 1.49 kHz and at Q = 100, the bandwidth is 160 Hz. This result further provides facts that as the quality factor of a band pass filter increases, its bandwidth decreases thereby making the filter more selective.

4 CONCLUSION
A second order band-pass filter is designed and simulated for different values of Quality factor using MATLAB. As the simulated results satisfy the filter requirements, it does provide an insight into the fact that as the quality factor of a Sallen-Key band pass filter increases, the filter selectivity increases thus drastically narrowing its bandwidth. This fact is extracted from the simulated result of Fig. 2 above.

5 REFERENCES