Analysis Of Formation Damage During The Drilling Of Horizontal Wells

I. Salaudeen, S. O. Isehunwa, D.A. Dauda, I. Ayuba

Abstract: Numerous studies have been carried out on Formation damage in vertical wells but less investigation was conducted in horizontal wells, this, therefore necessitates development of an approximate filtration theory through a compressible cake medium. A mathematical model was developed to describe the mechanisms and processes of filtration into the Formation through a compressible filter cake during drilling of horizontal wells. Goodman's integration technique was adopted for the solution, taking into consideration the effects of non-linearity and gravity. The result shows that the Cake thickness buildup at different values of constant of non-linearity is very small, also, depth of invasion was observed to increase at the beginning of drilling and reduced as time went by. The rapid decline in Solid pressure indicates more damage in horizontal wells than other models used for vertical wells. An approximate analytical solution combining both the steady and transient components has been successfully applied to solve the filtration theory equation developed for horizontal wells. The model is easily applicable to estimate important values needed to know the status of the formation during the drilling of horizontal wells.

Index Terms: diffusivity equation, filter-cake, filtration, heel, horizontal well, skin, solid pressure, thin reservoir, throughput

INTRODUCTION

Formation damage can be defined as any type of process which results in reduction of the flow capacity of an oil, water or gas bearing formation. This flow reduction in the reservoir can occur during drilling, completion or even workover operations. It is a zone of reduced permeability within the well vicinity of the wellbore because of foreign fluid or solid particles invasion into the reservoir rock. The additional pressure drop in this near wellbore region is termed "skin". Horizontal drilling is a special application of directional drilling, which involves directing a well laterally through a productive interval. In certain types of reservoirs, horizontal wells work to best advantage in thin reservoirs having a relatively high ratio of vertical to horizontal permeability (vertically fractured formations are prime candidates) and a potential for drawdown-sensitive production problems like water and gas coning. Other common candidates for horizontal drilling are: reservoirs that would otherwise be economically inaccessible, heavy oil reservoirs, channel sand and reef core reservoirs and coal bed methane reservoirs etc. Horizontal wells can trace their roots back to at least the 1930s (Ranny). But it was just in the 1980s that advances in directional drilling and formation evaluation have brought them into the mainstream of oil and gas operations. Since then, in a number of fields, they have significantly outperformed conventional wells in terms of increased productivity and lower overall development costs. Outmans (1963) was the first investigator to publish analytical approach to the filtration problem using theoretical – empirical non-linear diffusion equation. Some degree of non-linearity considered to have negligible effects on the final accuracy and he came up with the conclusion that the dynamic rate of filtration decreases when the drilling fluids becomes less viscous and also if the rate of circulation is reduced. Isehunwa (1982) developed an approximate analytical solution to the mathematical model using Goodman's integration technique as the method of approach to the solution.

- *S. O. Isehunwa, D.A. Dauda, I. Ayuba*
- *Ibraheem Salaudeen is currently an Assistant Lecturer at Abubakar Tafawa Balewa University Bauchi Nigeria.*
- *E-mail: 16763is@gmail.com, dikkodauda2002@yahoo.com, da.dikko@mail.ui.edu.ng*

Volume of filtration and cake build up were greatly affected by transient effects during filtration. The findings finally showed that viscosity affects volume of filtrate and should be controlled to prevent or minimized damage to the barest minimum. Barry (1980) developed a model used to estimate a depth of filtrate invasion where logs fail to yield this value and can also be used to examine the sensitivity of the variables in the filtration theory. However, the model seems too simple Mud properties, cake parameters and time were identified as major factors that control drilling mud filtration. "Isehunwa, (2012)" A mathematical model for analyzing mud filtration in deviated wells was developed by "Akinsete and Isehunwa, (2013)". The study determined extent of invasion under different conditions. Finally, numerous findings have been conducted on formation damage in horizontal wells but effect of length and the extent of damage at the heel part are yet to be properly considered and modeled. This model adopts the techniques used by "Isehunwa, (2012)" considering effect of gravity.

THEORY AND MATHEMATICAL FORMULATION

Whenever a liquid drilling fluid is used for hole cleaning and to serve other important functions during drilling of oil or gas wells, be it oil or water based mud, some invasions of the surrounding formations by mud filtrate inevitably takes place. Therefore, the permeability of this filter cake is a very complex function of many factors including time, temperature, mud circulation rate, mud composition and differential pressure. The fluid flow through a compressible media (filter cake) is governed by Law of conservation of mass, Equation of state and Darcy's law. Important simplifying assumptions employed in the development of the model include: effect of gravity since the filtration of the drilling mud through the filter cake flows in a linear vertical direction and the fluid flow is considered to be one dimensional. This model will therefore employ Terzaghi consolidation theory and Ruth and Tiller empirical relationship in conjunction with the stated governing equations to describe the mechanism of filtration process, Rate of mud cake build up and Volume of filtrate.

THE MECHANISM OF FILTRATION PROCESS

Consider an element of the porous, permeable, and compressible cake medium through which vertical flow of filtration is observed as shown in figure 1 below

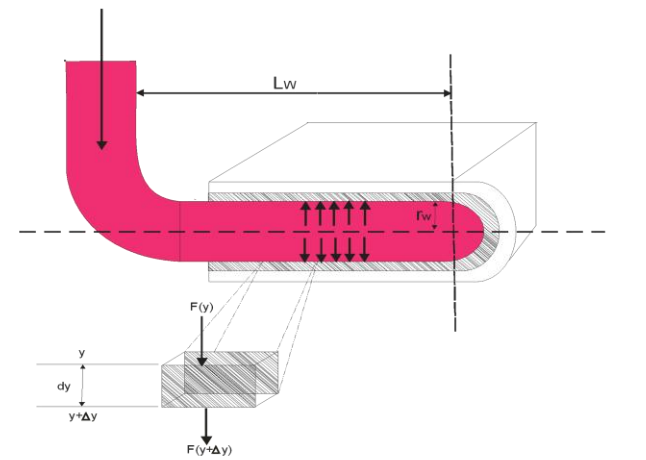

Fig. 1. *Elemental cake medium in horizontal well*

Applying continuity equation and using Taylor's Series expansion Mass influx – Mass out flux = Rate of Mass Accumulation

Results to:

$$\frac{\partial}{\partial y}\left(\rho.\left(-\frac{K}{\mu}(\frac{dp}{dy})\ +\ \rho g\right)\right)\Delta V = -\,\phi\frac{\partial}{\partial t}\left(\rho.\Delta V(y)\right)\ (1)$$

It can be observed from Terzaghi's stress theory that: Total filtration pressure = Solid pressure + Fluid pressure

$$P_t = \quad p_s \ + \quad P\ (2)$$

All measurable effects of stress changes are caused by the solid pressure and the total stress P_t and it is constant. Differentiating (2) with respect to y gives:

$$\frac{dP_s}{dp} = -\frac{\mathrm{dp}}{\mathrm{dy}}\ (3)$$

Relate (3) with the given cake medium under consideration in (2)

$$\frac{\partial}{\partial y}\left(\frac{\rho_s k}{\mu}.-\frac{dP_s}{dy}\right)\Delta V + \frac{\partial}{\partial y}\left(\frac{\rho_s^2 \mathrm{gk}}{\mu}\right)\Delta V = \ \phi\frac{\partial}{\partial t}(\rho_s\Delta V)\ (4)$$

Assume solid density to be constant and therefore divide (4) by ρ_s to give:

$$\frac{\partial}{\partial y}\left(\frac{k}{\mu}.\frac{dP_s}{dy}\right)\Delta V - \frac{\partial}{\partial y}\left(\frac{\rho_s \mathrm{gk}}{\mu}\right)\Delta V = \ -\phi\frac{\partial}{\partial t}(\Delta V)\ (5)$$

Expressing Cake compressibility under the influence of external pressure in terms of change in time as:

$$c\Delta V\frac{dP_s}{dt} = \ -d\left(\frac{\Delta V}{dt}\right)(6)$$

Substitute (6) into (5) and simplify

$$\frac{1}{\mu}\frac{\partial}{\partial y}\left(k.\frac{dP_s}{dy}\right) - \frac{g}{\mu}\frac{\partial}{\partial y}(\rho_s k\ ) = \ \phi\ c\frac{dp_s}{dt}\ \ (7)$$

According to Tiller (1975) and Grace (1953) k, ρ_s and c are expressed in terms of pressure.

The expression below can be obtained as:

$$\phi\ \ = \phi_0 P_s^{-f}\ (i)$$

$$k = \frac{1}{a_y\rho_s(1-\phi)}\ (ii)$$

$$a_y = a_0 P_s^n\ (iii)$$

$$1-\phi = BP_s^m\ (iv)$$

$$c = -\frac{1}{(1-\ \phi)}\frac{\partial\phi}{\partial P_s}\ (v)$$

From the above expressions, the following can be implied thus:

$$\frac{\partial\phi}{\partial p_s} = -f\phi_0 P_s^{-(f+1)}\ (8)$$

$$k = \frac{1}{a_0 B\rho_s P_s^{m+n}}\ (9)$$

$$c = f\phi_0 B^{-1} P_s^{-(f+m+1)}\ (10)$$

Stress function is therefore introduced and defined as:

$$\varepsilon(y,t) = \int_0^{P_s} c dP\ (11)$$

dε can be expressed as

$$d\varepsilon = (-)cdP_s\ (12)$$

By substitution,

$$\varepsilon(y,t) = \frac{f\phi_0 P_s^{-(f+m)}}{-B(f+m)}\ (13)$$

From (13),

$$P_s = \left(\frac{-B(f+m)}{f\phi_0}\right)^{-\left(\frac{1}{f+m}\right)}.(\varepsilon)^{-\left(\frac{1}{f+m}\right)}\ (14)$$

Differentiate P_s with respect to ε and simply to give:

$$dp_s = \left(\frac{B(f+m)}{f\phi_0}\right)^{-\left(\frac{1}{f+m}\right)}.\left(\frac{1}{f+m}\right).(\varepsilon)^{-\left(\frac{f+m+1}{f+m}\right)}d\varepsilon\ (15)$$

Substitute for k, cdP_s and P_s into (7), multiply through by μ and simplify to obtain (16) which gives the diffusivity equation for filtrate in Horizontal wells.

$$\boldsymbol{D_1\frac{\partial}{\partial y}(\varepsilon^{\alpha})\frac{\partial\varepsilon}{\partial y} - gD_2\frac{\partial}{\partial y}\varepsilon^{\beta} = \phi\mu\frac{\partial\varepsilon}{\partial t}\ (16)}$$

Where,

$j = \frac{m+n-1}{f+m}$, $\beta = \frac{m+n}{f+m}$ and $\alpha = \frac{n-f-1}{f+m}$

$$D_1 = \frac{1}{a_0 B\rho_s(f+m)}.\left(\frac{B(f+m)}{f\phi_0}\right)^j$$

$$D_2 = \frac{1}{a_0 B}.\left(\frac{B(f+m)}{f\phi_0}\right)^{\beta} =$$

This equation defines the non-linear diffusivity equation for the filtration mechanism in horizontal wells across porous and compressible cake medium. The effect of gravity is incorporated which can be readily solved for different conditions of filtration.

Rate of Mud Cake Build up

The rate of cake build up during filtration is directly proportional to the flow rate across the cake. It can be mathematically expressed as:

$$\frac{dh}{dt} = bq \ (17)$$

Where,
q = flow rate of filtrate
b = specific cake volume "Outmans, (1963)" constant of proportionality given as:

$$b = \frac{1 - \epsilon_s}{\epsilon_s - \epsilon_l} \ (18)$$

$$\frac{dh}{dt} = \text{rate of mud cake build up}$$

Recall from Darcy's law as $q_y = -\frac{KA}{\mu}\left(\frac{dP}{dy} + \rho g\right)$ and relating $\frac{dp}{dy} = -\frac{dp_s}{dy}$ with it to give:

$$q_y = \frac{KA}{\mu}\left(\frac{dP_s}{dy} - \rho g\right) (19)$$

Substitute (19) into (17) to obtain:

$$\frac{dh}{dt} = b\frac{KA}{\mu}\left(\frac{dP_s}{dy} - \rho g\right) (20)$$

Substitute (15) into (20) and simplify to obtain

$$\frac{dh}{dt} = b\left(\frac{1}{\mu}D_3\varepsilon^{\gamma}\frac{d\varepsilon}{dy} + D_4\right) \ (21)$$

Where,

$$D_3 = \left(\frac{kA}{f+m}\right)\left(\frac{B(f+m)}{f\phi_0}\right)^{-\left(\frac{1}{f+m}\right)}, \ D_4 = -\frac{KA}{\mu}\rho g,$$

$$\gamma = -\left(\frac{f+m+1}{f+m}\right)$$

Horizontal wells are exposed to drilling fluids longer than the vertical wells. This factor causes a larger cake build up near the vertical section of the wells

Volume of Filtrate

Since the filtrate throughput is given by q, then the cumulative volume after time t can be expressed as:

$$Q = \int_0^t q dt \ (22)$$

Using (19) and (14) to obtain:

$$Q = \frac{D_3}{\mu}\int_0^t \varepsilon^{\gamma}\frac{d\varepsilon(y,t)}{dy} dt + D_4 t \ (23)$$

Equation (23) describes the filtration volume and can be evaluated after the stress function of the cake $\varepsilon(y,t) is$ carefully defined.

Solution Development

The mechanism of filtration through any compressible cake medium described by the non-linear diffusivity equation given in (16) will apply to the three types of filtration identified depending on the necessary boundary conditions applied. The boundary conditions governing the filtration process in horizontal well include: inner boundary, outer boundary and moving boundary conditions.

Inner Boundary Condition

When filtration process starts, some mud cake is deposited at the wall of the bore. Under this condition, the fluid pressure is constant. (P = 0, P_t $= P_s$)

The stress condition now becomes:
ε (0, t) = ε (y_0) = ε =f(y) t ≥ 0

Where, ε_0 is defined as

$$\varepsilon_0 = \frac{f\phi_0 P_t^{-(f+m)}}{-B(f+m)}$$

Outer Boundary Condition

As footage in the drilling of horizontal well is made, the well is exposed to drilling fluids and no cake is formed before the filtration commences. This indicates that fluid pressure equal to filtration pressure (solid pressure remains constant - $P_t = P, \ P_s = 0$. We therefore have the stress function as

$$\left.\begin{array}{l}\varepsilon(y_0,t) = 0\\ \varepsilon(h,t) = 0\end{array}\right\} \text{For all values of } t \geq 0$$

The moving boundary condition is represented with the rate of mud cake build up equation. The diffusivity Equation describing the filtration mechanism, the rate of mud cake build up, the cumulative volume of filtrate are given in (16), (21) and (23) respectively.

Boundary conditions
$(i) \quad \varepsilon(h,t) = 0 \quad \forall\, t \geq 0$
$(ii) \quad \varepsilon(o,t) = \varepsilon_0 \quad \forall\, t \geq 0$
Where,

$$\varepsilon_0 = \frac{f\phi_0 P_t^{-(f+m)}}{-B(f+m)}$$

$$(iii)\frac{dh}{dt} = b\left(\frac{1}{\mu}D_3\varepsilon^{\gamma}\frac{d\varepsilon}{dy} - D_4\right)$$

Simplify (16), and apply Goodman's integration technique "Crank, (1975)"

$$\int_0^y \partial\left(\varepsilon^{\alpha}\frac{\partial\varepsilon}{\partial y}\right) - \frac{gD_2}{D_1}\int_0^y \partial(\varepsilon^{\beta}) = \frac{\phi\mu}{D_1}\frac{\partial\varepsilon}{\partial t}\int_0^y \partial y \ (24)$$

$$\varepsilon^{\alpha}\frac{\partial\varepsilon}{\partial y} - \varepsilon_0^{\alpha}\frac{\partial\varepsilon}{\partial y} - \frac{gD_2}{D_1}\left[\varepsilon^{\beta} - \varepsilon_0^{\beta}\right] = \frac{\phi\mu}{D_1}\frac{\partial}{\partial t}\int_0^y \varepsilon\partial y \ (25)$$

Apply the moving boundary condition to equation by assuming $\propto = \gamma$

$$\frac{\mu}{D_3}\left[\frac{dh}{bdt} - D_4\right] - \varepsilon_0^{\alpha}\frac{\partial\varepsilon}{\partial y} - \frac{gD_2}{D_1}\left[\varepsilon^{\beta} - \varepsilon_0^{\beta}\right] = \frac{\phi\mu}{D_1}\frac{\partial}{\partial t}\int_0^y \varepsilon\partial y \ (26)$$

Adopting the techniques used by Isehunwa (2012) for an approximate theory of static filtration of drilling mud in vertical wells, solution of the form
$\varepsilon(y,t) = \varepsilon S(y) + \varepsilon T(y,t)$ (27)

Would be assumed,

Where, εS(y) can be likened to elliptic steady state conduction equations while εT(y, t) to be parabolic transient heat condunction equation. A simple difference between these two types of equations is that for a steady state diffusion problem, a solution is required over an entire domain, whereas for the transient diffusion problem, it is possible to obtain a solution in small time interval.

Let εS(y) is of the form:

$$\varepsilon S(y) = A_1 + A_2 y \quad (28)$$

This gives the solution of equation with repeated root. Applying the boundary conditions to obtain

$$\varepsilon S(0) = \varepsilon_0 = A_1 + A_2(0) \ (29)$$

$$A_1 = \varepsilon_0$$

$$\varepsilon S(h) = \varepsilon_0 + A_2 h \quad (30)$$

$$A_2 = -\frac{\varepsilon_0}{h}$$

Therefore,

$$\varepsilon S(y) = \varepsilon_0 - \frac{\varepsilon_0 y}{h}$$

$$\varepsilon S(y) = \varepsilon_0 \left[1 - \frac{y}{h}\right] \ (31)$$

Substitute (30) into (28) to obtain

$$\varepsilon(y,t) = \varepsilon_0 \left[1 - \frac{y}{h}\right] + \ \varepsilon \mathrm{T(y,t)} \ (32)$$

To proffer a solution for the transient state, the diffusivity equation would be compared with transport equation of the form:

$$D\frac{\partial^2 c}{\partial x^2} - u\frac{\partial c}{\partial x} = \frac{\partial c}{\partial t} .. (33)$$

By transformation,

$$\left[\frac{D_1 \varepsilon^\alpha}{\phi\mu}\right]\frac{\partial^2 \varepsilon}{\partial y^2} - \frac{gD_2}{\phi\mu}\frac{\partial \varepsilon}{\partial y} = \quad (34)$$

Assumptions made here are that β should be made unity and the stress function to power of alpha be constant. Let $\varepsilon T(y,t)$ be of the form given below "Dennis, G. Z. (1999)"

$$\varepsilon T(y,t) = \left[C_1 \cos\frac{\sqrt{4\lambda^2 - \alpha^2}}{2}(y - h) + C_2 \sin\frac{\sqrt{4\lambda^2 - \alpha^2}}{2}(y - h)\right] C_3 e^{\frac{\alpha}{2}\left(y - h - \frac{D\alpha}{2}\right) - Dn^2\pi^2} (35)$$

Applying boundary conditions t (35)

(i) (h, t) = 0 = $C_1 \cos(0) + \ C_1 \sin(0)$

$C_1 \ = 0$

(ii) $\varepsilon(0,t) = \ \varepsilon_0 = \left[C_1 \cos\frac{\sqrt{4\lambda^2-\alpha^2}}{2}(h) - C_2 \sin\frac{\sqrt{4\lambda^2-\alpha^2}}{2}(h)\right] C_3 e^{-D\left(\frac{\alpha^2}{4}+n^2\pi^2\right)} 36)$

$$\sin\frac{\sqrt{4\lambda^2 - \alpha^2}}{2}(h) = 0$$

The solution to the transient state is given as:

$$\varepsilon \mathrm{T(y,t)} = \sum_{\mathrm{n=1}}^{\infty} \mathrm{A_n}\, e^{-D\left(\frac{\alpha^2}{4}+n^2\pi^2\right)} \sin n\pi \left(\frac{h - y}{h}\right) (37)$$

Therefore, combining (32) and (37) to obtain:

$$\varepsilon(y,t) = \varepsilon_0 \left[1 - \frac{y}{h}\right] + \sum_{\mathrm{n=1}}^{\infty} \mathrm{A_n}\, e^{-D\left(\frac{\alpha^2}{4}+n^2\pi^2\right)} \sin n\pi \left(\frac{h - y}{h}\right) \ (38)$$

Assuming the initial condition $\varepsilon(y,t) = \ \varepsilon_0$ implies that

$$\varepsilon_0 \ = \sum_{n=1}^{\infty} A_n e^{\alpha/2} . \sin n\pi y \ \ (39)$$

Solving for A_n and simplify to obtain:

$$\varepsilon(y,t) = \varepsilon_0 \left[\left[1 - \frac{y}{h}\right] + \frac{2}{\pi}\sum_{\mathrm{n=1}}^{\infty} \frac{n\left[e^{\alpha/2} - (-1)^n\right]}{\frac{e^{\alpha/2}}{4\pi^2}\left[\alpha^2 - n^2\right]} e^{-D\left(\frac{\alpha^2}{4}+n^2\pi^2\right)} . \sin n\pi \left(\frac{h - y}{h}\right)\right] (40)$$

Equation (40) gives the theoretical equation that defines the stress distribution in a static filtration process during drilling in horizontal wells. It is explicit in time variable through the presence of h which is time dependent. This can be shown to hold true by solving (26) through the use of(40)

Differentiating (40) gives:

$$\frac{\partial \varepsilon(0,t)}{\partial y} = -\frac{\varepsilon_0}{h} + \frac{2\varepsilon_0}{\pi}\sum_{\mathrm{n=1}}^{\infty} \frac{n\left[e^{\alpha/2} - (-1)^n\right]}{\frac{e^{\alpha/2}}{4\pi^2}\left[\alpha^2 - n^2\right]} e^{-D\left(\frac{\alpha^2}{4}+n^2\pi^2\right)} . \left(-\frac{n\pi}{h} . \left(\cos n\pi \left(\frac{h - y}{h}\right)\right)\right) (41)$$

Recall from power series "Speigel, (1981)" for:

$$\cos y = 1 \ - \frac{y^2}{2!} + \frac{y^4}{4!} - \frac{y^6}{6!} + \cdots \ldots \ldots (-1)^{n-1}\frac{y^{2n-2}}{2n-2} \ldots \ldots \ldots .. -\infty < y < \infty$$

Assuming $\cos n\pi \left(\frac{h-y}{h}\right)$ to be conditionally convergent and neglecting other terms,

Equation (41) is transformed to:

$$\frac{\partial \varepsilon(0,t)}{\partial y} = -\frac{\varepsilon_0}{h} - \frac{2\pi\varepsilon_0}{h\pi} \ (42)$$

$$\frac{\partial \varepsilon(0,t)}{\partial y} = -\frac{\varepsilon_0}{h}(1 + 2) \ (43)$$

$$\frac{\partial \varepsilon(0,t)}{\partial y} = -\frac{3\varepsilon_0}{h} \ (44)$$

Integrate (40) as:

$$\int_0^y \varepsilon = \int_0^y \varepsilon_0 \left[1 - \frac{\mathrm{y}}{\mathrm{h}}\right] \mathrm{dy} + \frac{8\varepsilon_0}{\pi^2 e^{\alpha/2}}\sum_{\mathrm{n=1}}^{\infty} \frac{n\left[e^{\alpha/2} - (-1)^n\right]}{\frac{1}{\pi^2}\left[\alpha^2 - 4n^2\right]} e^{-D\left(\frac{\alpha^2}{4}+n^2\pi^2\right)} . \int_0^y \sin n\pi \left(\frac{h - y}{h}\right) (45)$$

The expression with summation terms are constants, and therefore can be neglected to obtain:

$$\lim_{\substack{y\to h\\ n\to\infty}} \int_0^y \varepsilon dy = \varepsilon_0\left(h-\frac{h}{2}\right)+\frac{8\pi\varepsilon_0}{\pi^2 e^{\alpha/2}}\cdot\frac{h}{n\pi}\cos n\pi\left(\frac{h-y}{h}\right) \quad (46)$$

Expansion of cosine can still be regarded to be conditionally convergent, simplifying gives:

$$\left[\frac{\mu}{bD_3}-\frac{\phi\mu}{D_1}\varepsilon_0\left(\frac{1}{2}+\frac{8}{\pi^2 e^{\alpha/2}}\right)\right]\int_0^h hdh = \left[\frac{gD_2}{D_1}\left[\varepsilon^\beta-\varepsilon_0^\beta\right]h+\frac{hD_4\mu}{D_3}-\left(\frac{3\varepsilon_0^{\alpha+1}}{h}h\right)\right]\int_0^t dt \quad (47)$$

Recall that area of cylinder is given as $2\pi rh$ $(2\pi rl)$, height h in the right-hand side of (47) can be replaced with the horizontal section of the well, by integration and further simplification,

$$h=\sqrt{\frac{2l\left[\frac{gD_2}{D_1}\left[\varepsilon^\beta-\varepsilon_0^\beta\right]+\frac{D_4\mu}{D_3}-\frac{6\pi r_w\varepsilon_0^{\alpha+1}}{A_w}\right]}{\left[\frac{\mu}{bD_3}-\frac{\phi\mu}{D_1}\varepsilon_0\left(\frac{1}{2}+\frac{8}{\pi^2 e^{\alpha/2}}\right)\right]}t} \quad (48)$$

$$h=\sqrt{Rt} \quad (49)$$

Where, R is

$$\frac{2l\left[\frac{gD_2}{D_1}\left[\varepsilon^\beta-\varepsilon_0^\beta\right]+\frac{D_4\mu}{D_3}-\frac{6\pi r_w\varepsilon_0^{\alpha+1}}{A_w}\right]}{\left[\frac{\mu}{bD_3}-\frac{\varepsilon_0\phi\mu}{D_1}\left(\frac{1}{2}+\frac{8}{\pi^2 e^{\alpha/2}}\right)\right]}$$

Equation (49) gives the basic equation that relates the cake thickness to filtration time. Substituting (48) into (40), time dependency of the stress function distribution is observed.

$$\varepsilon(y,t)=\varepsilon_0\left[\left[1-\frac{y}{\sqrt{Rt}}\right]+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{n\left[e^{\alpha/2}-(-1)^n\right]}{\frac{e^{\alpha/2}}{4\pi^2}\left[\alpha^2-n^2\right]}e^{-D\left(\frac{\alpha^2}{4}+n^2\pi^2\right)}.\sin n\pi\left(1-\frac{y}{\sqrt{Rt}}\right)\right] \quad (50)$$

Solid Pressure Variation in the Cake

Substituting (50) into (15) gives the equation that relates the solid pressure within the cake to the stress function distribution

$$P_s=P_t\left(\left[1-\frac{y}{h}\right]+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{n\left[e^{\alpha/2}-(-1)^n\right]}{\frac{e^{\alpha/2}}{4\pi^2}\left[\alpha^2-n^2\right]}e^{-D\left(\frac{\alpha^2}{4}+n^2\pi^2\right)}.\sin n\pi\left(\frac{h-y}{h}\right)\right)^{-\left(\frac{1}{f+m}\right)} \quad (51)$$

Cumulative Volume of Filtration

Recall equation (23) that describes the filtration volume after a time interval t:

$$Q=\frac{D_3}{\mu}\int_0^t \varepsilon^\alpha\frac{d\varepsilon(y,t)}{dy}dt+D_4 t$$

Also recall that:

$$\frac{\partial\varepsilon}{\partial y}=\frac{3\varepsilon_0}{h}$$

Put this equation into (23) to obtain:

$$Q=\frac{D_3}{\mu}\int_0^t {\varepsilon_0}^\alpha\frac{3\varepsilon_0}{h}dt+D_4 t \quad (51)$$

Substituting for h to give:

$$Q=\frac{D_3}{\mu}\int_0^t {\varepsilon_0}^\alpha\frac{3\varepsilon_0}{\sqrt{Rt}}dt+D_4 t \quad (52)$$

Integrate:

$$Q=\frac{6D_3{\varepsilon_0}^{\alpha+1}}{\mu\sqrt{R}}t^{1/2}+D_4 t \quad (53)$$

Equation (53) is the basic equation that describes the cumulative filtrate volume per unit surface area. The effects of viscosity and specific cake volume are readily assumed from the equation.

Depth of Invasion

Equation (53) can simply be used to estimate the depth of invasion penetrated by mud filtrate since the volume of filtrate lost must be equal to the volume of invading fluid- a simple material balance. Under these assumptions, material balance yields the equation to calculate depth of invasion.

$$Q=\pi(r_i^2-r_w^2)h\phi s_i \quad (54)$$

Radius of invasion can be obtained thus:

$$r_i=\sqrt{\frac{Q}{\pi h\phi s_i}+r_w^2} \quad (55)$$

Depth of invasion (d_i) can therefore be defined as:

$$d_i=r_i-r_w \quad (56)$$

$$d_i=\sqrt{\frac{Q}{\pi h\phi s_i}+r_w^2}-r_w \quad (57)$$

Results and Discussion

Considering the equations developed and solved, equation (16) gives non-linear partial differential equation describing the filtration process of mud solid contents as well as the filtrate through the compressible filter cake in the formation. Equation (50) representing the stress function shows that stress function is a function of some parameters such as cake thickness, distance of the mud from the wall and a constant" w" which depends on the constant of non-linearity alpha. It can be clearly observed from (21) and (53) that the rate of buildup of mud cake as well as the cumulative volume of filtrate into the formation are both time dependent. So also, the depth of invasion can be implied from the invasion volume. For the purpose of this research work, the solution obtained was ran on oil well data under static filtration boundary conditions specified. The solid pressure distribution is given by (51), it can be clearly seen that the value depends on the ratio of the distance from the wall and the cake thickness which agrees with some previously developed model of researchers. The data set of tables 1 and the developed equations (48), (50), (51), (53) and (57) are used as the basis for the analysis with the aid of Excel spread sheet. The set of data used for the analysis is presented below in table 1

Table 1.0: *Set of Data Used for Analysis*

Symbol	Parameter	Value
m	Exponent	-0.26
f	Exponent	0.08
n	Exponent	0.13
μ	Mud viscosity	0.8
ρ_s	Mud density	12 ppg
d_w	Well diameter	0.5 ft
k	Cake permeability	0.42
K_x, k_y, k_z	Permeability in x, y and z directions	0.2, 0.14, 0.12
γ	Specific gravity	2.5
l	Hole depth	561 ft
a_0	Specific cake resistance	1×10^6 ft/lb
R	Radius of curvature	200ft
h	Pay thickness	20ft
ϕ_0	Initial cake porosity	0.3
ϕ	Formation porosity	0.24
b	Specific cake volume	0.23

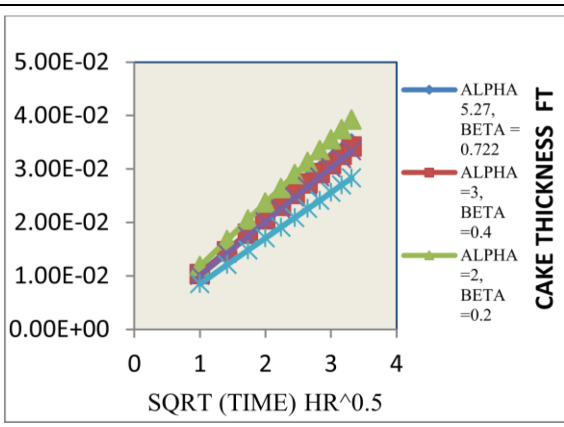

Fig. 2. *Cake Thickness Vs Sqrt of Time*

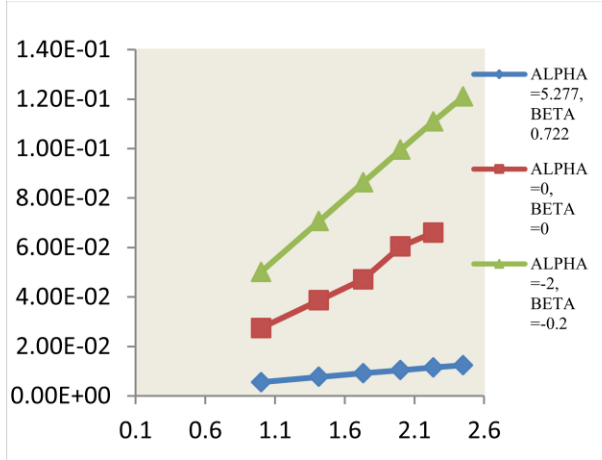

Fig. 3. *Filtrate Volume Vs SQRT of Time*

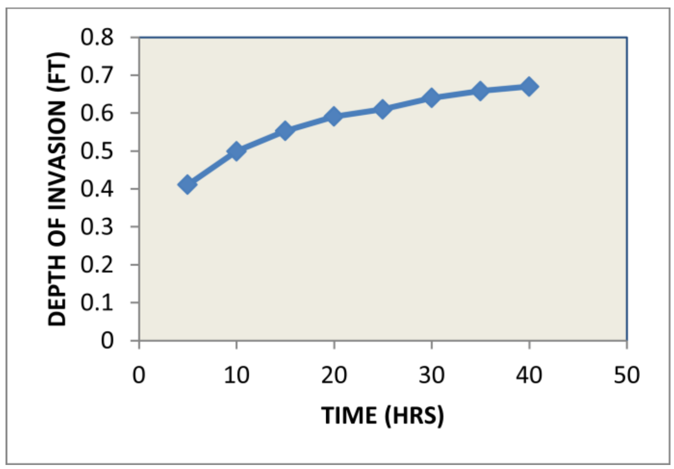

Fig. 4. *Depth of Invasion Vs Time for Horizontal Well*

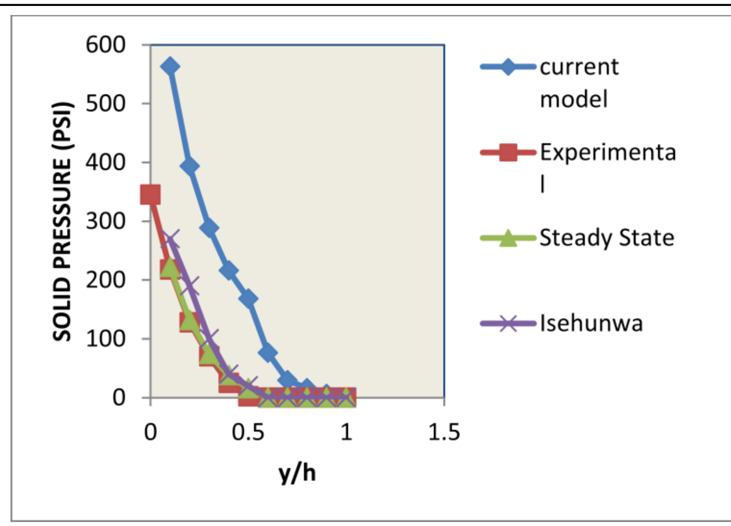

Fig. 5. *Solid Pressure Predictions for Different Models*

Figure 2. shows that the mud cake builds up gradually as time goes by. The graph was plotted at different values of constants of non-linearity alpha and Beta. This therefore shows the importance of non-linearity in the filtration mechanism. At this point, the mud cake is growing and filtrate loss rate can be considered to be proportional to the square root of time. But it should be noted that the mud cake properties will tend to be stable under the joint influences of hydrodynamic forces, differential filtration pressure and shear stress. Figure 3 shows the plot of cumulative filtration volume of mud against the square root of time. This indicates that the cumulative filtrate volume increases linearly with time at different values of alpha and Beta. The exposure time for the invasion of filtrate into the formation that result in severe damage are quite different from heel to toe of the horizontal segment of the well, therefore, exposure times at different horizontal segment must be properly estimated to ensure accurate value for volume of filtrate invasion Figure 4. shows the depth of invasion with time, at the initial time of drilling when mud cake is yet to be formed, spurt loss dominates filtrate invasion, but as the mud cake builds up, the volume of filtrates reduces and thereby depth of invasion also decreases. Figure 5 shows the variation of solid pressure generated and it can be shown that using the same set of data for both vertical and horizontal wells, the effect of damage tends to be more pronounced in the horizontal well than the vertical well. This is usually as a result of longer exposure time of horizontal wells to drilling fluid.

Conclusion

An approximate analytical solution combining both the steady and transient components has been successfully applied to solve the filtration theory equation developed for horizontal wells. From the model, the rate of mud cake build up at different values of constants of non-linearity Alpha and Beta can be conveniently obtained. Some important parameters such as Filtrate volume and depth of invasion have been shown to be sensitive to Alpha and Beta - Factors of non-linearity in the filtration equation. Volume of filtrate, depth of invasion at different part of the horizontal segment can easily be estimated. Control of volume of filtrate into the formation is achievable by controlling the viscosity of the mud, specific mud cake volume and other important mud properties that filtration volume depends on. Effect of gravity which cannot be ignored on filtrates and filtration of solid contents either from mud or particles from rock cuttings is considered in the model development as well as the solution obtained. By comparison of the solid pressure of this model with other models, it can be inferred that damage is more pronounced in the horizontal wells than any other wells due to some known reasons attributed to damage in horizontal drilling. Effect of length of the horizontal section of the well is readily observed in estimating the thickness of mud cake.The model is easily applicable to estimate important values needed to know the status of the formation during the drilling of horizontal wells.

Acknowledgment

This study was supported by grant from the Management of Abubakar Tafawa Balewa University Bauchi – Nigeria.

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