# Comparision Of Fourth Order Rk Method For Weight Loss Problem By Using Intuitionistic Fuzzy Differential Equations

**Dr.M. Mary Jansi Rani, K.Jamshida**

**Abstract** : The purpose of this article to paradigm Runge-Kutta (RK) methods to obtain the numerical solution of intuitionistic fuzzy differential equations (IFDEs) and Convergence RK methods for solving intuitionistic differential equations. Then fourth order RK methods have been compared Arithmetic mean (AM),Centroidal Mean (CeM) andContra-harmonic Mean (CoM) to solve the solving intuitionistic fuzzy initial value problems (IFIVPs). The absolute error results are compared with AM, CeMand CoMwhich show good accuracy.

**Keywords:** IFDE, IVP, RK methods, AM, CeM and CoM.

## 1. INTRODUCTION

The fuzzy sets theory was measuredby one of Intuitionistic fuzzy set (IFS) theory. Atanassov [2] was first proposed Intuitionistic fuzzy set theoryfound to be suitable to deal with new areas. Now-a-days, IFSs are being studied widely and being used in different fields of science and engineering. Abbasbandy and Allahviranloo[1] proposed solution of FDE by RK method with intuitionistic treatment.Melliani et al. [5, 6]have deliberated differential and partial differential equations under intuitionistic fuzzy environment. S. Lata and Amit Kumar [10] have introduced time dependent intuitionistic fuzzy linear differential equation.Mondal and Roy in [4, 9] have discussed the solution of intuitionistic fuzzy ordinary differential equation and introduced second order linear differential equations using the fuzzy boundary value. The convergence and stability of fuzzy initial value problems discussed byKelava [3], Ma et al. [7]. Nirmala and chenthur Pandian [8] proposed for solving IFDE under the differentiability. In this paper presents fourth order RK methodbased onAM, and CeM and CoM for solving intuitionistic fuzzy IVPs. The effectiveness of these methods has been demonstrated by numerical example. The RK methods are compared with AM, CeM and CoM.

## 2. PRELIMINARIES

DEFINITION 1

Let a set X be fixed. AnIFS $A$ inX is an entity having the form $A=\{<x,\mu_A(x),\vartheta_A(x)>0:x\in X\}$ where the $\mu_A(x):\mathrm{X}\to[0,1]$ and $\vartheta_A(x):\mathrm{X}\to[0,1]$ define the degree of membership and degree of non-membership respectively, of the element $x\in X$ to the set $A$ which is a subset of X, for every element of $x\in X,\ 0<\mu_A(x),\ \vartheta_A(x)\le 1$

DEFINITION 2

The $\alpha$ , $\beta$ -cut of an IFN $A=\{x,\mu_A(x),\vartheta_A(x)\setminus x\in X\}$ is defined as follows:

$$A=\{x,\mu_A(x),\vartheta_A(x)\setminus x\in X,\mu_A(x)\ge\alpha\, and\, \vartheta_A(x)\le 1-\alpha\}\ \forall x$$

The $\alpha$ -cut representation of IFN A generates the following pair of intervals and is denoted by $[\mathrm{A}]_\alpha=\{[\mathrm{A}_L^+(\alpha),\mathrm{A}_U^+(\alpha)],[\mathrm{A}_L^-(\beta),\mathrm{A}_U^-(\beta)]\}$.

**DEFINITION3 [7]**

A Triangular Intuitionistic Fuzzy Number (TIFN) U isan intuitionistic fuzzy set in R with the following membership function $\mu_A(x)$ and non-membership function $\vartheta_A(x)$. given as follows:


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$$\mu_A(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2}, & a_2\le x\le a_3\\ 0, & \text{otherwise}\end{cases}$$

$$\vartheta_A(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1'}, & a_1'\le x\le a_2\\ \dfrac{x-a_2}{a_3'-a_2}, & a_2\le x\le a_3'\\ 1, & \text{otherwise}\end{cases}$$

where $a_1'\le a_1\le a_2\le a_3\le a_3'$ and TIFN is denoted by $U=(a_1,a_2,a_3;\ a_1',\ a_2,\ a_3')$

Arithmetic operators over TIFNs can be found in.

Definition 4

A mapping $f:U\to W^n$ for some interval $U$ be an intuitionistic fuzzy function.The $(\alpha,\beta)-\text{cut of } f$ is given by

$$[\mathrm{f(t)}]_{\alpha,\beta}=\left\{[\underline{f}^{+}(\mathrm{t};\alpha),\overline{f}^{+}(\mathrm{t};\alpha)],[\underline{f}^{-}(\mathrm{t};\beta),\overline{f}^{-}(\mathrm{t};\beta)]\right\},$$

where

$$\underline{f}^{+}(\mathrm{t};\alpha)=\min\left\{f^{+}(\mathrm{t};\alpha)\,|\,\mathrm{t}\in U,\ 0\le\alpha\le 1\right\},$$
$$\overline{f}^{+}(\mathrm{t};\alpha)=\max\left\{f^{+}(\mathrm{t};\alpha)\,|\,\mathrm{t}\in U,\ 0\le\alpha\le 1\right\},$$
$$\underline{f}^{-}(\mathrm{t};\alpha)=\min\left\{f^{-}(\mathrm{t};\beta)\,|\,\mathrm{t}\in U,\ 0\le\beta\le 1\right\},$$
$$\overline{f}^{-}(\mathrm{t};\alpha)=\max\left\{f^{-}(\mathrm{t};\beta)\,|\,\mathrm{t}\in U,\ 0\le\beta\le 1\right\}.$$

## 3. Intuitionistic Fuzzy Cauchy Problem

Consider IFDE equation is of the form

$$\begin{cases}y'(t)=f(t,y(t)), & t\in[\mathrm{a,b}]\\ y(t_0)=y_0\end{cases}\tag{3.1}$$

where y is an intuitionistic fuzzy variable $t$, $f(t,y(t))$ is an intuitionistic fuzzy function of the crisp variable tand the intuitionistic fuzzy variable y and y' is the intuitionistic fuzzy derivative. If an initial value $y(t_0)=y_0$ . It is given by $y'(t)=f(t,y(t)),\ \ y(t_0)=y_0$

An equivalent system equationsas follows:

$$y'(t)=\left\{\underline{y'}^{+}(\mathrm{t};\alpha),\overline{y'}^{+}(\mathrm{t};\alpha)],[\underline{y'}^{-}(\mathrm{t};\alpha),\overline{y}^{-}(\mathrm{t};\alpha)]\right\}$$

, where

$$\underline{y}'^{+}(t;\alpha)=\underline{f}^{+}(t,y^{+})=\min\left\{f^{+}(t.u)\,|\,u\in[\underline{y}^{+},\overline{y}^{+}]\right\}$$
$$=F(t,\underline{y}^{+},\overline{y}^{+}),\quad \underline{y}^{+}(t_0)=\underline{y_0}^{+}\tag{3.2}$$
$$\overline{y}'^{+}(t;\alpha)=\overline{f}^{+}(t,y^{+})=\max\left\{f^{+}(t.u)\,|\,u\in[\underline{y}^{+},\overline{y}^{+}]\right\}$$
$$=G(t,\underline{y}^{+},\overline{y}^{+}),\quad \overline{y}^{+}(t_0)=\overline{y_0}^{+}\tag{3.3}$$

$$\underline{y}'^{-}(t;\beta)=\underline{f}^{-}(t,y^{-})=\min\left\{f^{-}(t.u)\,|\,u\in[\underline{y}^{-},\overline{y}^{-}]\right\}$$
$$=H(t,\underline{y}^{-},\overline{y}^{-}),\quad \underline{y}^{-}(t_0)=\underline{y_0}^{-}\tag{3.4}$$
$$\overline{y}'^{-}(t;\beta)=\overline{f}^{-}(t,y^{-})=\max\left\{f^{-}(t.u)\,|\,u\in[\underline{y}^{-},\overline{y}^{-}]\right\}$$
$$=I(t,\underline{y}^{-},\overline{y}^{-}),\quad \overline{y}^{-}(t_0)=\overline{y_0}^{-}\tag{3.5}$$

## 4. Fourth Order Runge-Kutta Methods for Solving Intuitionistic fuzzy Differential Equations

A first order IFDE is of the form

$$y'=f(t,y),\qquad y(\mathrm{t}_0)=y_0\tag{4.1}$$

The construction of Runge-Kutta methods is of the form

$$y_{n+1}=y_n+\int_0^h f(t_n+\tau,y(t_n+\tau))\,d\tau\tag{4.2}$$

Then the integral in (4.2) is approximated by a quadrature formula can be written as

$$y_{n+1}=y_n+h\sum_{i=1}^{s}b_iK_i\tag{4.3a}$$

where

$$K_i=f(t_n+\mathrm{c}_i\,\mathrm{h},\ \ y_n+\mathrm{h}\sum_{j=1}^{i-1}\mathrm{a}_{ij}K_j),\quad i=1,2,\cdots,s\tag{4.3b}$$

The parametric form of the system of IFDE is given by

$$\underline{y}'^{+}(\mathrm{t};\alpha)=\ \ F(t,\ \underline{y}^{+}(\mathrm{t};\alpha),\overline{y}^{+}(\mathrm{t};\alpha)),\quad \underline{y}^{+}(t_0;\alpha)=\underline{y_0}^{+}(\alpha)$$
$$\overline{y}'^{+}(\mathrm{t};\alpha)=G(t,\ \underline{y}^{+}(\mathrm{t};\alpha),\overline{y}^{+}(\mathrm{t};\alpha)),\quad \overline{y}^{+}(t_0;\alpha)=\overline{y_0}^{+}(\alpha)$$

$$\underline{y}'^{-}(\mathrm{t};\beta)=H(t,\ \underline{y}^{-}(\mathrm{t};\beta),\overline{y}^{-}(\mathrm{t};\beta)),\quad \underline{y}^{-}(t_0;\beta)=\underline{y_0}^{-}(\beta)$$
$$\overline{y}'^{-}(\mathrm{t};\beta)=I(t,\ \underline{y}^{-}(\mathrm{t};\beta),\ \overline{y}^{-}(\mathrm{t};\beta)),\quad \overline{y}^{-}(t_0;\beta)=\overline{y_0}^{-}(\beta)$$

for $\alpha,\beta\in[0,1]$.

Fourth order RK for intuitionistic fuzzy IVPs

$$\underline{y}^{+}(t_{n+1};\alpha)=\underline{y}^{+}(t_n;\alpha)+h\sum_{i=1}^{4}b_iK_i$$

$$\overline{y^{+}}(t_{n+1};\alpha)=\overline{y^{+}}(t_n;\alpha)+h\sum_{i=1}^{4}b_iL_i \quad \text{`(4.4a)}$$

$$\underline{y}^{-}(t_{n+1};\beta)=\underline{y}^{-}(t_n;\beta)+h\sum_{i=1}^{4}b_iM_i$$

$$\overline{y^{-}}(t_{n+1};\beta)=\overline{y^{-}}(t_n;\beta)+h\sum_{i=1}^{4}b_iN_i$$

where

$$K_i=F(t_n+c_i h,\ \underline{y}^{+}(t_n;\alpha)+h\sum_{j=1}^{i-1}a_jK_j),$$

$$L_i=G(t_n+c_i h,\ \overline{y}^{+}(t_n;\alpha)+h\sum_{j=1}^{i-1}a_jL_j),$$

$$M_i=H(t_n+c_i h,\ \underline{y}^{+}(t_n;\beta)+h\sum_{j=1}^{i-1}a_{ij}M_j),$$

$$N_i=I(t_n+c_i h,\ \overline{y}^{+}(t_n;\beta)+h\sum_{j=1}^{i-1}a_jN_j),\quad i=1,2,...,4.$$

(4.4b)

Where $b_i$'s and $c_i$'s are constants.

4.1 RK4 for Intuitionistic Fuzzy IVP

RK4Arithmetic mean formula for intuitionistic fuzzy IVP is given by

$$\underline{y}^{+}(t_{n+1};\alpha)=\underline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}\frac{K_i+K_{i+1}}{2}=\underline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}(AM)$$

$$\overline{y}^{+}(t_{n+1},\alpha)=\overline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}\frac{L_i+L_{i+1}}{2}=\overline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}(AM)$$

$$\overline{y^{-}}(t_{n+1};\beta)=\overline{y_n^{-}}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}\frac{M_i+M_{i+1}}{2}=\overline{y^{-}}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}(AM)$$

$$\underline{y}^{-}(t_{n+1};\beta)=\underline{y}^{-}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}\frac{N_i+N_{i+1}}{2}=\underline{y}^{-}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}(AM)$$

$$\underline{y}^{+}(t_{n+1};\alpha)=\underline{y}^{+}(t_n;\alpha)+\frac{h}{6}[K_1+2K_2+2K_3+K_4]$$

$$\overline{y}^{+}(t_{n+1};\alpha)=\overline{y}^{+}(t_n;\alpha)+\frac{h}{6}[L_1+2L_2+2L_3+L_4]$$

$$\underline{y}^{-}(t_{n+1};\alpha)=\underline{y}^{-}(t_n;\alpha)+\frac{h}{6}[M_1+2M_2+2M_3+M_4]$$

$$\overline{y}^{-}(t_{n+1};\alpha)=\overline{y}^{-}(t_n;\alpha)+\frac{h}{6}[N_1+2N_2+2N_3+N_4]$$

RK4Centroidal mean formula for intuitionistic fuzzy IVP is given by

$$\underline{y}^{+}(t_{n+1};\alpha)=\underline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}\frac{2(K_i^2+K_iK_{i+1}+K_{i+1}^2)}{3(K_i+K_{i+1})}=\underline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}(CeM)$$

$$\overline{y}^{+}(t_{n+1},\alpha)=\overline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}\frac{2(L_i^2+L_iL_{i+1}+L_{i+1}^2)}{3(L_i+L_{i+1})}=\overline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}(CeM)$$

$$\overline{y^{-}}(t_{n+1};\beta)=\overline{y_n^{-}}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}\frac{2(M_i^2+M_iM_{i+1}+M_{i+1}^2)}{3(M_i+M_{i+1})}=\overline{y^{-}}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}(CeM)$$

$$\underline{y}^{-}(t_{n+1};\beta)=\underline{y}^{-}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}\frac{2(N_i^2+N_iN_{i+1}+N_{i+1}^2)}{3(N_i+N_{i+1})}=\underline{y}^{-}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}(CeM)$$

RK4Contra-harmonic mean formula for intuitionistic fuzzy IVP is given by

$$\underline{y}^{+}(t_{n+1};\alpha)=\underline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}\frac{K_i^2+K_{i+1}^2}{K_i+K_{i+1}}=\underline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}(CoM)$$

$$\overline{y}^{+}(t_{n+1},\alpha)=\overline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}\frac{L_i^2+L_{i+1}^2}{L_i+L_{i+1}}=\overline{y}^{+}(t_n;\alpha)+\frac{h}{3}\sum_{i=1}^{3}(CoM)$$

(4.7)

$$\overline{y^{-}}(t_{n+1};\beta)=\overline{y_n^{-}}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}\frac{M_i^2+M_{i+1}^2}{M_i+M_{i+1}}=\overline{y^{-}}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}(CoM)$$

$$\underline{y}^{-}(t_{n+1};\beta)=\underline{y}^{-}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}\frac{N_i^2+N_{i+1}^2}{N_i+N_{i+1}}=\underline{y}^{-}(t_n;\beta)+\frac{h}{3}\sum_{i=1}^{3}(CoM)$$

where

$$K_i=F(t_n+c_i h,\ \underline{y}^{+}(t_n;\alpha)+h\sum_{j=1}^{i-1}a_{ij}K_j),$$

$$L_i=G(t_n+c_i h,\ \overline{y}^{+}(t_n;\alpha)+h\sum_{j=1}^{i-1}a_{ij}L_j),$$

$$M_i=H(t_n+c_i h,\ \underline{y}^{+}(t_n;\beta)+h\sum_{j=1}^{i-1}a_{ij}M_j),$$

$$N_i=I(t_n+c_i h,\ \overline{y}^{+}(t_n;\beta)+h\sum_{j=1}^{i-1}a_{ij}N_j),\quad i=1,2,3,4.$$

The missing elements in the matrix $A=(a_{ij})_{i,\,j=1,2,3,4.}$ are defined to be zero. The values of the parameters $a_{ij}$ for the above of means are listed in [11, 12].

## 5. Convergence of Runge-Kutta Methods for Intuitionistic Fuzzy differential equations

The solution is obtained by grid points at

$$a=t_0\le t_1\le\cdots\cdots\le t_N=b \text{ and } h=\frac{b-a}{N}=t_{n+1}-t_n$$

(5.1)

We define

$$F[t_n, y(t_n;a)] = \sum_{i=1}^{s} b_i K_i(t_n, y(t_n;a)) \quad G[t_n, y(t_n;a)] = \sum_{i=1}^{s} b_i L_i(, y(t_n;a))$$

$$H[t_n, y(t_n;\beta)] = \sum_{i=1}^{s} b_i M_i(t_n, y(t_n;\beta)) \quad I[t_n, y(t_n;\beta)] = \sum_{i=1}^{s} b_i N_i(t_n, y(t_n;\beta))$$

(5.2)

The exact and approximate solutions at $t_n,\ 0 \le n \le N$ are denoted by

$[\mathrm{Y}(\mathrm{t}_n)]_{\alpha,\beta} = [\underline{Y}^{+}(\mathrm{t}_n;\alpha), \overline{Y}^{+}(\mathrm{t}_n;\alpha), \underline{Y}^{-}(\mathrm{t}_n;\beta), \overline{Y}^{-}(\mathrm{t}_n;\beta)]$ and

$[\mathrm{y}(\mathrm{t}_n)]_{\alpha,\beta} = [\underline{y}^{+}(\mathrm{t}_n;\alpha), \overline{y}^{+}(\mathrm{t}_n;\alpha), \underline{y}^{-}(\mathrm{t}_n;\beta), \overline{y}^{-}(\mathrm{t}_n;\beta)]$.

We have

$$\begin{aligned}
\underline{Y}^{+}(t_{n+1};\alpha) &\approx \underline{Y}^{+}(t_n;\alpha) + h\,\mathrm{F}[\mathrm{t}_n, \underline{Y}^{+}(\mathrm{t}_n;\alpha), \overline{Y}^{+}(\mathrm{t}_n;\alpha)],\\
\overline{Y}^{+}(t_{n+1};\alpha) &\approx \overline{Y}^{+}(t_n;\alpha) + h\,\mathrm{G}[\mathrm{t}_n, \underline{Y}^{+}(\mathrm{t}_n;\alpha), \overline{Y}^{+}(\mathrm{t}_n;\alpha)],\\
\underline{Y}^{-}(t_{n+1};\beta) &\approx \underline{Y}^{-}(t_n;\beta) + hH[\mathrm{t}_n, \underline{Y}^{-}(\mathrm{t}_n;\beta), \overline{Y}^{-}(\mathrm{t}_n;\beta)],\\
\overline{Y}^{-}(t_{n+1};\beta) &\approx \overline{Y}^{-}(t_n;\beta) + hI[\mathrm{t}_n, \underline{Y}^{-}(\mathrm{t}_n;\beta), \overline{Y}^{-}(\mathrm{t}_n;\beta)].
\end{aligned}$$

and

$$\begin{aligned}
\underline{y}^{+}(t_{n+1};\alpha) &= \underline{y}^{+}(t_n;\alpha) + h\,\mathrm{F}[\mathrm{t}_n, \underline{y}^{+}(\mathrm{t}_n;\alpha), \overline{y}^{+}(\mathrm{t}_n;\alpha)],\\
\overline{Y}^{+}(t_{n+1};\alpha) &= \overline{y}^{+}(t_n;\alpha) + h\,\mathrm{G}[\mathrm{t}_n, \underline{y}^{+}(\mathrm{t}_n;\alpha), \overline{y}^{+}(\mathrm{t}_n;\alpha)],\\
\underline{Y}^{-}(t_{n+1};\beta) &= \underline{y}^{-}(t_n;\beta) + hH[\mathrm{t}_n, \underline{y}^{-}(\mathrm{t}_n;\beta), \overline{y}^{-}(\mathrm{t}_n;\beta)],\\
\overline{Y}^{-}(t_{n+1};\beta) &= \overline{y}^{-}(t_n;\beta) + hI[\mathrm{t}_n, \underline{y}^{-}(\mathrm{t}_n;\beta), \overline{y}^{-}(\mathrm{t}_n;\beta)].
\end{aligned}$$

Thus

$\underline{y}^{+}(\mathrm{t}_n;\alpha)$, $\overline{y}^{+}(\mathrm{t}_n;\alpha)$, $\underline{y}^{-}(\mathrm{t}_n;\beta)$, and $\overline{y}^{-}(\mathrm{t}_n;\beta)$ converges to $\underline{Y}^{+}(\mathrm{t}_n;\alpha)$, $\overline{Y}^{+}(\mathrm{t}_n;\alpha)$, $\underline{Y}^{-}(\mathrm{t}_n;\beta)$, and $\overline{Y}^{-}(\mathrm{t}_n;\beta)$ respectively whenever $h \to 0$.

Lemma 5.1

Let the sequence of numbers $\left\{W_n^{+}\right\}_{n=0}^{N}, \left\{W_n^{-}\right\}_{n=0}^{N}$ satisfy

$$|W_{n+1}| \le A|W_n| + B \quad 0 \le n \le N-1$$

for some given positive constants A and B. Then

$$|W_n| \le A^n |W_0| + B\frac{A^n - 1}{A-1}.$$

Lemma 5.2

Let the sequence of numbers $\left\{W_n\right\}_{n=0}^{N}$ , $\left\{V_n\right\}_{n=0}^{N}$ satisfy

$$|W_{n+1}| \le |W_n| + A.\max\left\{|W_n||V_n|\right\} + B,$$
$$|V_{n+1}| \le |V_n| + A.\max\left\{|W_n||V_n|\right\} + B,$$

for some given positive constants A and B and denote

$$U_n = |W_n| + |V_n|, \quad 0 \le n \le N.$$

Then

$$U_n \le \overline{\mathrm{A}}^n U_0 + \overline{\mathrm{B}}\,\frac{\overline{\mathrm{A}}^n - 1}{\overline{\mathrm{A}} - 1}, \quad 0 \le n \le N \text{ where } \overline{\mathrm{A}} = 1 + 2\mathrm{A} \text{ and } \overline{\mathrm{B}} = 2\mathrm{B}.$$

The domain where F and G are defined is therefore

$$K = \{(t,u,v)/0 \le t \le T, \quad -\infty < v < \infty, -\infty < u \le v\}.$$

Theorem 5.1

Let F (t, u, v) and G (t, u, v) belong to $c^P(K)$ and let the partial derivatives of F and G be bounded over K. Then for arbitrary fixed $\alpha,\beta : 0 \le \alpha,\beta \le 1$, the approximate solution of (4.1), $\left[\underline{y}^{+}(\mathrm{t}_n;\alpha),\ \overline{y}^{+}(\mathrm{t}_n;\alpha), \underline{y}^{-}(\mathrm{t}_n;\beta),\ \overline{y}^{-}(\mathrm{t}_n;\beta)\right]$ converges to the exact solution $\left[\underline{Y}^{+}(\mathrm{t}_n;\alpha),\ \overline{Y}^{+}(\mathrm{t}_n;\alpha), \underline{Y}^{-}(\mathrm{t}_n;\beta),\ \overline{Y}^{-}(\mathrm{t}_n;\beta)\right]$.

Numerical Examples

Example 1Weight-Loss problem:

Consider initial value weights for TrIFN (120,130,135:115,130,140) lb. Let us find the weight after one month (The constant of proportionality is k = 1/3500 lb/cal)

Solution: The time rate of reduction of weight y is given by $\frac{dy}{dt} = -ky$ with the initial condition $y(t_0)$=(120,130,135:115,130,140), an intuitionistic fuzzy number.

The $\alpha,\beta-$cut of $\mathrm{y}(t_0)$ of is given by

$$\mathrm{y}(t_0;\alpha) = y_0(\alpha) = \left\{[\underline{y_0^{+}}(\alpha), \overline{y_0^{+}}(\alpha)], [\underline{y_0^{-}}(\beta), \overline{y_0^{-}}(\beta)]\right\}$$

$$(i.e)\,\mathrm{y}(t_0;\alpha,\beta) = y_0(\alpha,\beta) = \{[120 + 10\alpha\ 135 - 5\alpha], [130 - 15\beta, 130$$

The exact solution is given by

$$\underline{y}^{+}(t;\alpha) = (120 + 10\alpha)e^{-kt} \qquad \overline{y}^{+}(\mathrm{t};\alpha) = (135 - 5\alpha)e^{-kt}$$
$$\underline{y}^{-}(t;\beta) = (130 - 15\beta)e^{-kt} \qquad \overline{y}^{-}(\mathrm{t};\beta) = (130 + 10\beta)e^{-kt}$$

The Error results for example 1 at t =1 and $(\alpha,\beta) = 1$ are shown in the Tables 1,2 and 3. The solution graphs are given in Figures1, 2 and 3.

***TABLE 1** (ARITHMETIC MEAN)*

| $\alpha,\beta$ | Absolute Error for IFRK4_AM at t=1 | | | |
|---|---|---|---|---|
| | $\underline{y}^{+}(t;\alpha)$ | $\overline{y}^{+}(\mathrm{t};\alpha)$ | $\underline{y}^{-}(t;\beta)$ | $\overline{y}^{-}(\mathrm{t};\beta)$ |
| 0.00 | 1.813e-09 | 2.040e-09 | 1.738e-09 | 2.116e-09 |
| 0.20 | 1.844e-09 | 2.025e-09 | 1.783e-09 | 2.085e-09 |
| 0.40 | 1.874e-09 | 2.010e-09 | 1.828e-09 | 2.055e-09 |
| 0.60 | 1.904e-09 | 1.995e-09 | 1.874e-09 | 2.025e-09 |

| | | | | |
|---|---|---|---|---|
| 0.80 | 1.934e-09 | 1.980e-09 | 1.919e-09 | 1.995e-09 |
| 1.00 | 1.964e-09 | 1.964e-09 | 1.964e-09 | 1.964e-09 |

***Table** 2 (Centroidal Mean)*

| $\alpha,\beta$ | Absolute Error for IFRK_CeM at t=1 | | | |
|---|---|---|---|---|
| | $\underline{y}^{+}(t;\alpha)$ | $\overline{y}^{+}(t;\alpha)$ | $\underline{y}^{-}(t;\beta)$ | $\overline{y}^{-}(t;\beta)$ |
| 0.00 | 1.266e-10 | 1.424e-10 | 1.213e-10 | 1.476e-10 |
| 0.20 | 1.286e-10 | 1.413e-10 | 1.244e-10 | 1.455e-10 |
| 0.40 | 1.307e-10 | 1.403e-10 | 1.276e-10 | 1.434e-10 |
| 0.60 | 1.329e-10 | 1.392e-10 | 1.307e-10 | 1.413e-10 |
| 0.80 | 1.350e-10 | 1.381e-10 | 1.339e-10 | 1.392e-10 |
| 1.00 | 1.370e-10 | 1.370e-10 | 1.370e-10 | 1.370e-10 |

***Table 3** (Contra-harmonic Mean)*

| $\alpha,\beta$ | Absolute Error for IFRK_CoM at t=1 | | | |
|---|---|---|---|---|
| | $\underline{y}^{+}(t;\alpha)$ | $\overline{y}^{+}(t;\alpha)$ | $\underline{y}^{-}(t;\beta)$ | $\overline{y}^{-}(t;\beta)$ |
| 0.00 | 9.124e-10 | 1.026e-09 | 8.744e-10 | 1.064e-09 |
| 0.20 | 9.276e-10 | 1.019e-09 | 8.972e-10 | 1.049e-09 |
| 0.40 | 9.428e-10 | 1.011e-09 | 9.200e-10 | 1.034e-09 |
| 0.60 | 9.581e-10 | 1.004e-09 | 9.428e-10 | 1.019e-09 |
| 0.80 | 9.732e-10 | 9.961e-10 | 9.656e-10 | 1.004e-09 |
| 1.00 | 9.884e-10 | 9.884e-10 | 9.884e-10 | 9.884e-10 |

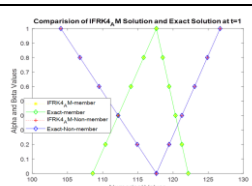


***Fig. 1** (Arithmetic Mean)*

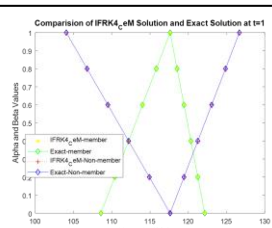


***Fig. 2** (Centroidal Mean)*

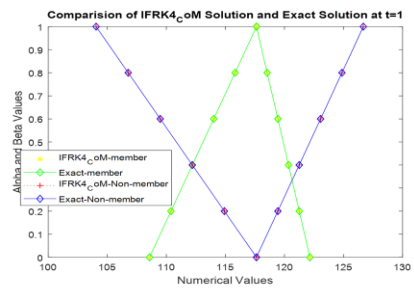


***Fig. 3** (Contra-harmonic Mean)*

## CONCLUSION

In this paper, fourth order RK methods have been constructed to solve the intuitionistic fuzzy IVPs. The effectiveness of these methods had illustrated via example of intuitionistic fuzzy IVPs. The error results had compared with the AM, CeM and CoM. From these tables, wecompared with CeM better than good accuracy AM and CoM. The absolute error is negligible small. Hence, we have observedthat the RK method is suitable for solving intuitionistic fuzzy IVPs.

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