Evaluation Of Slack Based Efficiency Of A Decision Making Unit

B. Venkateswarlu, B. Mahaboob, C. Narayana, C. Subbarami Reddy

Abstract: Data Envelopment Analysis has manifested itself as an outstanding data-oriented performance evaluation method when multiple outputs and inputs appear in a set of peer Decision Making Units (DMUs).This research article primarily focuses on the estimation of slack based measure of efficiency of a DMU. In terms of Shepard's distance functions, Farrell Efficiency (FE), Output Pure Technical Efficiency (OPTE), Output Overall Technical Efficiency (OOTE) and Output Scale Efficiency (OSE) has been proposed here. A Fractional Programming Problem (FPP) has been evaluated in order to compute the slack based measure of efficiency of a DMU and the definition of output distance function and its properties are also presented. Furthermore this paper proposes a slack based efficiency measurement problem as a fractional programming problem which is transformed into a linear programming problem.

Index Terms: Decision Making Unit (DMU), Output pure Technical efficiency (OPTE), Out Overall Technical efficiency (OOTE), Out Put Scale Efficiency (OSE), Slack based efficiency, Technical Efficiency(TE), Technical Efficiency Scores(TESs),Weighted Per Capita(WPC),Overall Performance (OP)

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1. INTRODUCTION

H. Golshani et.al, in 2019[1],suggested a slack-based network model of super efficiency considering the optimal value of intermediate measures in the objective function and right-hand set for ranking overall efficient DMUs to have more accurate and comprehensive ranking DMUs with internal structures. Kristina Kocisova et.al,[2] in 2018,in their research article examined the relative efficiency of agriculture in the European Union using DEA for the period 2005-2015.Tessa Vaxina Soetanto et.al [3] examined the efficiency of Indonesian manufacturing industry listed in the Indonesian stock Exchange during the period of 2010-2014 using non-parametric output oriented super slack-based model. Barbara A. Mark. et.al [4] estimated the technical efficiency of general medical, general surgical and combined medical-surgical nursing units and to incorporate into the DEA model several relevant indicators of patient safety and quality. S Nuti et.al [5] looked into the connections among technical efficiency scores, weighted per capita cost and overall performance. Gahe Zing Samuel Yank et.al [6] applied DEA to compute technical assessment in banking sectors. There are two types of models in DEA: radial and non-radial. Radial models are represented by the CCR (Charnes-Cooper-Rhodes) model. Basically they deal with propositional changes of inputs or outputs. On the other hand non-radial models, e.g. the black based measure of efficiency model, handle input or output slacks directly and do not assume propositional changes of inputs or outputs. On the other hand non-radial models, e.g. the black based measure of efficiency model, handle input or output slacks directly and do not assume propositional changes of inputs or outputs. The input distance function can be parametrically postulated and estimated. A second order approximation by Taylor's series of an arbitrary input distance function is as follows.

$$\ln D_i(u,x) = \alpha_0 + \sum_{i=1}^{m} \alpha_i \ln x_i + \sum_{r=1}^{s} \beta_k \ln u_r + \frac{1}{2}\sum_{i=1}^{m}\sum_{k=1}^{m} \alpha_{ik} (\ln x_i)(\ln x_k) + \frac{1}{2}\sum_{r=1}^{s}\sum_{p=1}^{s} \beta_{rp} (\ln u_r)(\ln u_p) + \sum_{i=1}^{m}\sum_{r=1}^{s} \delta_{ir} (\ln x_i)(\ln u_r)$$

We have,

(i) $D_i(u,x) \geq 1$, $\ln D_i(u,x) \geq 0$

(ii) $D_i(u,x)$ is linear homogeneous in inputs.

(iii) $D_i(u,x)$ Increases monotonically in inputs

$$\frac{\partial D_i(u,x)}{\partial x_k} > 0$$, where k= 1, 2 … s

The input distance function in translog form can be estimated by linear programming.

(i) $$\alpha_0 + \sum_{i=1}^{m} \alpha_i \ln x_{ij} + \sum_{r=1}^{s} \beta_r \ln u_{rj} + \frac{1}{2}\sum_{i=1}^{m}\sum_{k=1}^{m} \alpha_{ik} (\ln x_{ij})(\ln x_{kj}) + \frac{1}{2}\sum_{r=1}^{s}\sum_{p=1}^{s} \beta_{rp} (\ln u_{rj})(\ln u_p) + \sum_{i=1}^{m}\sum_{r=1}^{s} \delta_{ir} (\ln x_{ij})(\ln u_{rj}) \geq 0$$

(ii) Linear Homogeneity conditions

$$\sum_{i=1}^{m} \alpha_i = 1$$

$$\alpha_{1k} + \alpha_{2k} + \ldots.\alpha_{mk} = 0 \ where\ k\ runs\ over\ \{1,2\ldots.m\}$$

$$\alpha_{i1} + \alpha_{i2} + \ldots..\alpha_{im} = 0 \ where\ i\ runs\ over\ \{1,2\ldots..m\}$$

(iii) Conditions of symmetry: $\alpha_{ik} = \alpha_{ki}, \forall i \neq k, \beta_{rp} = \beta_{pr}$ $\forall p \neq r$

Objective Function:

$$\alpha_0 + \sum_{i=1}^{m} \alpha_i \overline{\ln x_i} + \sum_{r=1}^{s} \beta_r \overline{\ln u_r} + \frac{1}{2}\sum_{i=1}^{m}\sum_{k=1}^{m} \alpha_{ik} \overline{\ln x_i}\ \overline{\ln x_k} + \frac{1}{2}\sum_{r=1}^{s}\sum_{p=1}^{s} \beta_{rp} \overline{\ln u_r}\ \overline{\ln u_p} + \sum_{i=1}^{m}\sum_{r=1}^{s} \delta_{ir} \overline{\ln x_i}\ \overline{\ln u_r}$$

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The objective function is minimized with respect to the unknown parameters subject to the constraints (i), (ii) and (iii)

2 .OUTPUT DISTANCE FUNCTION:

Shephard's output distance function $D_0(x_0, u_0)$ is related to the output level sets p(x) [1].$P(x_0)$ = {u: u can be produced by x_0}. The input and output level sets are dualistically related.
P(x) = {u: x □ L (u)} and L (u) = {x: u □ P(x)}

$$D_0(x_0,u_0) = \left[Max\ \{\theta : \theta u_0 \in p(x_0)\}\right]^{-1}$$
$$= \left[F_0(x_0,u_0)\right]^{-1}$$

(i) Shephard's output distance function is inversely related to Farrell's output measure of technical efficiency.

$$\left[D_0(x_0,\delta u_0)\right]^{-1} = Max\ \{\theta : \theta\delta u_0 \in p(x_0)\}$$
$$= \delta^{-1} Max\ \{\delta\theta : \delta\theta u_0 \in p(x_0)\}$$
$$= \delta^{-1} Max\ \{\eta : \eta u_0 \in p(x_0)\}$$
$$= \delta^{-1}\left[D_0(x_0, u_0)\right]^{-1}$$
$$D_0(x_0,\delta u_0) = \delta D_0(x_0,u_0)$$

(iii) $0 \le D_0(x_0,u_0) \le 1$

In one output and multi-inputs setting, the output distance function takes the form:

$$D_0(x_0,u_0) = \frac{u}{\varphi(x)} \text{ and } u_0 \in R_1^+,\ \ x_0 \in R_m^+$$

The output level sets induced by the distance function may be expressed as,

$$p_\varphi(x) = \left\{u : \frac{u}{\varphi(x)} \le 1\right\} = \{u : \varphi(x) \ge u\}$$

If the frontier production function is Cobb-Douglas, the output sets are $p_\varphi(x) = \left\{u : r\prod_{i=1}^{m} x_i^{\alpha_i} \ge u\right\}$

For translog production frontier,

$$p_\varphi(x) = \left\{u : \alpha_0 + \sum_{i=1}^{m}\alpha_i x_i + \frac{1}{2}\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha_{ik}(\ln x_i)(\ln x_k) \ge \ln u\right\}$$

In terms of piece-wise linear technology,

$$\left[D_0(x_0,u_0)\right]^{-1} = Max\ \ \theta$$
$$subject\ to$$
$$\sum_{j=1}^{n}\lambda_j x_{ij} \le x_{i0}\ ,\ \ i = 1, 2, \ldots\ldots\ldots, m$$
$$\sum_{j=1}^{n}\lambda_j u_{rj} \ge \theta\ u_{r0}\ ,\ \ r = 1, 2, \ldots\ldots\ldots, s$$
$$and\ \ \lambda_j \ge 0$$

This problem implicitly assumes scale efficiency. To account for variable returns to scale, we can append the constraint, $\sum_{j=1}^{n}\lambda_j = 1$, to the above linear programming problem

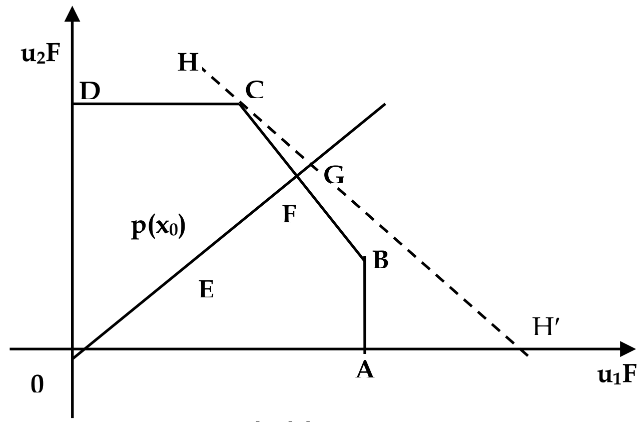

Fig (1)

P (xo) is output level set. The producer who operates at E is inefficient. He can attain technical efficiency by radial magnification of his output from E to F.

$$F_0(x_0,u_0) = \frac{OF}{OE} \text{ and } 0 \le F_0(x_0,u_0) \le 1\ .$$

Where $F_0(x_0,u_0)$ Farrell's output measure of technical efficiency. The straight line HH□ represents the revenue line. Maximum potential revenue can be estimated solving the following optimization problem.

$$R(x,r) = \underset{u}{Max}\ \ \{ru : u \in p(x_0)\}$$

Where $r \in R_+^s$ is the vector of output prices. With respect to piecewise linear technology, this problem may be expressed as follows:

$$R(x,r) = \underset{u}{Max}\ \ ru \downarrow$$
$$subject\ to$$
$$\sum_{j=1}^{n}\lambda_j x_{ij} \le x_{i0}\ ,\ \ i = 1, 2, \ldots\ldots, m$$
$$\sum_{j=1}^{n}\lambda_j u_{rj} \ge u_r\ ,\ \ r = 1, 2, \ldots\ldots, s$$

Where x_{i0} are the observed inputs of DMU0, u_r values have to be determined by optimizing the problem.

Revenue efficiency: $RE = \dfrac{R(x_0, r)}{ru_0}$ where the denominator stands for realized revenue. In terms of fig (1), $RE = \dfrac{OG}{OE}$.

Maximum revenue occurs at C, which belongs to the iso-product curve. Revenue efficiency can be decomposed into the product of output technical efficiency and output allocative efficiency.

3. PURE OUTPUT TECHNICAL EFFICIENCY

In fig (2) two output sets are displayed, viz., $p^{v}(x_0)$ and $p^{k}(x_0)$. These output level sets respectively model variable and constant returns to scale $p^{v}(x_0) \subseteq p^{k}(x_0)$. DMU0 represented by the point P is output technical inefficient. Augmenting its inputs from P to Q radially, it can achieve pure output technical efficiency.

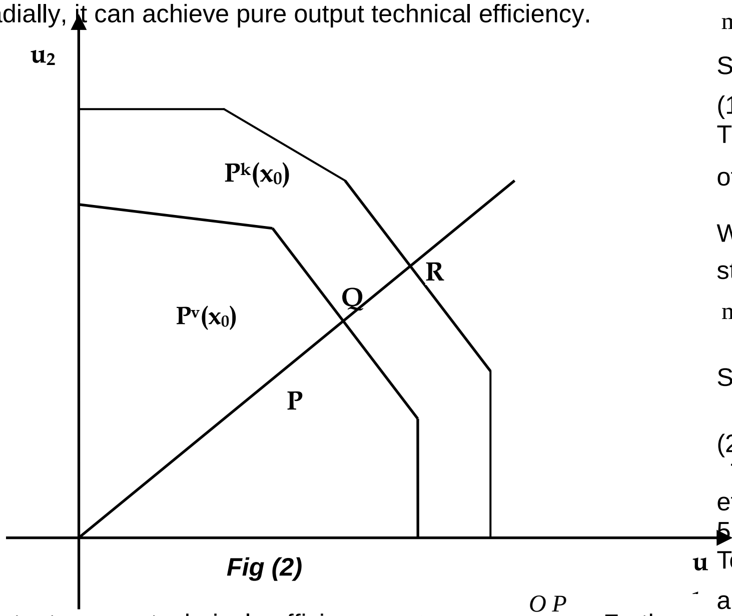

Fig (2)

Output pure technical efficiency: $OPTE = \frac{OP}{OQ}$ Further augmenting its inputs from Q to R, the DMU can achieve overall technical efficiency.

Output overall technical efficiency: $OOTE = \frac{OP}{OR}$. The output scale efficiency of DMU0 is the ratio, $OSE = \frac{OR}{OQ}$ and

$$OOTE = \frac{OP}{OQ}\frac{OQ}{OR} = (OPTE)(OSE)$$

In terms of Shephard's distance functions [1],

$$OPTE = \left[D_0^{v}(x_0, u_0)\right]^{-1}$$

Where $\left[D_0^{v}(x_0, u_0)\right]^{-1} = \underset{\theta}{Max}\left\{\theta : \theta u_0 \in p^{v}(x_0)\right\}$

$$OOTE = \left[D_0^{k}(x_0, u_0)\right]^{-1} = \underset{\theta}{Max}\left\{\theta : \theta u_0 \in p^{k}(x_0)\right\}$$

Where $p^{v}(x_0)$ and $p^{k}(x_0)$ respectively represent output distance functions, modeling variable and constant returns to scale.

$$OSE = \frac{OOTE}{OPTE} = \frac{\left[D_0^{k}(x_0, u_0)\right]^{-1}}{\left[D_0^{v}(x_0, u_0)\right]^{-1}} = \frac{\left[D_0^{v}(x_0, u_0)\right]^{-1}}{\left[D_0^{k}(x_0, u_0)\right]^{-1}}$$

In terms of Farrell efficiency [2], $OPTE = F_0^{v}(x_0, u_0)$

$OOTE = F_0^{k}(x_0, u_0)$ and $OSE = \frac{F_0^{k}(x_0, u_0)}{F_0^{v}(x_0, u_0)}$

4. SLACK BASED EFFICIENCY

Technical efficiency Vs Mix-Efficiency

$\min \rho$

Such that $x\lambda \le \rho x_0$ and $y\lambda \ge y_0, e\lambda = 1, \lambda \ge 0$

(1)

This optimization problem evaluates technical efficiency of $DMU_0(x_0, y_0)$.). Let the optimal solution be (ρ^{*}, λ^{*}).

Where ρ^{*} measures technical efficiency, which is radial. In stage (2) we maximize the sum of all slacks.

$\max = es^{-} + es^{+}$

Such that $x\lambda + s^{-} = \rho^{*} x_0$

$y\lambda + s^{+} = y_0, e\lambda = 1$

(2)

The optimal solution of the above problem gives mix efficiency.

5. SLACK BASED MEASURE OF EFFICIENCY [9]

To estimate the slack based measure of efficiency of a $DMU_0(x_0, y_0)$, we formulate the following fractional program in λ, s^{-} and s^{+}.

$$Min \quad \rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{m}\left(\frac{s_i^{-}}{x_{i0}}\right)}{1 + \frac{1}{s}\sum_{r=1}^{s}\left(\frac{s_r^{+}}{y_{r0}}\right)}.$$

Subject to $\lambda_1 x_1 + \lambda_2 x_2 + \ldots\ldots + \lambda_n x_n + s^{-} = x_0$

$\lambda_1 y_1 + \lambda_2 y_2 + \ldots\ldots + \lambda_n y_n - s^{+} = y_0$

Where $\lambda, s^{-}, s^{+} \in [0, \infty)$

(3)

The additive model: slack based efficiency:

$\max(z) = es^{-} + es^{+}$

$x\lambda + s^{-} = x_0$

$y\lambda + s^{-} = y_0, e\lambda = 1, s^{+} \ge 0$

(4)

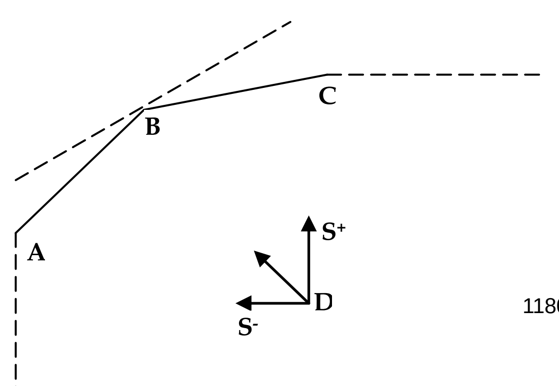

Fig (3)

In the above figure we have,

(1) D is an inefficient DMU (2) S-=0, maximization es^{+} of vertically projects D on the frontier. The outcome is output based BCC problem

(3) $s^{+} = 0$, maximization of es^{-} horizontally projects D on the frontier. The outcome is input based BCC problem

(4) Maximization of $es^{-} + es^{+} = 1$ compares D with B, a point on the frontier production function.

Additive Efficiency: A DMU is said to be additive efficient if and only if, $s^{+} = 0$ and $s^{-} = 0$. Let $s^{-}_{*}, s^{+}_{*}, \lambda_j^{*}$ be an optimal solution to the additive model.

6. THE CCR EFFICIENCY Vs THE SLACK BASED EFFICIENCY

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$$Min\ \theta$$

$$subject\ to \sum_{j=1}^{n} \lambda_j x_j + \omega^{-} = \theta x_0$$

$$and \sum_{j=1}^{n} \lambda_j y_j - \omega^{+} = y_0,\ \lambda_j, \omega^{+}, \omega^{-} \geq 0$$

(5)

Let $\left(\theta^{*}, \ \lambda_j^{*}, \ \omega_{*}^{-}, \ \omega_{*}^{+}\right)$ be the optimal solution of the CCR problem, we have, $\sum_{j=1}^{n} \lambda_j^{*} x_j + \omega_{*}^{-} = \theta^{*} x_0$ and

$$x_0 - \theta^{*} x_0 + \sum_{j=1}^{n} \lambda_j^{*} x_j = x_0$$

$$x_0 = \left(1 - \theta^{*}\right) x_0 + \sum_{j=1}^{n} \lambda_j^{*} x_j + \omega_{*}^{-}$$

(6)

$$y_0 = \sum_{j=1}^{n} \lambda_j^{*} x_j - \omega_{*}^{+}$$

(7)

The expression (6) and (7) may be expressed as follows

$$\sum_{j=1}^{n} \lambda_j^{*} x_j + s^{-} = x_0 \ \text{and} \sum_{j=1}^{n} \lambda_j^{*} y_j - s^{+} = y$$

Thus $\left(\lambda_j^{*}, \ s^{-}, \ s^{+}\right)$ is a solution which is feasible for the slack based efficiency problem $\rho^{*} \leq \theta^{*}$.

(8)

Let $\left(\rho^{*}, \ \lambda_j^{*}, \ s_{*}^{-}, \ s_{*}^{+}\right)$ be an optimal solution to the slack based efficiency problem

$\lambda_1^{*} x_1 + \lambda_2^{*} x_2 + \ldots\ldots\lambda_n^{*} x_n + s^{-} = x_0$ and

$$\theta x_0 + \sum_{j=1}^{n} \lambda^{*}{}_j x_j + \left(\theta - 1\right) x_0 + s$$

$y_0 = \sum_{j=1}^{n} \lambda_j^{*} y_j - s_{*}^{+}, \ \sum_{j=1}^{n} \lambda_j^{*} x_j + \omega_{*}^{-} = \theta x_0$ and

$\sum_{j=1}^{n} \lambda_j^{*} y_j - \omega_{*}^{+} = y_0$. Where $\omega_{*}^{-} = \left(\theta - 1\right) x_0 + s_{*}^{-}$ and

$\omega_{*}^{+} = \lambda_{*}^{+}$

$\left(\lambda_j^{*}, \ \theta, \ s_{*}^{-}, \ s_{*}^{+}\right)$ is a feasible solution of the CCR problem

$$\rho^{*} \geq \theta^{*} \qquad (9)$$

Combine (8) and (9) to get, $\rho^{*} = \theta^{*}$

Thus, a DMU is CCR efficient if and only if it is slack based efficient.

7. CONCLUSION AND FUTURE RESEARCH

In the above discussion the input distance function in translog form has been estimated by linear programming and an optimization problem is proposed in order to evaluate maximum potential revenue. A problem whose optimal solution gives mix efficiency has been formulated. In the context of future research one can propose different types of efficiency stability regions and their infeasibility in DEA. Furthermore for extremely efficient DMUs super efficiency can be estimated.

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