How Do Students' Mathematical Epistemological Beliefs Affect Their Critical Thinking Tendencies?

Rahaju, Purwanto, I Nengah Parta, Swasono Rahardjo

Abstract: The study aimed to investigate the tendency of students to think critically and explore the relationship between the students' critical thinking with their mathematical epistemological beliefs. This descriptive qualitative study involved 32 students from the Department of Mathematics Education. The result showed that subjects who were non-critical thinkers exhibited carelessness in understanding, recognizing errors, and overviewing the tasks because they believed that mathematical tasks are always right and problem-solving with formulas and procedures results in correct solutions. On the other hand, critical-thinker students were more careful in comprehending the tasks, checking the errors, and doing an overview. The belief that mathematical tasks can contain errors and that conceptual understanding is highly crucial in mathematical problem-solving increased the tendency of the students to think critically. Besides, the students also had to highlight that numbers in mathematical problems have meaning and logical relationships.

Index Terms: critical thinking tendecies, closed ended problem, ill-logical problems, mathematical epistemological beliefs.

1 INTRODUCTION

Critical thinking is a skill needed to face challenges in life. Critical thinking helps individuals test, connect, and evaluate all aspects of a situation or a problem [1] to carry out their duties and responsibilities [2],[3]. In the globalization era, critical thinking is used to filter information that is disseminated through various media [4],[5],[6]. However, research shows that the majority of students in Asia have poor critical thinking performance. Some studies even proved that Malaysian [7], Indonesian [8], and Middle Eastern [9] high school students were unable to achieve satisfactory scores in critical thinking. In addition, a great deal of mathematics teacher candidates cannot be categorized into critical thinkers [10] and had their low critical-thinking dispositions [11]. Rasiman [12] examined students' critical thinking dispositions using closed-ended problems and identified three levels of critical thinkers. Types of problems, such as closed-ended problems, implicit open-ended problems, ill-logical problems, and incomplete information problems require different critical thinking tendencies [10]. This study aimed to explore students' critical thinking tendencies by using ill-logical mathematics problems and closed-ended problems. Futhermore, investigated mathematical beliefs that supported the tendencies. It is hoped that the results of this study can provide preliminary information about students' mathematical epistemological beliefs that support the tendency of the students to think critically.

2 METHOD

The subjects of this descriptive qualitative study consisted of 32 students from the Department of Mathematics Education of a private university in Malang who had learned and solved various triangle problems.

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Data of the study were gathered using two instruments, triangle problem-solving tasks (PSTt) and an interview guideline. PSTt contained ill-logical mathematics problems and closed-ended problem that were used to pinpoint the subjects' critical thinking tendencies (Fig. 1 and Fig. 2).

An equilateral triangle has a side length 3 and the height 5. If possible:
a. Draw the triangle!
b. Determine the area of the triangle!

Fig. 1 PSTt 1

Look at the XYZ triangle. If possible, determine:
a. The area of the triangle!
b. The perimeter of the triangle!

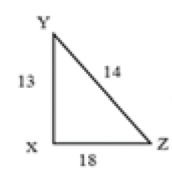


Fig. 2 PSTt 2

Data collection was conducted by asking the participants to solve PSTt 1. Two weeks later, the participants were re-invited to solve PSTt 2. An interview was conducted to explore the subjects' thinking process when solving PSTt and elicit the reasons why they chose a specific way to solve the problems. The interview was carried out after the participants completed PSTt tasks. The subjects' answers to PSTt problems were scored and the data were classified. After that, an interview guideline was developed. The subjects' thinking process was analyzed to investigate their critical thinking tendencies and mathematical epistemological beliefs.

3 RESULTS

The results of the analysis were categorized into two groups: critical thinker subjects (TCT subjects) and non-critical thinker subjects (NTCT subjects). The categorization of subjects was presented on Table 1 dan 2.

TABLE 1

THE CATEGORIZATION OF THE PSTT 1 SUBJECTS

Group	Number of Subject	
	PSTt	Interview
TCT	3	2
NTCT	29	4

TABLE 2

THE CATEGORIZATION OF THE PSTT 2 SUBJECTS

Group	Number of Subject	
	PSTt	Interview
TCT	2	2
NTCT	30	3

3.1 The Non-Critical Thinkers' Problem-Solving

There were 29 NTCT subjects on PSTt 1 and 30 subjects on PSTt 2. All of the NTCT subjects provided the same solution to PSTt 1 and PSTT 2. The result of the interview with four NTCT subjects after PSTt 1 completion and with three NTCT subjects after PSTt 2 completion revealed that the subjects used similar thinking process and same strategies to solve the problems. Therefore, the description of the data can be represented by one subject only, namely subject S1. Subject S1 read PSTt 1, drew an equilateral triangle ABC, and placed side length of the triangle 3 cm and height 5 cm. Then, S1 wrote a formula to find the area of the triangle and inserted the known values into the formula. S1 spent only little time to complete PSTt 1. The PSTt 1 result by S1 was presented in Fig, 3.

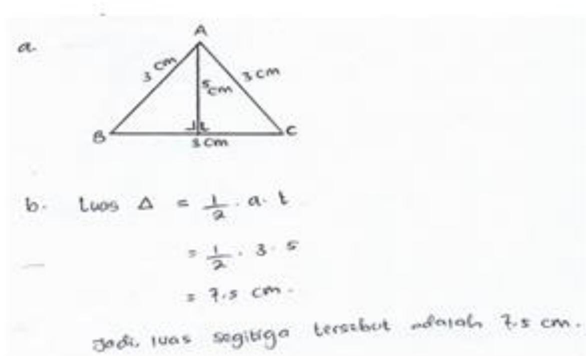


Fig. 3 The Results of PSTt 1 Problem Solving by S1

S1 had been very familiar with PSTt 1 since elementary school. S1 classified the task into a trivial task since it can be completed by using a single formula. Instead of re-checking the logic of the problem, S1 simply followed the common procedure to complete PSTt 1 that was by writing the formula for the area of a triangle and entering the known values. S1 explained that every task assigned by mathematics teachers was always correct and solvable. After solving PSTt 1, S1 was given the opportunity to review the solution. However, without checking it, S1 directly said "correct already". S1 felt that he had written the right formula and carried out the right procedure to solve such type of problem. The result of the interview uncovered the fact that mathematics teachers did not usually do a thorough checking on students' work. As a result, the belief that formula substitution is the most effective way to solve mathematical problems has been implanted into the students' mind. S1 also used the same strategy to solve PSTt 2. S1 read the task and wrote down the formula of the area of a triangle. S1 added all the lengths of the base (18) and the height (13) of the triangle. The result was presented in Fig. 4.

Fig. 4 The Results of PSTt 2 Problem Solving by S1

The interview revealed that subject S1 considered XYZ as a right triangle because XYZ triangle resembled a right triangle on text books. Therefore, XZ was thought to be the base of the triangle and XY was thought to be the height of the triangle. This finding suggests that S1 paid lack of attention to the triangle picture and to the instructions provided on PSTt 2. S1 missed the information "if possible" and thus perceived XYZ as a right triangle. S1 also failed to notice that XYZ triangle did not contain any symbol that indicates the right angle.

3.2 The Critical Thinkers' Problem-Solving

There were 3 TCT subjects and two different patterns on PSTt 1. The result of the interview showed that the subjects also exhibited two distinguished ways of thinking. Therefore, data description on NCT subjects was represented by two subjects, namely S2 and S3. In contrast, the two TCT subjects on PSTt 2 performed the same strategy to solve the task. The result of the interview also indicated a lot of similarities in the subjects' problem-solving techniques. Therefore, data description on PSTt 2 problem-solving was referred to one subject, namely S3.

Problem-Solving by S2

S2 drew the ABC triangle based on the known length of sides and height of the triangle provided on PSTt 1. Firstly, S2 drew the base of the triangle with a length of 3 cm, and then dragged the height (CD) 5 cm. S2 connected poin A and C, then B and C to create the ABC triangle (Fig. 5). According to Fig. 5, it was known that S2 realized that the length of AB was different from the length of AC and BC. Therefore, S2 measured the length of both lines and concluded that the ABC triangle was an equilateral triangle because neither AC nor BC had a length of 3 cm. S2 examined the length of AC and the length of BC by applying the Phytagorean Theorem. The subject found that BC had a length of 5.22. What S2 did was not in line with what was instructed in PSTt 1, that was to draw an equilateral triangle. The activity conducted by S2 is called overview. PSTt 1 problem-solving process carried out by S2 was presented in Fig. 5.

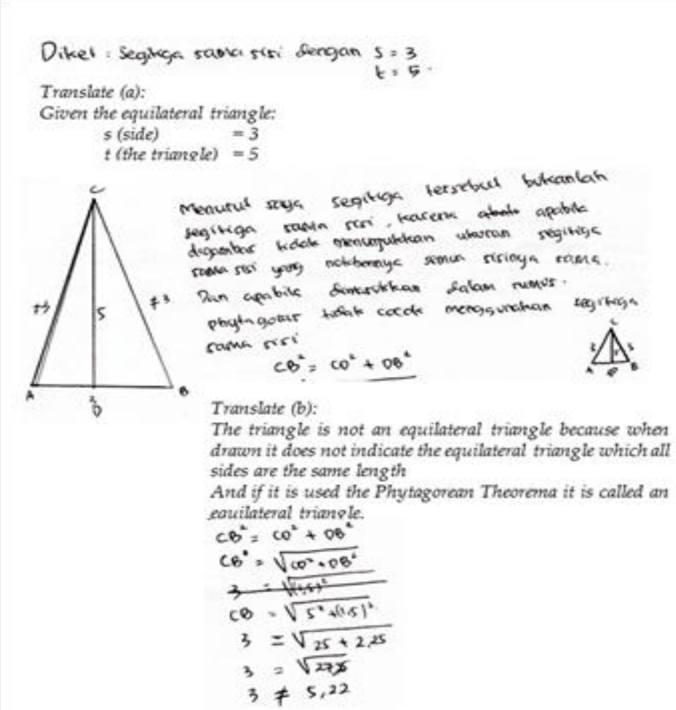


Fig. 5 The Results of PSTt 1 Problem Solving by S2

S2 found the area of the ABC triangle using the formula $The\ area\ of\ the\ triangle\ (A) = \frac{1}{2} \times base \times heigth$. S2 wrote 7.5 as the area of the triangle and added some notes that the area of the triangle can only be obtained without considering Fig. 5. The answer of S2 was shown in Fig. 6.

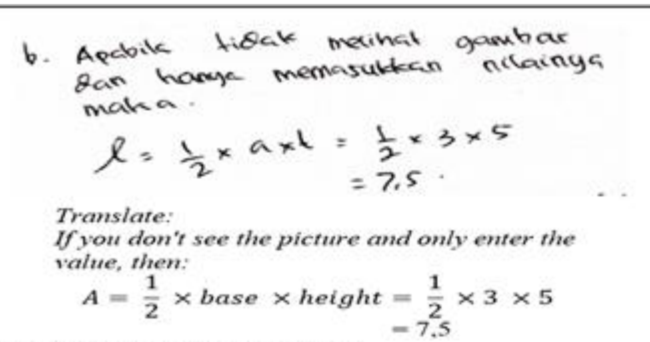


Fig. 6 The Area of The Triangle by S2

The result of the interview showed that S2 was accustomed to adjusting a geometric shape with given sizes. Therefore, S2 drew the ABC triangle by following the instructions provided on PSTt 1. However, the triangle created was not exactly like what was instructed in the task because according to S2, it was impossible to draw an equilateral triangle with the length of sides equal to 3 and the length of height to 5. In fact, S2 was hesitant to draw the triangle because the given size seemed illogical. S2 mentioned that mathematical tasks usually contained sizes in standardized measurement, for example in centimeter or meter. Therefore, the non-standardized measurement given in PSTt 1 made S2 confused and unable to decide whether it was the solution that was inaccurate or it was the task that was mistaken.

Problem-Solving by S3

S3 started by drawing an equilateral triangle. S3 noticed a mismatch between the length of the sides and the height of the triangle. Therefore, S3 used the Pythagorean Theorem to check the side length (AB). The result of the Pythagorean Theorem application showed that AB was not equal to 3. S3 said that the problem in PSTt 1 was not related to an equilateral triangle. Considering the height of the triangle, it was impossible that AB or CB was equal to 3. Therefore, the base length of the ABC triangle was not 3 and the height of the ABC triangle was not 5. The result of problem resolution S3 are presented in Fig. 7.

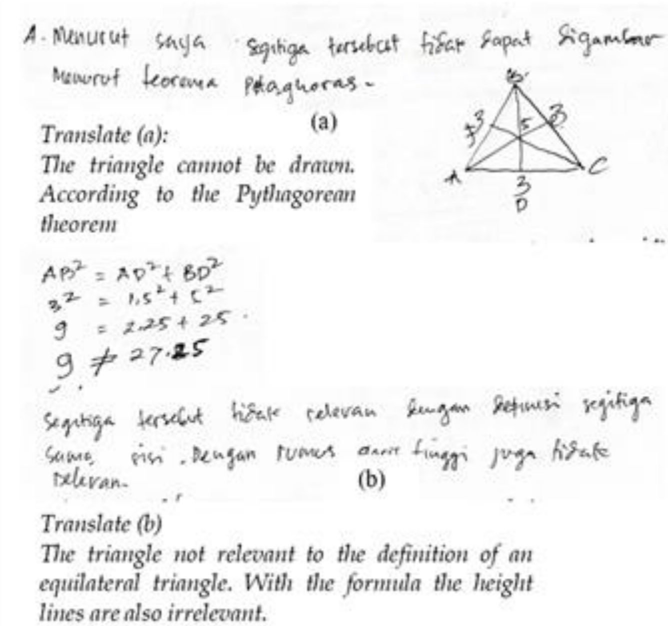
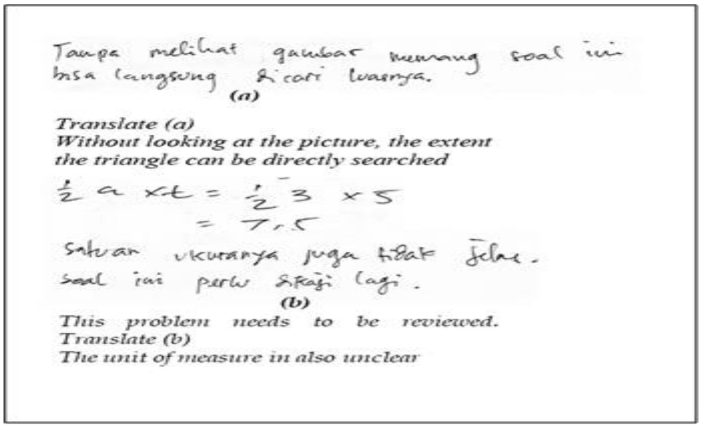


Fig. 7 The Results of PSTt 1 Problem Solving by S3

S3 completed task number 2 by explaining "without considering Fig. 7, the area of the ABC triangle can be searched by the formula $A = \frac{1}{2} \times base \times heigth$ and hence the area is equal to 7.5". According to S3, the length of the sides and the length of the height of the triangle were unclear are ambiguous because they did not specify units of measure, such as centimeters or meters. The result of the interview showed that the absence of units of measurement on the sides and height of the triangle made the assignment incomplete. Therefore, S3 firmly said that this task had to be reviewed. The answer of S3 to assignment number 2 was presented in Fig. 8.



S3 read PSTt 2 carefully and explained that XYZ triangle was not a right triangle because there was no explicit information about it and there was no symbol on the triangle that indicates a right angle. Further examination was conducted by connecting the XYZ triangle sides. If the XYZ triangle was a right triangle, then the hypotenuse of the triangle would be much longer than the other two sides. The length of the XYZ triangle hypotenuse can be obtained from the root of the sum of the squares of its sides. The result of the Pythagorean Theorem examination showed that the longest side of the XYZ triangle was not the same as the root of the number of its sides $18 \neq \sqrt{13^2 + 14^2}$. S3 not only paid close attention to the XYZ triangle, but also did an overview to validate the accuracy of the information provided in the task. Based on the above overview, S3 dealt with a difficulty determining which part was the base and height of the XYZ triangle. It suggests that the area of the XYZ triangle cannot be determined using the formula $A = ½ \times base \times height$ Therefore, S3 used Heron's Theory to figure out the area of the XYZ triangle (Fig. 9).

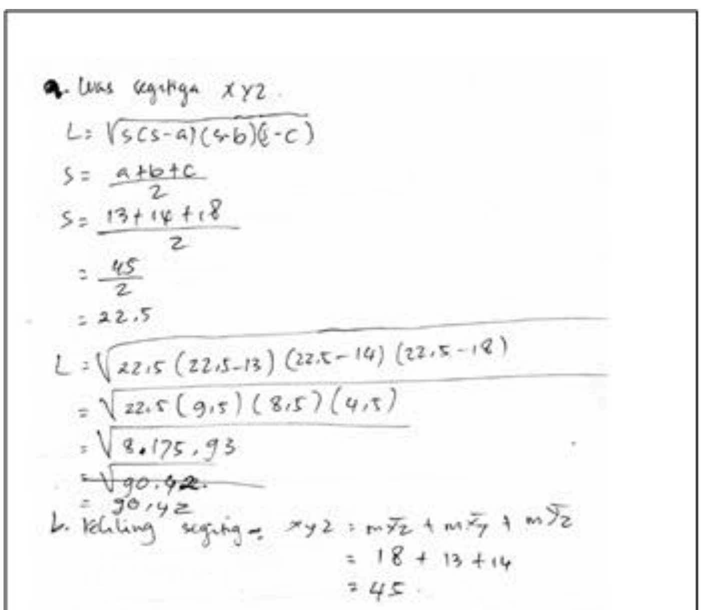


Fig. 9 The Results of PSTt 2 Problem Solving by S3

The result of the interview uncovered the fact that S3 was aware of the existence of the "If possible" statement in PSTt 2. This statement can also be found in PSTt 1. As a matter of fact, PSTt 1 had zero possibility to solve. Therefore, S3 came to an understanding that the "If possible" statement was actually a clue provided by the test maker to help the test taker comprehend PSTt 2. S3 admitted that he often found triangle tasks that required Pythagorean Theorem application. S3 believes that mathematics contains a collection of meaningful numbers and that the elements of a plane are logically correlated. Thus, the length of each element of a plane cannot be placed arbitrarily. Through the interview, S3 also revealed his experience studying Heron's Theory that can be used to find out the area of a triangle if the base length and the height cannot be identified. This finding suggests that S3 has more ways to find the area of a triangle.

4 DISCUSSION

During the PSTt 1 and PSTt 2 problem-solving processes, S1 showed no sign of using reasoning to check the accuracy of the tasks. Therefore, S1 can be categorized as a non-critical thinker as he did not exhibit a critical attitude toward a problem-solving scheme [10]. Besides, S1 ignored the "If possible" statement. As a result, the student failed to understand the instructions of the tasks and failed to focus on what was being asked by the tasks. This behavior exhibited by S1 is in contrast with the behavior of a critical thinker who is highly observant to problems [13]. S1 was reluctant to re-check his work upon completion despite being reminded by the researcher. Instead, S1 confidently stated that he had found the correct answer to the question. S1 is not used to overviewing. Overviewing is the act of reflecting and reviewing a decision [14]. In solving mathematical problems, overviewing can be done by re-checking the result or applying other theories to re-examine the solution. Since S1 assumed that PSTt was a common triangle task, he used a common procedure to solve it, the procedure taught by the teacher in the classroom. This has seemingly led to his lack of tendency to think critically. S1 has never seen his mathematics teacher check the accuracy and completeness of a task. As a result, S1 is confident that all the tasks given by teacher are always correct and require no further examination on their accuracy or completeness. At the last stage of problem-solving, S1 did not perform an overviewing activity because his experience has shaped the belief that all mathematical problems can be solved using particular mathematical formulas and procedures. S1 considered using a formula as an effective way to solve a mathematical problem because he was never taught about the importance of overviewing and/or reviewing a mathematical answer/solution. Ashton [15] explains that the biggest barrier to the development of students' critical thinking skills is the teacher's poor knowledge on critical thinking. Critical thinking needs to be improved through hands-on learning experiences [9]. On the contrary, S2 and S3 solved PSTt tasks differently. As S2 began to follow the instruction to draw a triangle, he was suddenly noticing an inconsistency between the question and the solution based on the given sizes. On the other hand, S3 found the ambiguity after sketching the triangle image. This finding suggest that S2 and S3 are critical thinkers because they are able to discover "what is wrong" with the tasks [10]. Critical thinkers have a tendency to exhibit a truth-seeker and curious attitude (Kökdemir in [16]). These two critical thinkers (S2 and S3) show different critical thinking tendencies. Unlike S2 who doubted his work, S3 confidently said that something was wrong with PSTt1, so PSTt 1 had to be reviewed. These differences in critical thinking tendencies are based on different learning experiences. S2 Figured out the inconsistency in PSTt 1 from an image he drew according to the specified size. In addition, S2 has learned how to find the height of a triangle using the Pythagorean Theorem. However, S2 has never been faced with an ill-logical problem such as found in PSTt 1. This forms the belief that all mathematical assignments given by the teacher are always right. Since the solution to the PSTt 1 task was not in accordance with his mathematical epistemological belief, S2 was unsure about his answer. S3 using a sketch of image and an examination of concepts that have been studied to find out the logic of PSTt 1. S3 had a deep understanding of triangle concepts. A rich diet of learning experiences has formed S3's belief that understanding concepts is very important in solving a mathematical problem. Therefore, with fully confidence, S3 said that there was an illogical relationship between the question and the answer provided in PSTt 1. S3 is a critical thinker who has strong confidence (Kökdemir in [16]) In PSTt 2, S3 was not affected by the XYZ triangle image that resembles a right triangle. S3 carefully read the instructions in PSTt 2 and finally noticed that there was no symbol indicating that the XYZ triangle had a right angle. This led S3 to check his answer using the Pythagorean Theorem. The accuracy shown by S3 is the behavior of critical thinkers who are highly focused on problems and the situation that surround them [13]. As a critical thinker, S3 has tried to gather and assess relevant information using abstract ideas to interpret them effectively [17]. S2 and S3 have different learning experiences that result in different mathematical beliefs. The belief that mathematical tasks are always flawless causes S2 to doubt that PSTt 1 has no solution. On the other hand, S3 has a belief that problem-solving does not only depend on formulas or procedures, but requires conceptual understanding. In addition, a wide range of mathematics learning experiences have shaped the belief that mathematics is not just a collection of numbers. Numbers in mathematics have logical meanings and relationships. Therefore, S3 expressly stated the illogicality of PSTt 1 and said that the XYZ triangle on PSTt 2 was not a right triangle. Although visually it looks like a right triangle, the results of an examination of the length of the sides have shown that the XYZ triangle is not a right triangle.

5 CONCLUSION

Students who exhibit lack of tendency to think critically are unable to point out an inconsistency in a task through overviewing or to understand the task. This is due to their mathematical epistemological belief that mathematical assignments given by the teacher are always free from errors and the problem-solving merely relies on formulas or procedures. Therefore, non-critical thinkers are reluctant to do an overview of the logic of the task and of the solution to the problem. In contrast, critical thinkers tend to show a careful attitude in understanding the instructions of the tasks. This is due to the belief that not all mathematical tasks are correct and complete. It is possible that the assignment is incomplete, wrong, or illogical. Therefore, critical thinkers always do an overview to check the accuracy of a task. When doing an overview, critical thinkers refer to various concepts. This is motivated by the belief that conceptual understanding is very important in problem-solving and that numbers in mathematics have meaningful and logical relationships. The results of this study serve as preliminary information for future research that is going to explore the causes of students' low critical thinking tendencies, especially those related to mathematical beliefs. In addition, the results of this study can be used as a reference in designing various tasks to increase students' critical thinking tendencies.

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REFERENCES

[1]. S. Krulik and J. A. Rudnick, The New Sourcebook for Teaching Reasoning and Problem Solving in Elementary School. Boston: Allyn and Bacon, 1995.

[2]. E. Aizikovitsh-Udi and D. Cheng, "Developing Critical Thinking Skills from Dispositions to Abilities: Mathematics Education from Early Childhood to High School.," Creative Eduction, vol. 6, pp. 455–462, 2015, doi: 10.4236/ce.2015.64045.

[3]. S. M. Jacob, "Mathematical achievement and critical thinking skills in asynchronous discussion forums," Procedia - Social and Behavioral Sciences, vol. 31, pp. 800–804, 2012, doi: 10.1016/j.sbspro.2011.12.144.

[4]. M. Bruniges, 21st Century Skills for Australian Students. New South Wales, 2012.

[5]. Pasific Policy Research Center, 21st Century Skills for Students and Teachers. Honolulu, 2010.

[6]. B. J. Thayer-Bacon, Transforming Critical Thinking: Thinking Constructively. New York: Teachers College Press, 2000.

[7]. L. M. Foong, R. S. Shariffudin, and N. Mislan, "Pattern and Relationship Between Multiple Intelligences, Personality Traits and Critical Thinking Skills Among High Achievers in Malaysia," Singapura: IACSIT Press, vol. 27, pp. 205–209, 2012.

[8]. E. Elisanti, S. Sajidan, and B. A. Prayitno, "The Profile of Critical Thinking Skill Students in XI Grade of Senior High School," in Advances in Social Science, Education and Humanities Research, 2017, vol. 218, pp. 205–209, doi: https://doi.org/10.2991/icomse-17.2018.36.

[9]. M. Aliakbari and A. Sadeghdaghighi, "Teachers' Perception of the Barriers to Critical Thinking," Procedia - Social and Behavioral Sciences, vol. 70, pp. 1–5, Jan. 2013, doi: 10.1016/j.sbspro.2013.01.031.

[10]. A. R. As'ari, A. Mahmudi, and E. Nuerlaelah, "Our Prospective MathematicTeachers are not Critical Thinkers Yet," Journal on Mathematics Education, vol. 8, no. 2, pp. 145–156, 2017, doi: http://dx.doi.org/10.22342/jme.8.2.3961.145-156.

[11]. A. C. Biber, A. Tuna, and L. Incikabi, "An investigation of critical thinking dispositions of mathematics teacher candidates," Educational Research, vol. 4, no. 2, pp. 109–117, 2013.

[12]. Rasiman, "Leveling of Critical Thinking Abilities of Students of Mathematics Education in Mathematical Problem Solving," Journal on Mathematics Education, vol. 6, no. 1, pp. 40–52, 2015, doi: 10.22342/jme.61.40.

[13]. P. A. Facione, Critical Thinking: A Statement of Expert Consensus for Purposes of Educational

Assessment and Instruction. New York: The California Academy Press, 1990.

[14]. R. H. Ennis, Critical Thinking. USA: Prentice Hall, Inc, 1996.

[15]. G. S. Aktaş and M. Ünlü, "Critical Thinking Skills of Teacher Candidates of Elementary Mathematics," Procedia - Social and Behavioral Sciences, vol. 93, pp. 831–835, Oct. 2013, doi: 10.1016/j.sbspro.2013.09.288.

[16]. E. Emir, "Contributions of teachers' thinking styles to critical thinking dispositions (Istanbul- Fatih Sample)," Educational Sciences: Theory & Practice, vol. 13, no. 1, pp. 337–347, 2013.

[17]. P. A. Facione, Critical Thinking: What It Is and Why It Counts. Insight Assessment (Online), 2009.