Peak Ageing Inequality In Some Asian Countries

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Abstract: In this paper, an attempt has been made to use three different modified inequality indices namely Gini coefficient (G), Logarithmic transformation of Geometric equivalent of Gini coefficient (LTGEG) and Trigonometric measures of Gini coefficient (TMG) for measuring peak ageing inequality. The study is based on a secondary population data of Bangladesh as well as Asian countries which is taken from an international data base, US census Bureau. From the analysis it is observed that the Gini coefficient shows equally sensitivity at all levels. The coefficient is more concern for the country which are closed to the line of absolute equality. For example, Gini coefficient is more concern for India than in China. The sensitivity levels of the LTGEG is higher for India than in China which indicates that this index is more sensitive to the countries whom are far from the line of absolute equality like India. From the analysis it is also found that the trigonometric measure of Gini coefficient satisfies transfer principle as well as shows higher sensitivity. Finally, based on this analysis it can be conclude that the trigonometric measure of the Gini coefficient is the best measure of peak aging inequality among the other measures.

Keywords: Inequality, Peak Ageing, Gini Index, Geometric Measure, Trigonometric Measure, Asia, Bangladesh

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1. INTRODUCTION

In spite of being a long subject of interest to sociologists, the term inequality has been cautiously specified by a very few scholars. The difference between perfect equality and a state of inequality is quite easy to understand. But in case of some social regard when one considers two different unequal distributions then how does one decide which distribution is the more unequal? The answer to this question would seem to be a prerequisite for any theory of the determinants and consequents of social inequality. Yet even Lenski's influential theory on the effects of economic surplus and democracy on inequality fails to include a definition of the dependent variable [1]. In recent times the necessity for adopting some precise measures of inequality including Gini index or standard deviation has been consulted by test of hypothesis while examining the inequality among several societies [2]–[4]. Due to the unavailability of specific ground for selecting the perfect inequality measure among the numerous measures of inequality, the researchers picked a measure on the basis of convenience, familiarity, or on vague methodological criteria. Nevertheless, the decision to rank one distribution is more unequal than another has both theoretical and methodological implications. Actually, the selection of an inequality measure is completely considered as a choice among alternative definition of inequality rather than a choice among alternative ways of measuring a single theoretical construct [5]. This choice can make a difference. Although some have reported high correlations among different inequality measures [6], such correlations are relevant only for the particular populations and variables for which they are computed [5]. And in one case of particular interest, it has been shown that the rank ordering of countries by income inequality can differ substantially with different measures of inequality [7], [8]. In spite of being vastly used for the comparison among nations, cities, and other social units, the properties of alternative measures of inequality have received little attention in the sociological literature. A major exception to the failure of sociological theorists to specify what they mean by greater and lesser inequality is the recent work of Peter Blau [9], [10]. He assumes inequality as a fundamental characteristics of all graduated social parameters (quantitative status variables), and proposes that it can be conceptualized as ''the average difference in the status between any pairs relative to the average status'' [9]. Nothing that the Gini index is an appropriate algebraic specification of this concept, he proceeds to consider in detail how changes in the distribution affect the Gini index [10]. The overall level of inequality in a country, region or population group and more generally the distribution of consumption, income or other attributes is also in itself an important dimension of welfare in that group. Allison suggested that inequality measures not only can be calculated for any distribution including consumption, income or other monetary variables but also for land and other continuous and cardinal variables [5]. Measures of inequality have applied to entire populations with group data [5]. In literature, it has found that the traditional measure of population ageing has two short comings. First one is the use of cutoff point for both old and young age of population. For illustration, 65 is considered for developed and 60 for developing countries as a cutoff point for old age. Likewise, the cutoff point of young age is 15 and 20 in developing country and developed country respectively. Secondly, the precision of any measure increases along with the observable range of variability enhances. The main lacking of the traditional measure is that it only considers the change of age cohorts while overlooking the total pattern of the age structure of population. For overcoming this short coming Kii suggested to apply the regression coefficient (the slope of trend line) [11]. Nath et al. observed that ageing is one of the most emerging problems of Bangladesh while studying the ageing process of some Asian countries [12]. The consequences of this problem is vast and a clear indication of increasing trend of Bangladesh demographic ageing process. Although the Gini index satisfies the basic criteria of scale invariance and the principle of transfers, the other measures namely the coefficient of variation and Theil's measure are usually preferable. In real life for interval-level data some situations arise where the above mentioned measures are not completely appropriate but valid comparison can also be made by adopting social welfare function as an alternative approach to generate measures of inequality and methods of estimation, testing, and decomposition [5]. The Gini index is vastly used by the economist to rank income distribution in empirical studies along with the risk analysis and financial theory. Therefore, we won't surprise to see Gini index as a measure of dispersion in portfolio analysis [7], [13]–[15]. An empirical example is provided by Blau's reanalysis of data reported by Hauser et al. who give the frequency distribution among 12 occupational categories of U.S. men at selected ages in

1952, 1962 and 1972 [9], [16]. Using the Duncan SEI scores corresponding to the occupational categories, he calculated the Gini index for men aged 35-44 during the three periods were 0.353, 0.300 and 0.318, a clear decline in inequality over time. Blau (1978) as an example of nonnegative ratio scale data for applied to test on occupational status of blacks and whites reported by Farley (1977). In 1969, black females had a coefficient of variation of 0.773 and a mean of 28.6; white females had a coefficient of variation of 0.491 and a mean of 42.6. He concluded that in this sample black females were more unequal in occupational status than white females. The measures, the coefficient of variation (CV) and the Gini index (G) in statistics texts are only appropriate for variables which have a theoretically fixed zero point that means ratio scale variable such as income [17]. Shalit defined Gini index of inequality for individual data as a simple measure of dispersion and stated that its' field of application is not limited to income distribution [15]. From the beginning to present time, the Gini index is interpreted as a measure of dispersion and statistically it is a function of Gini's mean difference [18]. Pyatt discussed it deeply and interpret as the average gain to be expected, if each and every one is allowed to compare ones' income with the income of another one and to keep the income that is higher. By neglecting the transfer of income and how much poor or rich an individual is, Dalton stated that measures of inequality should increase when income is transferred from poorer to richer individual. Considering a suggestion from Pigou, he also constructed a condition and mentioned that a transfer of income from a richer to a poorer individual, until that transfer does not reverse the ranking of the two, will result in decrease inequity. This principle is known as the Pigou–Dalton principle [19]. In social science, Gini coefficient is the most popular statistical measure of diversity or inequality and vastly applied by the economist as a standard measure of inter-individual or inter-household inequality in income or wealth [5], [7], [19], [20]. From literature, it has also observed that it is employed to measure variability in levels of mortality among socio-economic groups and analyze the variation in degree of people's inequality in the face of death overtime and across countries [21]. In literature other measures including variance, coefficient of variation and standard deviation have also been considered but their performance is not up to the mark because of their total concentration on differences around mean and the violation of Pigou-Dalton principle. The Pigou-Dalton principle states that any transfer from smaller group (poor group) to higher group (rich group), other thing remaining the same, would always increase the inequality measure. Similarly, it is thought that any transfer from higher group (rich group) to smaller group (poor group), other thing remaining the same, would always decrease the inequality measure [22]. Although Kelly et al. and Kolm argue that proportional increases in income represent an increase in inequality [4], [23]. Taking the opposite point of view, Dalton and Sen suggest that inequality should decrease when all incomes are increased proportionately [22], [24]. The change of inequality over time can also be analyzed. For a specific sector of economy, one can estimate the change among different groups of the population to visualize whether inequality changes have been similar for all that have taken place. Despite of increasing average income between 1983 and 1991, the inequality is also increased, especially among the poor in rural Tanzania. The Gini coefficient support this findings because it also increases from 0.52 to 0.72 [25]. In the above discussion, most of the literatures are related to income inequality. Very few articles were focused on population ageing inequality all over the world. In Bangladesh, Taj Uddin et al., 2012 conducted an excellent study on ''A new index for measuring ageing inequality'' which is considered to be one of the most important baseline studies of ageing inequality of older people in Bangladesh [26]. Majumder proposed several alternative measures of economic based on the existing formulas of Gini coefficient and used them to the data set of 96 countries on distribution of income or consumption from world development indicators 1999 [27]. Taj Uddin et al., 2012 mainly proposed a new index for measuring the alternative measures of ageing inequality of existing formulas of Gini coefficient which was introduced by Majumder, 2007 in income distribution. They consider the whole population into three groups like as proportion of older people, proportion of persons aged less than 15 years and proportion of person aged between 15 and 59 years to the Bangladesh population. Although the Gini coefficient is an ideal measure of income inequality, he has been developed an alternative measures of ageing inequality to the Bangladesh population as well as 50 Asian countries based on Gini index. From the analysis he observed that the Gini coefficient shows equally sensitivity at all levels. The coefficient is more concern for the country which are closed to the line of absolute equality. For example, the sensitivity level in the Gini coefficient is observed much higher in Israel than in Qatar. Finally he suggest that the trigonometric measure of the Gini coefficient is the best measure of ageing inequality among the measures considered of his study [26]. The above studies provide invaluable initial information regarding ageing inequality issues in Bangladesh. So it is necessary to use the modified different inequality indices namely Gini coefficient (G), Logarithmic transformation of Geometric equivalent of Gini coefficient (LTGEG) and Trigonometric measures of Gini coefficient (TMG) for measuring peak ageing inequality. Since much of the literature is concerned with income inequality, in this study it is assumed that population peak ageing is the variable of interest. It is assumed that peak ageing population consist in three groups like as proportion of youngest old person (upper end) (P_{60-69}), proportion of old-old person (middle end) (P_{70-79}) and proportion of the oldest old person (lower end) (P_{80+}). In this paper, an attempt has been made to use these above three modified indices of population peak ageing inequality on the basis of Gini coefficient by using secondary data for Bangladesh and some other selected Asian countries.

2. DATA AND METHODOLOGY

The study is based on a secondary population data which is taken from an international data base, US census Bureau (www.census.gov/population/data/idb) for 2018. For comparison of the study it has also used the same sources of data in 2000 and 2010. Three different modified alternative ageing measures have been implemented along with conventional ageing indices (HCR): Proportion of youngest old person (upper end) (P_{60-69}), Proportion of old-

old person (middle end) (P_{70-79}), Proportion of the oldest old person (lower end) (P_{80+}) to the Bangladesh population as well as 52 Asian countries. For convenience of the analysis, it is assumed that z_1, z_2 and z_3 represents lower end, middle end and upper end of the distribution respectively. To observe the sensitivity level of the modified alternative ageing measures, special focus is given on Bangladesh, Hong Kong, China, Sri Lanka, Bhutan, Malaysia, India, Mongolia, Burma, Nepal and Pakistan.

2.1 Gini-Coefficient of Inequality

The most popular measure of distributional inequality namely Gini coefficient is expressed as Lorenz curve [28]. Lorenz curve and Gini coefficient illustrate any kind of distributional inequality by graphs and numeric values. Though in literature researchers commonly apply Gini coefficient for income inequality but Allison suggested that it can also be used to other quantitative variables [5]. The value of Gini coefficient lies between 0 and 1 where 0 and 1 represent complete equality and inequality respectively. In graph, the area between Lorenz curve and the line of equality recites the Gini coefficient. Despite being vastly used, it has some demerits such as the total Gini of a society is not equal to the sum of the Gini's for its sub-groups. Scholars derived Gini coefficient as a measure of inequality because it satisfies most of the criteria of a good measure of inequality and expressed it differently along with its' interpretation. Several existing formula of Gini coefficients are mentioned below:

$$G = \frac{\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|z_i - z_j\right|}{2\mu} \qquad (1)$$

$$G = \frac{2}{\mu n^2}\sum_{i=1}^{n} i z_i - \frac{n+1}{n} \qquad (2)$$

$$G = \frac{n+1}{n} - \frac{1}{2n^2\mu}\sum_{i=1}^{n} i z_i \qquad (3)$$

where, $z_i (i = 1,,2,..., n)$ and $z_j (j = 1,2,..., n)$ represent the income/age of i-th person and j-th person respectively, the average income/age is denoted by μ and $z_1 \le z_2 \le .... \le z_n$. Kendall et al. defined Gini coefficient as a measure of dispersion by taking the average of absolute difference between all pairs of individuals and divide it by twice of mean [29]. Their proposed formula is given in equation (1) and usually known as Gini coefficient of mean difference. We will use this measure to estimate the peak ageing inequality of population. For individual data another type form for Gini coefficient is proposed by Dasgupta et al. which is computationally suitable and mathematically tractable, defined in equation (2) [30]. It contains weighted sum of all the scores, where the weight applied to each score is its rank in the distribution. Equation (3) is derived by Sen and it explains the income-waiting system in the welfare function behind the Gini coefficient [22].

2.2 Modification of the formula of Gini Coefficient

In recent times, scholars in the field of measurement of inequality have always been in the quest of presenting of simple and easy way to calculate Gini coefficient keeping its objective [18], [31]–[33]. We are trying to use the three different modified formula of Gini coefficient which was developed by Taj Uddin et al., 2012 [26]. Anand reported that the formula given in equation (1), (2) and (3) are almost same [20]. Therefore, we can choose any one of them for simplification. We choose equation (1) for simplification among existing formulas. In this paper we are working with the distribution of peak age. Let z_i and z_j is the share/proportion of persons belonging to one particular age group and we assume that $z_1 \le z_2 \le .... \le z_n$. Thus, mean of z_i is $\mu = \frac{1}{n}\sum_{i=1}^{n} z_i = \frac{1}{n}$, because $\sum_{i=1}^{n} z_i = 1$.

The equation (1) can be simplified as follows:

$$G = \frac{\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|z_i - z_j\right|}{2(\frac{1}{n})} = \frac{1}{2n}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|z_i - z_j\right| \qquad (4)$$

We categorize the age structure of peak ageing population into three different groups namely youngest old, old-old and oldest old elderly.

Since the data set consists of 3 different age groups ($i = j = 3$), the expression (4) can be rewritten as

$$G = \frac{1}{6}\sum_{i=1}^{3}\sum_{j=1}^{3}\left|z_i - z_j\right| \qquad (5)$$

After some straight forward implication, the expression (5) can be written as follows:

$$G = \frac{2}{3}(z_3 - z_1) \qquad (6)$$

Ignoring the multiplier, the expression becomes

$$G = (-1)z_1 + (0)z_2 + (1)z_3 \qquad (7)$$

Where z_1, z_2 and z_3 represent the proportion of the oldest old (P_{80}) , the proportion of old-old (P_{70-79}) and the proportion of youngest old (P_{60-69}) as these supports for Bangladesh as well as developing countries.

2.3 Logarithmic Transformation of Gini Coefficient

In order to make Gini coefficient more rational in terms of sensitivity, we take the natural logarithm and simplify the equation (4) and (5) as follows:

$$LTG = \frac{1}{2n}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\log(z_i) - \log(z_j)\right| = \frac{1}{2n}\sum_{i=1}^{n}\sum_{j=1}^{n}\log(z_i / z_j)$$

i.e. $LTG = \frac{1}{2n}\sum_{i=1}^{n}\sum_{j=1}^{n}\log(z_i / z_j)$ (8)

For 3 different groups, the expression becomes

$$LTG = \frac{1}{6}\sum_{i=1}^{n}\sum_{j=1}^{n}\log(z_i / z_j) \quad (9)$$

Since it doesn't satisfy the Pigou-Dalton assumption, we should look for other suitable measures to fulfil our objective.

2.4 Geometric Equivalent of Gini Coefficient

Majumder provides a geometric formula of Gini coefficient based on the Lorenz curve and the line of absolute equality [27]. For the sake of simplicity, we will look forward to the following alternative geometric measures of inequality as follows:

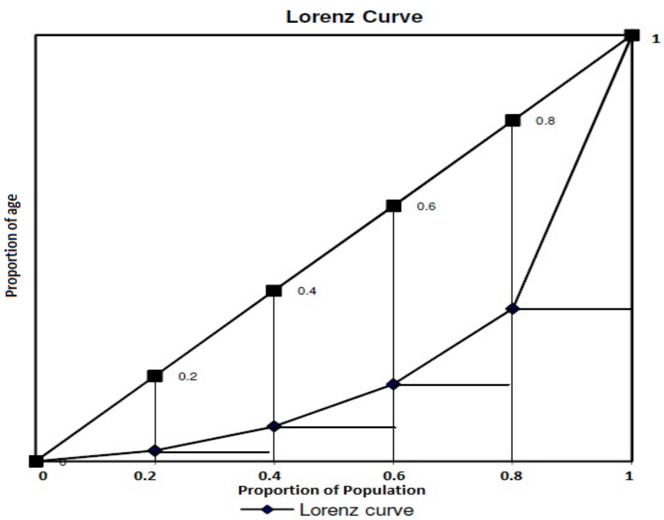

Figure 1: *Lorenz curve of age distribution [26].*

From the "Figure 1", it is evident that the diagonal line has divided the rectangle into two equal triangles. For each triangle, base=height=1, as sum of proportion equal to unity. The area of triangle is

$$\frac{1}{2}\times base \times height = \frac{1}{2} unit$$

It also shows that the area beyond the Lorenz curve is the sum of area of n small triangles and

($n-1$) rectangles. The area of each triangle is:

$$\frac{1}{2}.\frac{1}{n}.z_i = \frac{1}{2n}z_i$$

and the sum of n triangles is

$$\sum_{i=1}^{n}\frac{1}{2n}z_i = \frac{1}{2n}\sum_{i=1}^{n}z_i = \frac{1}{2n}, \text{ since } \sum_{i=1}^{n}z_i = 1$$

Similarly, the sum of ($n-1$) rectangles is

$$z_1.\frac{1}{n} + \frac{1}{n}(z_1+z_2) + ... + \frac{1}{n}(z_1+z_2+....+z_{n-1})$$

$$= \frac{1}{n}\{z_1 + (z_1+z_2) + ... + (z_1+z_2+....+z_{n-1})\}$$

$$= \frac{1}{n}[(n-1)z_1 + (n-2)z_2 + (n-3)z_3 + ...\{n-(n-1)\}z_{n-1}]$$

$$= \frac{1}{n}\sum_{j=1}^{n-1}(n-j)z_j$$

The total area beyond the Lorenz curve is

$$\frac{1}{2n} + \frac{1}{n}\sum_{j=1}^{n-1}(n-j)z_j$$

Now the area between the diagonal line and the Lorenz curve is

$$\frac{1}{2} - \{\frac{1}{2n} + \frac{1}{n}\sum_{j=1}^{n-1}(n-j)z_j\} = \frac{1}{2} - \frac{1}{2n} - \frac{1}{n}\sum_{j=1}^{n-1}(n-j)z_j$$

$$= \frac{n-1-2\sum_{j=1}^{n-1}(n-j)z_j}{2n}$$

$$= \frac{(n-1)-2\sum_{j=1}^{n-1}(n-j)z_j}{2n} \quad (10)$$

We may standardize the above expression (10) with the multiplier, $1/2\mu(= n/2)$.

Therefore, equation (10) may be considered as an alternative geometric measure of Gini's coefficient and written as

$$GEG = \frac{(n-1)-2\sum_{j=1}^{n-1}(n-j)z_j}{2n}\times\frac{n}{2}$$

$$\text{Therefore, } GEG = \frac{n-1}{4} - \frac{1}{2}\sum_{j=1}^{n-1}(n-j)z_j \quad (11)$$

For 3 different groups, the expression (11) becomes

$$GEG = \frac{3-1}{4} - \frac{1}{2}\sum_{j=1}^{3-1}(3-j)z_j = \frac{1}{2} - \frac{1}{2}\sum_{j=1}^{2}(3-j)z_j = \frac{1}{2} - \frac{1}{2}(2z_1+z_2)$$

$$\text{Therefore, } GEG = \frac{1}{2}\{1-2z_1-z_2)\} \quad (12)$$

The Gini coefficient (G) and the Geometric Equivalent of Gini coefficient (GEG) are identical with almost similar properties except the minimum and maximum value of GEG which are -100 and +100 respectively (after multiplying by 100). When all resources are given to the smallest age group/individual (z_1) then the Geometric Equivalent of Gini coefficient will be minimum that is -100. On the other hand, the maximum value of the Gini coefficient is always 100. It can be understood that when all resources are given to (z_1)

the concept of Lorenz curve breakdowns. This situation is captured by GEG instead of G. Now, we will check whether the logarithmic transformation of it works well or not.

2.5 Logarithmic Transformation of Geometric Equivalent of Gini Coefficient (LTGEG)

Taking logarithm, the expression (11) and (12) becomes

$$LTGEG = \frac{n-1}{4} - \frac{1}{2}\sum_{j=1}^{n-1}(n-j)\log z_j \qquad (13)$$

$$LTGEG = \frac{1}{2}\{1 - 2\log(z_1) - \log(z_2)\} \qquad (14)$$

Hypothetical minimum and maximum values of LTGEG range from 0 to ∞ (infinity). However, like the natural logarithmic when any age group (at the extreme) has zero share to total age then the index value will tend to infinity which arises no problem conceptually.

2.6 Trigonometric Measures of Gini Coefficient (TMG)

Though the arithmetic and geometric derivation of Gini coefficient have been applied for measuring peak ageing inequality but for a simple and alternative measure, we use the measure namely trigonometric measure of inequality for measuring peak ageing inequality which are also used by Taj Uddin et al. for estimating ageing inequality [26] and Majumder for income inequality [27]. Here, we are trying to demonstrate in a formalized manner how to apply the TMG to the peak age distribution. It is observed that the number of right-angled triangles below the Lorenz curve is same as the number of individuals/groups. Assume that we have n individuals/groups, so we have n-numbers of right-angled triangles. Cosecant or cotangent is estimated by the triangle divided by the perpendicular of it and add them to get a measure of inequality. Therefore, the Trigonometric measure based on cotangent (of left-hand side complementary angle) of a triangle is as follows (in general form):

$$TMG = \sum_{i=1}^{n}\frac{\mu}{z_i} = \sum_{i=1}^{n}\frac{\frac{1}{n}}{z_i} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{z_i} \qquad (15)$$

We can standardize the expression (15) by subtracting n from it and multiplying by $1/2\mu(= n/2)$. Thus we have

$$TMG = \{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{z_i} - n\}\times\frac{n}{2} = \frac{1}{2}(\sum_{i=1}^{n}\frac{1}{z_i} - n^2)$$

i.e., $$TMG = \frac{1}{2}(\sum_{i=1}^{n}\frac{1}{z_i} - n^2) \qquad (16)$$

For 3 different individual/age groups, the TMG becomes

$$TMG = \frac{1}{2}(\sum_{i=1}^{3}\frac{1}{z_i} - 3^2) = \frac{1}{2}(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} - 9)$$

i.e., $$TMG = \frac{1}{2}(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} - 9) \qquad (17)$$

Hypothetical minimum and maximum values of TMG are same as LTGEG and if share of any age group becomes 0, like as LTGEG the value of TMG will also tend to infinity.

3. RESULTS AND DISCUSSION

Bangladesh and other Asian countries population which satisfy the rank order condition $(z_1 \le z_2 \le z_3)$ are considered for this study to estimate the peak ageing inequality through the three different modified alternative ageing inequality measure for the year 2018. Table 1 contain the results of the measures of peak ageing inequality of few selected countries. The sensitivity levels of the measures have been presented from Table 2 to 4. The compare and sensitivity levels of the measurement in various countries in different years have been presented from Table 5 to 6. The projection of the different measures of inequality in Bangladesh has also been presented in Table 7. We assume that z_1, z_2 and z_3 represents lower end, middle end and upper end of the distribution respectively for the sake of convenience.

3.1 Gini Coefficient (G)

The Gini coefficient is computed by adopting equation (6). From Table 1, it is observed that the calculated values of the Gini coefficient satisfy the hypothetical values of it because the minimum and maximum observed value of G for selected countries is 0.0356 (in Pakistan) and 0.0883 (in Hong Kong) respectively. The sensitivity level for few selected Asian countries are given in Table 2 to 4 by considering 0.5 percent transfer of ages from one group to another group in upward and downward directions. It is also observed that the value of G increases when reorganization of ages takes place from the smaller group to higher group. Similarly, the value of G decreases as the reorganization of ages takes place from the higher group to smaller group which proves that Gini coefficient satisfies Pigou-Dalton condition. In Bangladesh for 0.5 percent transfer of ages between two consecutive groups G changes by 10.79 percent in both directions in Tables 2 to 3 and it suggest that Gini coefficient is equally sensitive at all levels in both directions. The other Asian countries also depict the similar results. So, it is evident that the sensitivity of Gini coefficient within a country or for a particular distribution is constant at all levels. Thus, it satisfies the property of Gini coefficient which states that equal transfer of amount (income/ages) between any two successive groups/individuals changes the Gini coefficient in the same manner but for different countries changes may be different. The Gini coefficient is higher for a country where share at the lower end are comparatively higher than the other. In this study, it is observed that the G value of China, India and Pakistan are 0.0837, 0.0504, and 0.0356 respectively where their corresponding shares of the lower end are 0.0199, 0.0087, and 0.0068 respectively (Table 1). So, we can say that the higher value of G indicates the oldest old ageing society. From Table 2 and 3, we find that in India and China, G changes by 9.93 percent and 5.98 percent respectively when transfer of ages (0.5 percent) takes place between any two consecutive groups in these two countries. So, the changes of G may be higher in a country where the values of G are comparatively smaller than others. Table 4 also indicates that G changes by 19.86 percent and 11.95 percent respectively in these two countries when reorganizations take place between highest group and lowest group. Based on this result, it can be say that Gini coefficient is more concern for India than in China.

Therefore, it is concluded that Gini coefficient expresses more concern for countries, which are comparatively in better position or closer to the line of absolute equality.

3.2 Logarithmic Transformation of Geometric Equivalent of Gini Coefficient (LTGEG)

In this study, LTGEG is calculated by using the expression (14) and its' minimum and maximum values are 9.8509 (in Hong Kong) and 14.828 (in Pakistan) that satisfies the hypothetical values as displayed in Table 1. Like as Gini coefficient, the LTGEG index also satisfies Pigou-Dalton condition. It increases as the reorganization of ages takes place from the smaller group to higher group and vice versa. In Table 2, the sensitivity analysis estimate that LTGEG increases by 4.04 percent in China and 11.007 percent in India for 0.5 percent transfer of ages from z_1 to z_2. The sensitivity of LTGEG decreases gradually when similar transfers of two consecutive groups in upward direction. This indicates that the index is not equally sensitive at all levels within a country, but the Gini coefficient is equally sensitive at all level. Also, from Table 4, we observe that the changes of LTGEG is lower where the variation (inequality) is higher than other for 0.5 percent transfer from lowest group (z_1) to highest group (z_3). For example, LTGEG increases by 4.88 percent (in China) and 12.12 percent (in India). The sensitivity levels of the LTGEG is higher for India than China which indicates that this index is more sensitive to the countries whom are far from the line of absolute equality like India. The sensitivity level gradually increases, in case of downward direction of ages from z_3 to z_2, z_2 to z_1 and so on (Table 3). In Bangladesh, LTGEG increases and decreases by 9.11 percent and 4.61 percent for 0.5 percent transfer of ages from z_1 to z_2 and z_2 to z_1 respectively (Table 2 and Table 3). It indicates that the extent of increase and decrease in both directions are not equal. From our study, we have observed that LTGEG satisfies the rank order condition ($z_1 \leq z_2 \leq z_3$).This study indicates that the lower end is comparatively decreases as the change of LTGEG increases. "Figure2" supports out findings because the changes of LTGEG in Pakistan and China are 16.38 and 4.04 respectively whereas, the share at the lower end are 0.0068 and 0.0199 for these two countries. Finally, we can say that the share of the oldest old population (lower end) is higher when the sensitivity of index is smaller and LTGEG is more sensitive than G.

3.3 Trigonometric Measures of Gini Coefficient (TMG)

The results of Trigonometric measures of Gini coefficient is shown in Table 1. The minimum and maximum values of trigonometric measure (TMG) are 35.1262 (in Hong Kong) and 208.256 (in Pakistan) respectively which are computed by using equation (17) and it satisfies the hypothetical values. It increases by 20.76 percent, 94.78 percent and 188.76 percent in China, India and Pakistan respectively for 0.5 percent transfer from z_1 to z_2 (Table 2). This result suggests that TMG is most sensitive measure of inequality than the others. From Table 1, we have learnt that Pakistan has the most unequal distribution of age because of the higher value of the measures. This also indicates that for 0.5 percent transfer of age's takes place from z_1 to z_3, the changes are almost similar (slightly higher). TMG is more concern for the most unequal distribution of age because Table 4 shows that percentage figures are 22.90 percent, 97.05 percent and 191.88 percent for China, India and Pakistan respectively. The sensitivity analysis also suggests that TMG is higher when transfer takes place between lowest group and highest group than the transfer of consecutive groups. It is evident that the sensitivity is smaller where the share of the lower end is comparatively higher. This indirectly implies that China has the oldest old elderly than in India and Pakistan. Again, like as the previously discussed measures the trigonometric measure satisfies the Pigou-Dalton's conditions along with the rank order condition ($z_1 \leq z_2 \leq z_3$).

3.4 Comparison among different Measures of Inequality for various years in selected Asian countries

Table 5 shows the Gini coefficients for different years in selected Asian countries. Gini coefficient for different countries in different years satisfies Pigou-Dalton condition. When reorganization of ages takes place from the smaller group to higher group, value of G increases and vice versa. It may be realized that the value of G may be higher in a country where share at the lower end are comparatively higher than the other. Our data support this claim. For example, the G value of China and Bangladesh are 0.0509 and 0.0328 in 2000, 0.0570 and 0.0394 in 2010 and 0.0837 and 0.0463 in 2018 respectively (Table 5). Their corresponding shares of the lower end are 0.00861 and 0.00406 in 2000, 0.01471 and 0.00673 in 2010 and 0.0199 and 0.0099 in 2018 respectively ("Figure 3"). It is clear from the analysis that the higher value of G implies the oldest old ageing society. We also observed that the changes of G may be higher in all countries where the values of G are comparatively smaller than others. Our data also support this claim. For example, when transfer of ages (0.5 percent) takes place between any two consecutive groups in Bangladesh, G changes by 15.25, 12.69 and 10.79 percent (Table 6) where the value of G are 0.0328, 0.0394 and 0.0463 respectively according to years 2000, 2010 and 2018 as shown in "Figure 4". Another index (LTGEG) for different countries in different years satisfies Pigou-Dalton condition. When reorganization of ages takes place from the smaller group to higher group, value of LTGEG increases and vice versa. From the Table 5 and "Figure 5", we see that all the selected countries the value of LTGEG are gradually decreasing for the years 2000, 2010 and 2018 respectively. It is also observed that change of LTGEG is higher, where share at the lower end is comparatively smaller. For example, the changes of LTGEG in Hong Kong and Sri Lanka are 3.64 and 9.57 in 2000, 2.10 and 5.54 in 2010 and 1.51 and 4.09 in 2018 respectively (Table 6) whereas, the share at the lower end are 0.02197 and 0.00957 in 2000, 0.0363 and 0.01451 in 2010 and 0.04725 and 0.01924 in 2018 for these two countries ("Figure 6"). Therefore, it can be said that share of the oldest old population is higher where the sensitivity of index is smaller. From the Table 5, we also observed that all the selected countries the value of TMG are gradually decreasing over the years 2000, 2010 and 2018 respectively. TMG shows more sensitivity than any other measures as presented in the paper. For example, for 0.5 percent transfer from z_1 to z_2, the index increases by 100.87, 33.44 and 20.76 percent for China in 2000, 2010 and 2018, 822.43, 162.77 and 119.14 percent for Mongolia in 2000, 2010 and 2018 and 5031.0, 646.71 and 188.76 percent for Pakistan in 2000, 2010 and 2018 (Table 6). Pakistan has the most unequal distribution of age because of the higher value of the measures. Almost similar (slightly higher) changes can be observed when 0.5 percent transfer of ages takes place from z_1 to z_3. The percentage figures are 102.74, 35.17 and 22.90 percent for China in 2000, 2010 and 2018, 826.53, 165.33 and 121.89 percent for Mongolia in 2000, 2010 and 2018 and 5034.0, 649.35 and 191.88 percent for Pakistan in 2000, 2010 and 2018. So, it is clear that TMG is more concern for the most unequal distribution of age. It is also clear that the sensitivity of TMG is higher when transfer takes place between lowest group and highest group than the transfer of consecutive groups. From the sensitivity analysis of the index, one can say that the sensitivity is smaller where the share of the lower end is comparatively higher. This indirectly implies that China has oldest old elderly than in Mongolia and Pakistan. Again, it is clear that Trigonometric measure (TMG) for different countries in different years satisfies the Pigou-Dalton's conditions. Because when 0.5 percent transfer of ages take place in upward direction, TMG increases and in downward direction the index decreases.

3.5 Applications of peak ageing inequality in different measures of Bangladesh for projected data

Table 7 show the different values of Gini coefficient (G), logarithmic transformation of the geometric equivalent of Gini coefficient (LTGEG) and trigonometric measures of Gini coefficient (TMG) for projected age structure data of Bangladesh from the year 2011 to 2061. It is observed that the value of G is gradually increasing over the years "Figure 7", LTGEG and TMG are decreasing over the years "Figure 8" and "Figure 9". Which implies that the oldest old population in Bangladesh is gradually increasing.

4. CONCLUSION

In this study, it is used modified three different inequality indices namely Gini coefficient (G), Logarithmic transformation of Geometric equivalent of Gini coefficient (LTGEG) and Trigonometric measures of Gini coefficient (TMG) for measuring peak ageing inequality. All the modified measures satisfy the three basic properties of an ideal index including rank-order condition, mean or scale independence and population size independence [20]. Due to satisfy these properties, we can use these measures of peak ageing inequality which was first introduced in ageing inequality by Taj Uddin et al.,2012 [26]. From the analysis it is observed that the Gini coefficient shows equally sensitivity at all levels. The coefficient is more concern for the country which are closed to the line of absolute equality. For example, Gini coefficient is more concern for India than in China. The logarithmic transformation of Gini coefficient and the geometric measure of Gini coefficient does not work well because it violates the transfer principle. On the other hand, the logarithmic transformation of geometric equivalent of the Gini coefficient works better because it shows more sensitivity than the Gini coefficient and satisfies the transfer principle. The sensitivity levels of the LTGEG is higher for India than in China which indicates that this index is more sensitive to the countries whom are far from the line of absolute equality like India. From the analysis it is also found that the trigonometric measure of Gini coefficient satisfies transfer principle as well as shows higher sensitivity. It is evident that the sensitivity is smaller where the share of the lower end is comparatively higher. This indirectly implies that China has oldest old elderly than in India and Pakistan. The comparative study shows that the performance of LTGEG is better than G because it overcomes all the difficulties faced by G. On the other hand, based on the degree of sensitivity, TMG is the best result among all the measures (G, LTGEG, and TMG) considered in this study which was the similar result in Taj Uddin et al., 2012 [26]. Finally, based on this analysis it can be conclude that the trigonometric measure of the Gini coefficient is the best measure of peak aging inequality among the other measures.

***Table 1**: Distribution of ages and different measures of inequality for selected countries*

Country	Distribution of age			Measures of Inequality				
	z_1	z_2	z_3	G	LTGEG	TMG	SD	CV
Hong Kong	0.0473	0.0642	0.1356	0.0883	9.8509	35.1262	0.0469	0.5695
China	0.0199	0.0478	0.1035	0.0837	11.877	71.8889	0.0426	0.7463
Sri Lanka	0.0192	0.0433	0.0838	0.0646	12.041	77.9934	0.0326	0.6688
Bhutan	0.0116	0.0301	0.0498	0.0381	13.411	130.266	0.0191	0.6253
Bangladesh	0.0099	0.0297	0.0562	0.0463	13.749	143.563	0.0233	0.7282
Malaysia	0.0098	0.0275	0.0633	0.0535	13.839	144.935	0.0272	0.8121
India	0.0087	0.0297	0.0591	0.0504	13.996	155.986	0.0253	0.7783
Burma	0.0080	0.0261	0.0566	0.0486	14.313	172.595	0.0246	0.8136
Nepal	0.0071	0.0244	0.0509	0.0438	14.622	193.461	0.0221	0.8039
Mongolia	0.0078	0.0193	0.0466	0.0389	14.658	192.647	0.0200	0.8126
Pakistan	0.0068	0.0212	0.0424	0.0356	14.828	208.256	0.0179	0.7626

***Table 2:** Sensitivity of different measures of Inequality for upward direction*

	0.5 percent transfer from z_1 to z_2				0.5 percent transfer from z_2 to z_3			
Country	G	LTGEG	TMG	CV	G	LTGEG	TMG	CV
Hong Kong	5.6612	1.5087	3.9199	0.5835	5.6612	0.8238	3.0045	0.617
China	5.977	4.0411	20.761	0.7794	5.977	0.9304	2.7815	0.8064
Sri Lanka	7.7445	4.0902	20.326	0.713	7.7445	1.019	3.0039	0.7368
Bhutan	13.117	7.2224	46.001	0.719	13.117	1.3566	3.6876	0.7238
Bangladesh	10.789	9.1074	68.553	0.8073	10.789	1.3422	3.7469	0.8277

Malaysia	9.3498	9.0734	68.957	0.872	9.3498	1.4489	4.7688	0.9169
India	9.9293	11.007	94.777	0.8535	9.9293	1.3187	3.5343	0.8768
Mongolia	12.869	12.417	113.54	0.8926	12.869	2.051	8.3777	0.9637
Burma	10.282	12.585	119.14	0.888	10.282	1.4879	4.4438	0.9255
Nepal	11.415	15.619	175.18	0.8915	11.415	1.5673	4.5434	0.9247
Pakistan	14.038	16.377	188.76	0.8702	14.038	1.812	5.7822	0.905

Table 3: *Sensitivity of different measures of Inequality for downward direction*

Country	0.5 percent transfer from z_3 to z_2				0.5 percent transfer from z_2 to z_1			
	G	LTGEG	TMG	CV	G	LTGEG	TMG	CV
Hong Kong	-5.6612	-0.7619	-2.4048	0.5247	-5.6612	-1.2182	-2.0132	0.5617
China	-5.977	-0.8377	-2.0746	0.6922	-5.977	-2.8478	-10.664	0.7224
Sri Lanka	-7.7445	-0.9075	-2.0944	0.6105	-7.7445	-2.8176	-9.8769	0.6382
Bhutan	-13.117	-1.1474	-1.9192	0.5585	-13.117	-3.9722	-14.716	0.5646
Bangladesh	-10.789	-1.1327	-2.1771	0.6516	-10.789	-4.6078	-18.878	0.6766
Malaysia	-9.3498	-1.2063	-2.9201	0.723	-9.3498	-4.4973	-18.125	0.7766
India	-9.9293	-1.1128	-2.1143	0.7001	-9.9293	-5.1431	-22.289	0.7283
Mongolia	-12.869	-1.5748	-4.221	0.6891	-12.869	-4.7102	-16.565	0.779
Burma	-10.282	-1.226	-2.5848	0.7225	-10.282	-5.3206	-22.783	0.7682
Nepal	-11.415	-1.2742	-2.4909	0.7098	-11.415	-5.7643	-24.967	0.7511
Pakistan	-14.038	-1.4268	-2.8038	0.6593	-14.038	-5.6034	-22.79	0.7046

Table 4: *Sensitivity of different measures of Inequality for extreme direction*

Country	0.5 percent transfer from z_1 to z_3				0.5 percent transfer from z_3 to z_1			
	G	LTGEG	TMG	CV	G	LTGEG	TMG	CV
Hong Kong	11.3223	2.2706	6.38189	0.627	-11.3223	-2.042	-4.96047	0.5127
China	11.9541	4.8788	22.8988	0.8325	-11.9541	-3.7782	-13.3827	0.6605
Sri Lanka	15.4891	4.9977	22.5293	0.7704	-15.4891	-3.8367	-12.7715	0.5676
Bhutan	26.2339	8.3698	48.2353	0.7893	-26.2339	-5.3288	-18.0892	0.4613
Bangladesh	21.5779	10.24	70.9272	0.8843	-21.5779	-5.9501	-22.4278	0.5724
Malaysia	18.6997	10.28	72.0135	0.9588	-18.6997	-5.9462	-22.7569	0.6664
India	19.8587	12.119	97.0473	0.9315	-19.8587	-6.4619	-25.6672	0.6254
Mongolia	25.7375	13.992	118.017	1.0118	-25.7375	-6.7612	-24.6837	0.6164
Burma	20.5644	13.811	121.886	0.9776	-20.5644	-7.3316	-27.066	0.6233
Nepal	22.829	16.893	177.868	0.9851	-22.829	-7.4154	-29.3116	0.5516
Pakistan	28.0756	17.804	191.879	0.9744	-28.0756	-2.042	-28.254	0.5127

Table 5: *Comparison among G, LTGEG and TMG for various years in selected Asian countries*

Country	G			LTGEG			TMG		
	2000	2010	2018	2000	2010	2018	2000	2010	2018
Hong Kong	0.0543	0.0506	0.0883	11.611	10.419	9.8509	69.2179	46.2945	35.1262
China	0.0509	0.057	0.0837	13.947	12.62	11.877	155.021	97.005	71.8889
Sri Lanka	0.043	0.052	0.0646	13.822	12.825	12.041	148.439	103.732	77.9934
Bhutan	0.0406	0.0398	0.0381	15.505	14.292	13.411	262.895	174.915	130.266
Bangladesh	0.0328	0.0394	0.0463	16.062	14.753	13.749	321.735	203.797	143.563
Malaysia	0.033	0.0395	0.0535	15.471	14.7	13.839	258.571	197.944	144.935
India	0.0393	0.0442	0.0504	15.547	14.753	13.996	268.842	203.784	155.986
Mongolia	0.0284	0.0375	0.0486	15.567	14.615	14.313	267.409	195.05	172.595
Burma	0.0364	0.0363	0.0438	15.302	15.327	14.622	249.325	249.237	193.461
Nepal	0.034	0.0275	0.0389	16.061	15.162	14.658	321.229	236.592	192.647
Pakistan	0.0313	0.0303	0.0356	15.56	15.296	14.828	269.774	248.754	208.256

Table 6: *Comparison among the sensitivity of G, LTGEG and TMG for various years in selected Asian countries for upward direction*

Country	0.5 percent transfer from z_1 to z_2								
	Change of G			Change of LTGEG			Change of TMG		
	2000	2010	2018	2000	2010	2018	2000	2010	2018
Hong Kong	9.214	9.881	5.661	3.643	2.095	1.509	16.85	6.8714	3.9199
China	9.814	8.773	5.977	11.42	5.685	4.041	100.87	33.443	20.761
Sri Lanka	11.63	9.623	7.745	9.567	5.54	4.09	73.786	31.442	20.326
Bhutan	12.32	12.55	13.12	-	12.57	7.222	-	116.6	46.001
Bangladesh	15.25	12.69	10.79	-	17.1	9.107	-	206.78	68.553
Malaysia	15.14	12.65	9.35	32.49	15.01	9.073	926.05	161.35	68.957
India	12.71	11.31	9.929	-	18.36	11.01	-	240.24	94.777
Mongolia	17.6	13.32	10.28	31.21	15.13	12.59	822.43	162.77	119.14
Burma	13.74	13.76	11.41	36.12	33.41	15.62	1250.2	998.24	175.18
Nepal	14.7	18.16	12.87	-	20.37	12.42	-	288.33	113.54
Pakistan	15.97	16.47	14.04	53.05	28.64	16.38	5031	646.71	188.76

Table 7: *Projection of the different Measures of inequality in Bangladesh*

Country	Year	Distribution of age			Measures of Inequality		
		z_1	z_2	z_3	G	LTGEG	TMG
Bangladesh	2011	0.01120	0.02155	0.041953	0.03076	13.82123	150.5
Bangladesh	2016	0.00818	0.02196	0.043621	0.035445	14.43177	181.8
Bangladesh	2021	0.00980	0.02532	0.047331	0.03753	13.92629	153.6
Bangladesh	2026	0.00890	0.02598	0.056424	0.04753	14.09511	159.6
Bangladesh	2031	0.01158	0.02969	0.064767	0.05319	13.43404	126.5
Bangladesh	2036	0.01141	0.03618	0.074638	0.06323	13.26586	119.7
Bangladesh	2041	0.01485	0.04321	0.080612	0.06576	12.56131	93.89
Bangladesh	2046	0.01789	0.05119	0.093502	0.07561	12.01875	77.11
Bangladesh	2051	0.02284	0.056707	0.100921	0.07809	11.42878	62.34
Bangladesh	2056	0.02853	0.06840	0.096314	0.06778	10.79602	51.05
Bangladesh	2061	0.03407	0.07560	0.106860	0.07279	10.34095	42.94

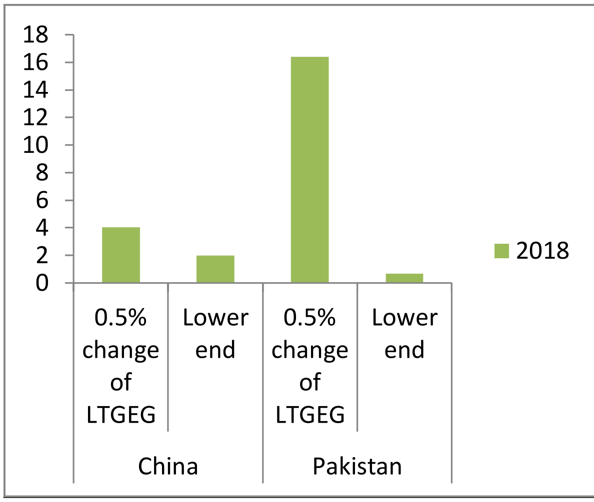

Figure 2: *Sensitivity of LGTEG*

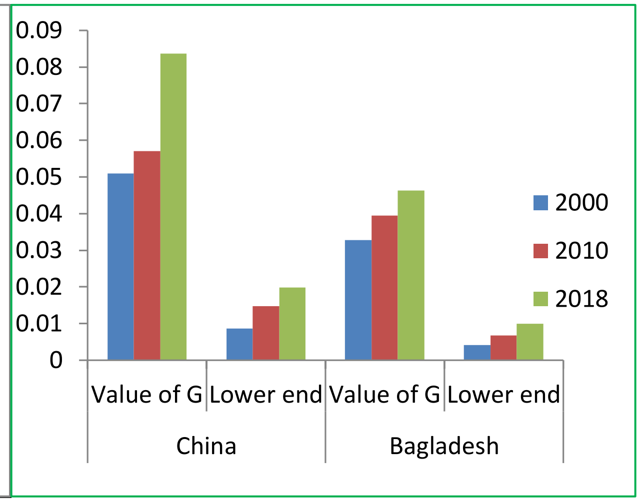

Figure 3: *The Value of G*

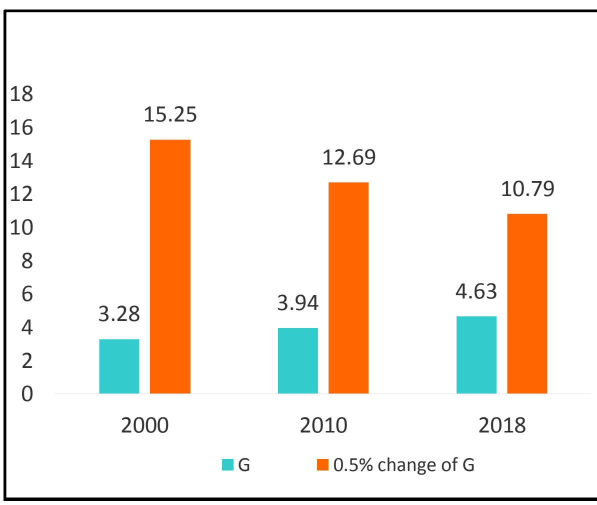

Figure 4: *Sensitivity of G value of Bangladesh from 2000-2018*

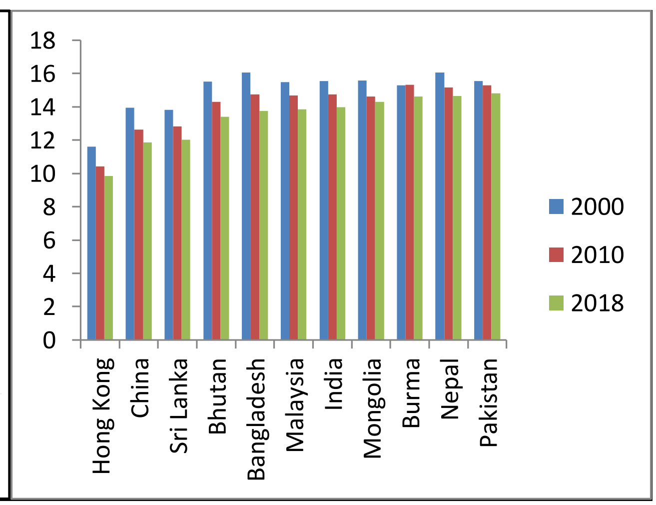

Figure 5: *The value of LTGEG from 2000-2018*

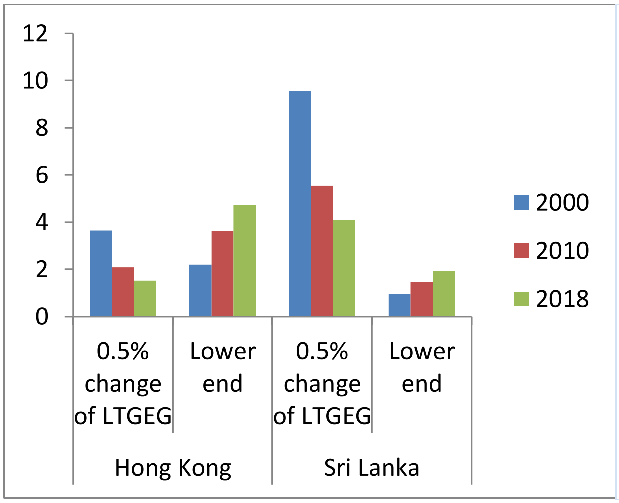

Figure 6: *Sensitivity of LTGEG*

Figure 7: *Projected value of G*

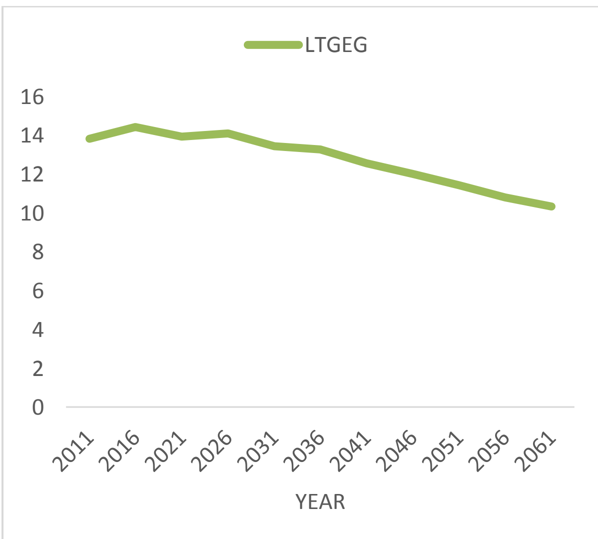

Figure 8: *Projected value of LTGEG*

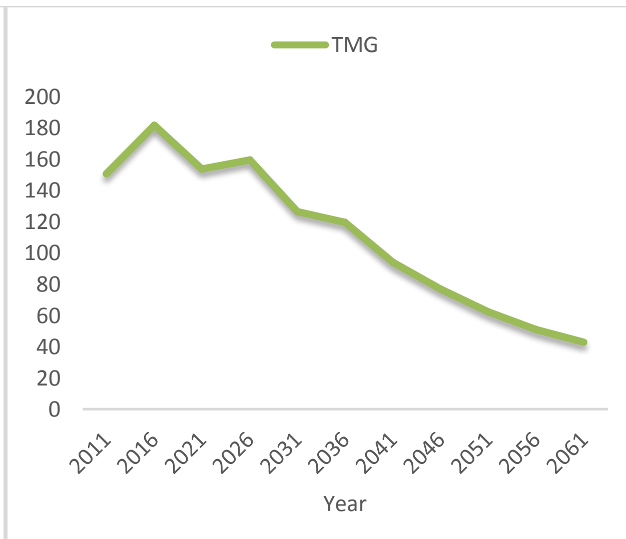

Figure 9: *Projected value of TMG*

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