# Inverse Majority Neighborhood Number For Cartesian Product Of Graphs

**T. Dhivya, I. Paulraj Jayasimman ,J. Joseline Manora**

**Abstract**: If $S_M$ be the majority set of $G$ if $V-S_M$ contains a majority neighborhood set $S_M^{'}$ of $G$ then $S_M^{'}$ be the inverse majority neighborhood set of $G$ with respect to $S_M$. In this article the inverse majority neighborhood number $n_M^{-1}(\mathrm{G})$ of $G$ are determined for cartesian product of graphs.

**Index Terms**: Majority Neighborhood Set, Majority Neighborhood Number, Inverse Majority Neighborhood Number.

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## 1 INTRODUCTION

The concept of majority neighborhood set has been studied by Prof.V.Swaminathan and J. Joselin Manora[9]. The neighborhood parameters are studied in the articles[11,][15], [16], [17] ,[21] ,[22].Further inverse majority neighborhood set introduced by I.Paulraj Jayasimman[19],[20]. A set $S$ of vertices in a graph $G$ is a neighborhood set if $G=\bigcup_{v\in S}\langle N[v]\rangle$ ,where $\langle N[v]\rangle$ is the subgraph of $G$ induced by $v$ and all vertices adjacent to $v$. The neighborhood number $n_0(G)$ of $G$ is the minimum number of vertices in a neighborhood set of $G$ [7]. A set $S \subseteq V(G)$ is called a majority neighborhood set if $G_M=\bigcup_{v\in S}\langle N[v]\rangle$ contains at least $\left\lceil \frac{p}{2}\right\rceil$ vertices and at least $\left\lceil \frac{q}{2}\right\rceil$ edges. A majority set $S$ is called a minimal majority neighborhood set if no proper subset of $S$ is a majority neighborhood set. The minimum cardinality of a majority neighborhood set is called the majority neighborhood number of $G$ and is denoted by $N_M(G)$[9]. If $S_M$ be the majority neighborhood set of $G$. If $V-S_M$ contains a majority neighborhood set $S_M^{'}$ of $G$, then $S_M^{'}$ is called an inverse majority neighborhood set of $G$ with respect to $S_M$. The inverse majority neighborhood number $n_M^{-1}(\mathrm{G})$ of $G$ is the minimum cardinality of an inverse majority neighborhood set of $G$[19].Cartesian product of graph operation has play vital role in chemical graph and network structure.


- *T. Dhivya, Assistant professor,Department of Mathematics,AMET Deemed to be University,Chennai.*
- *Email address: dhivya.t@ametuniv.ac.in*
- *I. Paulraj Jayasimman, Associate professor, Department of Mathematics, AMET Deemed to be University,Chennai.*
- *Corresponding Author:Email address: ipjayasimman@gmail.com*
- *J. Joseline Manora, PG & Research Department*
- *of Mathematics, T.B.M.L College, Porayar.*
- *Email address: joseline_manora@yahoo.co.in*


## 2 INVERSE MAJORITY NEIGHBORHOOD NUMBER OF CARTESIAN PRODUCT OF GRAPHS

Theorem 2.1. For $G=P_2\times P_m$ with $m\geq 2$ then $n_m^{-1}(\mathrm{G})=\left\lceil \frac{m}{2}\right\rceil$

Proof. Let $G=P_2\times P_m$ with $m\geq 2$ and the vertex set $V(\mathrm{G})=\{v_{11},v_{12},...,v_{1r},v_{21},v_{21},...,v_{2s}\}$ . $\mathrm{d}(v_{1r})=\mathrm{d}(v_{2s})=2$ $|V(\mathrm{G})|=p=2m$ and $|\mathrm{E(G)}|=q=2(\mathrm{m}-1)+\mathrm{m}$ .

Claim: $S_M^{'}$ be the inverse majority neighborhood set with cardinality $\left\lceil \frac{m}{2}\right\rceil$

Case(i). m=even. Let $S_M^{'}$ be the inverse majority neighborhood set

then $S_M^{'}=\{v_{13},v_{15},...,v_{1r-1},v_{21}\}$ $|S_M^{'}|=\left\lfloor\left(\frac{m-1}{2}\right)\right\rfloor+1\Rightarrow$

$$|\langle N[S_M^{'}]\rangle|=3\left(\left\lfloor\frac{m-1}{2}\right\rfloor\right)+2\geq\left\lceil\frac{2(m-1)+m}{2}\right\rceil=\left\lceil\frac{3m-2}{2}\right\rceil=\left\lceil\frac{q}{2}\right\rceil.$$

$|N[S_M^{'}]|=\left\lceil\frac{3m}{2}\right\rceil>\left\lceil\frac{p}{2}\right\rceil=\left\lceil\frac{2m}{2}\right\rceil=m$ .Therefore

$|S_M^{'}|\leq n_m^{-1}(\mathrm{G})$ .Suppose $S_M^{'}=\{v_{13},v_{15},...,v_{1r-1}\}$ then

$$|\langle N[S_M^{'}]\rangle|=\left\lceil\frac{3m}{2}\right\rceil-3\leq\left\lceil\frac{2(\mathrm{m}-1)+1}{2}\right\rceil=\left\lceil\frac{2\mathrm{m}-2+\mathrm{m}}{2}\right\rceil$$

$\leq\left\lceil\frac{q}{2}\right\rceil$ . $|N[S_M^{'}]|=\left\lceil\frac{3m}{2}\right\rceil-2<\left\lceil\frac{p}{2}\right\rceil$ .Therefore $|S_M^{'}|\geq n_m^{-1}(\mathrm{G})$ .

Hence $|S_M^{'}|=n_m^{-1}(\mathrm{G})$

Case(ii).If $m=odd$

SubCase(i).If m=3 then $S'_M=\{v_{22},v_{23}\}$. $|S'_M|=\left\lceil\frac{m-1}{2}\right\rceil+1$

$|\langle N[S'_M]\rangle|=2\left(\left\lceil\frac{m+1}{2}\right\rceil\right)=\left\lceil\frac{q}{2}\right\rceil.|N[S'_M]|=2\left(\left\lceil\frac{m+1}{2}\right\rceil\right)+1>\left\lceil\frac{p}{2}\right\rceil.$

$|S'_M|\le n_m^{-1}(\text{G})$ .Suppose $S'_M=\{v_{22}\}$ then

$|S'_M|=\left\lceil\frac{m-1}{2}\right\rceil\Rightarrow|\langle N[S'_M]\rangle|=\left\lceil\frac{m-1}{2}\right\rceil+2\le\left\lceil\frac{q}{2}\right\rceil$

$|S'_M|\ge n_m^{-1}(\text{G})$ . Hence $|S'_M|=n_m^{-1}(\text{G})$ .

Sub Case(i): If $m>3$ then $S'_M=\{v_{13},v_{15},...v_{1r},v_{21}\}$ then $|S'_M|=\left\lfloor\frac{3m}{2}\right\rfloor-2$ and $|\langle N[S'_M]\rangle|=3\left(\left\lceil\frac{m-3}{4}\right\rceil\right)$ $+4=\left\lceil\frac{q}{2}\right\rceil$. $|N[S'_M]|=\left(\left\lceil\frac{3m}{2}\right\rceil\right)+1>\left\lceil\frac{p}{2}\right\rceil$.Therefore $|S'_M|\le n_m^{-1}(\text{G})$ . Suppose $S'_M=\{v_{13},v_{15},...v_{1r}\}$ then $|S'_M|=\left\lfloor\frac{m}{2}\right\rfloor$and $|\langle N[S'_M]\rangle|=\left\lfloor\frac{3m}{2}\right\rfloor-2<\left\lceil\frac{q}{2}\right\rceil$and $|S'_M|\ge n_m^{-1}(\text{G})$. Hence $|S'_M|=n_m^{-1}(\text{G})$ .

Theorem 2.2. If the graph $G=P_3\times P_m$ with $m\ge3$ then $n_m^{-1}(\text{G})=\left\lceil\frac{2m}{3}\right\rceil$

Proof. Let $G=P_3\times P_m$ be the vertex set. $V_1(\text{G})=\{v_{11},v_{12},...,v_{1r}\}$ and $V_2(\text{G})=\{v_{21},v_{22},...,v_{1s}\}$ $V_3(\text{G})=\{v_{31},v_{32},...,v_{1t}\}$ . $|V(\text{G})|=p=3m$. $|\text{E(G)}|=q=3(2\text{m}-1)$. $\text{d}(v_{1r})=\text{d}(v_{2s})=\text{d}(v_{3t})=3$ Let $S'_M$ be the inverse majority neighborhood set with cardinality $\left\lceil\frac{2m}{3}\right\rceil$.

Case(i).If $m$ is odd then $|S'_M|=\left\lceil\frac{2m}{3}\right\rceil\Rightarrow|\langle N[S'_M]\rangle|=$ $5\left\lceil\frac{m-1}{2}\right\rceil+1>\left\lceil\frac{3(2m-1)}{2}\right\rceil=\left\lceil\frac{6m-3}{2}\right\rceil=\left\lceil\frac{q}{2}\right\rceil$.

Suppose $|\langle N[S'_M]\rangle|-1=\left\lceil\frac{5m}{2}\right\rceil-2>\left\lceil\frac{3(2m-1)}{2}\right\rceil=\left\lceil\frac{q}{2}\right\rceil$.

Therefore $|S'_M|\le n_m^{-1}(\text{G})$ .

Case(ii).If m is even then $|\langle N[S'_M]\rangle|=\left\lfloor\frac{5m}{2}\right\rfloor-1$ $<\left\lceil\frac{n(m-1)+m(n-1)}{2}\right\rceil=\left\lceil\frac{2nm-n-m}{2}\right\rceil<\left\lceil\frac{q}{2}\right\rceil$.

Therefore $|S'_M|\ge n_m^{-1}(\text{G})$ .Hence $n_m^{-1}(\text{G})=\left\lceil\frac{2m}{3}\right\rceil$.

Theorem 2. 3. For graph $G=P_4\times P_m$ with $m\ge4$,

$$n_m^{-1}(G)=\begin{cases}\left\lceil\frac{7m-4}{8}\right\rceil+1 & if\ m\le4\\ \left\lceil\frac{7m-4}{8}\right\rceil & if\ m>4\end{cases}$$

Proof: Let $G=P_4\times P_m$ with the vertex set $V_1(\text{G})=\{v_{11},v_{12},...,v_{1r}\}$ $V_2(\text{G})=\{v_{21},v_{22},...,v_{1s}\}$ and $V_3(\text{G})=\{v_{31},v_{32},...,v_{3t}\}$ $V_4(\text{G})=\{v_{41},v_{42},...,v_{4l}\}$. $|V(\text{G})|=p=4m$ and $|\text{E(G)}|=q=7m-4$ .

Case(i).If $m\le4$ then $|\langle N[S'_M]\rangle|=2\left\lceil\frac{6m}{4}\right\rceil+1\ge\left\lceil\frac{7m-4}{8}\right\rceil$ $=\left\lceil\frac{q}{2}\right\rceil$. $|S'_M|\le n_m^{-1}(\text{G})$ . Suppose $|\langle N[S'_M]\rangle|-1=2\left\lceil\frac{3m}{2}\right\rceil$ $\le\left\lceil\frac{7m-4}{8}\right\rceil=\left\lceil\frac{q}{2}\right\rceil$. Therefore $|S'_M|\ge n_m^{-1}(\text{G})$ . Hence $|S'_M|=n_m^{-1}(\text{G})$ .

Case(ii). If $m>4$ then $|S'_M|=\left\lceil\frac{7m-4}{8}\right\rceil\Rightarrow|\langle N[S'_M]\rangle|=2$ $\left\lceil\frac{7m-4}{8}\right\rceil\ge\left\lceil\frac{7m-4}{2}\right\rceil=\left\lceil\frac{q}{2}\right\rceil$. $|S'_M|\le n_m^{-1}(\text{G})$ . Suppose $|\langle N[S'_M]\rangle|-1=2\left\lceil\frac{7m-4}{4}\right\rceil-1\le\left\lceil\frac{7m-4}{2}\right\rceil=\left\lceil\frac{q}{2}\right\rceil$.Therefore $|S'_M|=n_m^{-1}(\text{G})=\left\lceil\frac{7m-4}{4}\right\rceil$.

Theorem 2.4. If the graph $G=P_5\times P_m$ with $m\ge5$ then

$$n_m^{-1}(G)=\begin{cases}\left\lceil\frac{9m-5}{8}\right\rceil+1 & if\ m=5\\ \left\lceil\frac{9m-4}{8}\right\rceil & if\ m>5\end{cases}$$

Proof. For the graph $G=P_5\times P_m$ with $|V(\text{G})|=p=5m$ and $|\text{E(G)}|=q=9m-5$. $d(v_{1r})=d(v_{5f})=2$ and $d(v_{2s})=d(v_{3t})=d(v_{4l})=3$ .

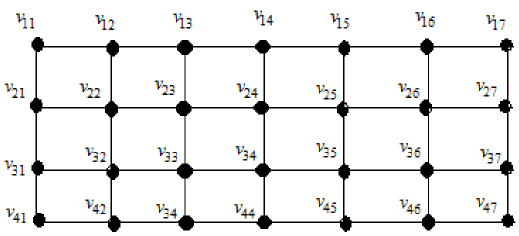
Case(i). If $m=5$ then $|S'_M|=\left\lceil\frac{9m-4}{8}\right\rceil+1\Rightarrow|\langle N[S'_M]\rangle|$ $=2\left\lceil\frac{9m-5}{4}\right\rceil+1>\left\lceil\frac{9m-5}{2}\right\rceil>\left\lceil\frac{q}{2}\right\rceil$. $|N[S'_M]|=2\left\lceil\frac{9m-5}{4}\right\rceil-1$ $>\left\lceil\frac{5m}{2}\right\rceil>\left\lceil\frac{p}{2}\right\rceil$. $|S'_M|<n_m^{-1}(\text{G})$

Case(ii). If $m>5$ then $|S'_M|=\left\lceil\frac{9m-5}{8}\right\rceil\Rightarrow|\langle N[S'_M]\rangle|=2$ $\left\lceil\frac{9m-5}{4}\right\rceil>\left\lceil\frac{9m-5}{2}\right\rceil>\left\lceil\frac{q}{2}\right\rceil$. $|S'_M|<n_m^{-1}(\text{G})$ . Suppose $|S'_M|-1=\left\lceil\frac{9m-5}{8}\right\rceil-1$ then $|\langle N[S'_M]\rangle|=2\left\lceil\frac{9m-5}{4}\right\rceil$ $-1<\left\lceil\frac{9m-5}{4}\right\rceil<\left\lceil\frac{q}{2}\right\rceil$. $|S'_M|>n_m^{-1}(\text{G})$. Hence $|S'_M|=n_m^{-1}(\text{G})$

$=\left\lceil \frac{9m-5}{8} \right\rceil$.

Theorem 2.5. For graph $G=P_n \times P_m$ with $m>5$ then $n_m^{-1}(G)=\left\lceil \frac{2nm-n-m}{8} \right\rceil$.

## III EXAMPLE




For the graph $G=P_4 \times P_7$ with $p=4m=4(7)=28$ and $q=7m-4=7(7)-4=45$ then $n_m^{-1}(G)=6$.

Theorem 2.7. If the graph $G=C_3 \times P_m$ with $m \geq 3$ then

$$n_m^{-1}(G)=\begin{cases} \left\lceil \frac{2(m-1)}{3} \right\rceil +1 & if\ m \leq 3 \\ \left\lceil \frac{3(2m-1)}{8} \right\rceil & if\ m>3 \end{cases}$$

Proof. For the graph $G=C_3 \times P_m$ with $|V(G)|=p=3m$ and $|E(G)|=q=3(m-1)+3m$

Case(i). If $m \leq 3$ then $|S_M^{'}|=\left\lceil \frac{2(m-1)}{3} \right\rceil +1 \Rightarrow |\langle N[S_M^{'}]\rangle|$ $=4\left\lceil \frac{3m}{2} \right\rceil -7>\left\lceil \frac{3(m-1)+3m}{2} \right\rceil=\left\lceil \frac{3m-3+3m}{2} \right\rceil=$ $\left\lceil \frac{3(2m-1)}{2} \right\rceil=\left\lceil \frac{q}{2} \right\rceil$. $|N[S_M^{'}]|=\left\lceil \frac{3m}{2} \right\rceil+2>\left\lceil \frac{3m}{2} \right\rceil=\left\lceil \frac{p}{2} \right\rceil$.

Therefore $|S_M^{'}| \leq n_m^{-1}(G)$. Suppose $|S_M^{'}|-1$ then $|\langle N[S_M^{'}]\rangle|=3\left\lceil \frac{3m}{2} \right\rceil-6<\left\lceil \frac{q}{2} \right\rceil$. $|N[S_M^{'}]|=3\left\lceil \frac{m+1}{2} \right\rceil+1<\left\lceil \frac{p}{2} \right\rceil$

Therefore $|S_M^{'}| \geq n_m^{-1}(G)$. Hence $|S_M^{'}|=n_m^{-1}(G)$.

Case(ii). If $m>3$ then $|S_M^{'}|=\left\lceil \frac{3(2m-1)}{8} \right\rceil$.

Sub case(i). If m is odd then $|\langle N[S_M^{'}]\rangle|=6\left\lceil \frac{m}{2} \right\rceil+2<\left\lceil \frac{q}{2} \right\rceil$.

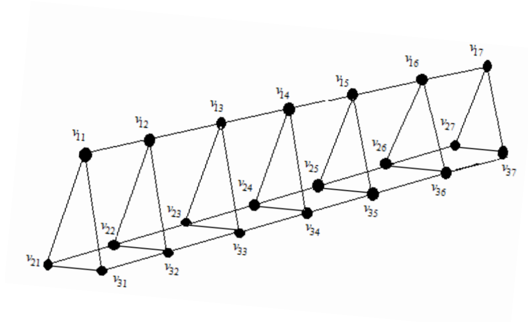
Suppose $|S_M^{'}|-1$ then $m=5,9,13,17,...$ then $|\langle N[S_M^{'}]\rangle|=6\left\lceil \frac{m-1}{2} \right\rceil<\left\lceil \frac{q}{2} \right\rceil$. Suppose $m=7,11,15,19,...$ then $|\langle N[S_M^{'}]\rangle|=4\left\lceil \frac{2m}{3} \right\rceil<\left\lceil \frac{q}{2} \right\rceil$. $|S_M^{'}| \geq n_m^{-1}(G)$.

Sub case(ii). If m is even. Suppose $m=4i(for\quad i=1,2,3,...)$ then $|\langle N[S_M^{'}]\rangle|=6\left\lceil \frac{m}{2} \right\rceil>\left\lceil \frac{q}{2} \right\rceil$. Therefore $|S_M^{'}| \leq n_m^{-1}(G)$.

Hence $n_m^{-1}(G)=|S_M^{'}|=\left\lceil \frac{3(2m-1)}{8} \right\rceil$.

## IV EXAMPLE

For $G=C_3 \times P_m$ with $p=3m=3(7)=21$ and $q=3(m-1)+3m=3(7-1)+3(7)=39$ then $n_m^{-1}(G)=5$




## V CONCLUSION

The research results obtained to find the inverse majority neighborhood number for Cartesian product of graphs.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