The Tragedy Of Commons Mimo Homogeneous Network And Sic Technology

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Abstract: The perpetually mounting demand of reliable higher data rates, better quality service, enlarged coverage with limited available RF spectrum and existing transmission problems caused by various factors viz. fading and multipath distortion etc. are the key challenges faced by wireless system designers now a days. These needs motivate to introduce novel technique named as Multi-Input-Multi-Output (MIMO) technology that promises a cost effective way to provide an efficient solution to reach the goal by improving spectral efficiency, operational reliability and assisting fading link reliability without sacrificing bandwidth efficiency. In this work, an efficient spectrum allocation is done of a MIMO network using game theoretical model commonly known as the tragedy of Commons to detect that amount of unused spectrum efficiently. Game theory practices rational choice theory along with assumptions of players' common knowledge in order to envisage utility-maximizing decisions. Users can detect the unused spectrum commonly known as cognitive spectrum by using log likelihood ratio test where a threshold value is detected using log-likelihood ratio model and the unused spectrum is allocated to the user who needs an extra band for some higher usage. Interference is avoided in this work using successive interference cancellation (SIC) approach.

Index Terms: MIMO, Game Theory, Tragedy of Commons, Log-likelihood ratio test, Successive Interference Cancellation (SIC),Utility Function ,Resource Sharing .

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1. INTRODUCTION

WIRELESS communication devises are increasing each day but the spectrum amount is limited. A proficient spectrum detection and allocation is explained in this work using game theoretical approach. Game theory is a mathematical model of strategy where an optimal approach is used in different engineering and science application to enhance the performance of Multiple Input Multiple Output (MIMO) wireless networks by efficient resource allocation [1]. Successive interference cancellation (SIC) method is used to reduce interference in a heterogeneous wireless network [2]. If SIC is minimized interference in wireless communication network then performance of the network will be improved [3]-[9]. Efficient spectrum allocation is done by using only different game strategy [10] but “Tragedy of Commons” game theoretical approach is not used in spectrum allocation by the above-mentioned article. The tragedy of Commons is a game theoretic approach where a common resource is shared between two players as per their self-interest [11]. Tragedy of Commons game theoretical approach considers two mobile tower in separate and in combined way; a fixed band of frequencies were allocated to both the towers in a normalized scale of 1. The frequency allocation strategy is done by using game theory model commonly known as “Tragedy of Commons”. In this work, two mobile towers were considered as two different players and frequency is shared among them equally in the ratio of (1/3,1/3) when the towers are used separately or (1/4,1/4) when the towers are used in combined way. If the spectrum allocation is deviated from this condition then there is a loss of equilibrium condition that means frequency spectrum can’t be shared among them equally to serve an equal number of users under the service of each tower, this condition is known as a Nash Equilibrium (NE) in game theory. A fixed number of users are served by that allocated frequency spectrum. There was some unused frequency in those two towers which were not allocated to any user. The probability of each mobile tower is denoted by Poisson distribution [12]. The probability of detection and probability of false alarm is detected by using threshold value, the threshold is detected by using the Log-Likelihood ratio test [13]. The probability of detection and probability of false alarm is compared among two different mobile tower strategy and it’s observed that probability of detection performs better when the mobile towers are used in a combine fashion. The unused spectrum can be allocated to some user who needs extra band to download some high definition (HD) video or to some extra user entered in the mobile tower range during busy hour otherwise else, this unused band can be reserved for any home/office user using Wi-Fi, by using this extra band their internet surfing can be done in a faster way. The paper is arranged as follows section II describes contribution and motivation of the work in section III, Successive Interference Cancellation (SIC) is explained in section IV, Section V described results and discussion, the conclusion part is explained section VI.

2 MOTIVATION & CONTRIBUTION OF PROPOSED WORK

The spectrum allocation is one of the major problem of modern wireless 5G communication. Many research work have done to improve the performance of wireless communication by using efficient spectrum allocation [1].But in the in the above mentioned article the spectrum allocation is done by using “user pairing” & by “channel assignment “technique but those methods were difficult to implement as the optimization problem is generated by the researcher by considering the number of users and allocated power. Spectrum allocation is done in some research work [14]-[15], but those works is suitable only for downlink in wireless networks. A game theoretic approach is used along with interference cancellation used in one of earlier research work [16]. But efficient spectrum allocation is not mentioned in that work. In our work a simple game theoretical approach known as “Tragedy of commons is applied” where frequencies are allocated in simple a scale of 1 both for uplink and downlink of a wireless network, also the probability of detection and probability is calculated by varying the number of users which was not mentioned in the previous articles.

3 ALGORITHM OF THE PROPOSED MODEL

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Algorithm 1: Unused spectrum value detection
Initialization of inputs: Utility function for user 1{ $U_1(e_1,e_2)$}, Utility function for user 2{$U_2(e_2,e_1)$}, Joint utility function for user 1 and user {$U_t(e_t)$},Normalized frequency band for mobile tower $S_1${$e_1$}, Normalized frequency band for mobile tower $S_2$ {$e_2$},Normalized frequency band for mobile towers $S_1$ and $S_2$ {$e_t$}
Output: Normalized unused frequency band {$e_u$}
Step 1: Define utility function for user 1 and user 2, joint utility function by $U_1(e_1,e_2)$= $e_1$(1-( $e_1+e_2$)) and $U_2(e_2,e_1)$= $e_2$(1-($e_1+e_2$)), $U_t(e_t)$= $e_t$(1-$e_t$) respectively. Where $e_1+e_2$= $e_t$.
Step 2: Detect the optimum value of normalized utilized spectrum by differentiating utility function and equating to zero, which is $e_1^*$= $\frac{1}{3}$ and $e_2^*$= $\frac{1}{3}$ , $e_t^*$= $\frac{1}{2}$ for the mobile towers $S_1$ and $S_2$ ,joint mobile tower $S_1$ and $S_2$ respectively.
Step 3: Find the value of normalized unused spectrum by $e_u$=1- [($e_1^*$= $\frac{1}{3}$)+( $e_2^*$= $\frac{1}{3}$)] = $\frac{1}{3}$ mobile tower $S_1$ and $S_2$ individually and normalized unused spectrum for combined is given by 1-($e_t^*$= $\frac{1}{2}$) = $\frac{1}{2}$.
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Algorithm 2: Unused spectrum threshold detection
Initialization of inputs: No of users for mobile tower $S_1$is {$k_1$},No of users for mobile tower $S_2$ is {$k_2$},Power allocated for mobile tower $S_1$is {$\lambda_1$} and Power allocated for mobile tower $S_2$ is {$\lambda_2$}
Output: The threshold value for unused spectrum is{ $\gamma$}
Step 1: The probability density of mobile tower $S_1$ is given by P($S_1$)=$e^{-\lambda_1}{\lambda_1}^{k_1}/k_1!$, The probability density of mobile tower $S_2$ is given by P($S_2$)=$e^{-\lambda_2}{\lambda_2}^{k_2}$ /$k_2$!.Respectively.
Step 2: Detect the overall utility function of the mobile tower $S_1$ is given by P ($S_1U_1$)=( $e^{-\lambda_1}{\lambda_1}^{k_1}/k_1!$) $*U_1$,the probability utility function of the mobile tower $S_2$ is given by P($S_2U_2$)=( $e^{-\lambda_2}{\lambda_2}^{k_2}$ /$k_2$!)* $U_2$ Respectively.
Step 3: probability utility function of the empty spectrum is given by P(E)=1-[ P ($S_1U_1$)+ P($S_2U_2$)].
Step 4: Determine threshold value by log-likelihood ratio test.
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4 THE PROPOSED GAME THEORETICAL NETWORK DESCRIPTION

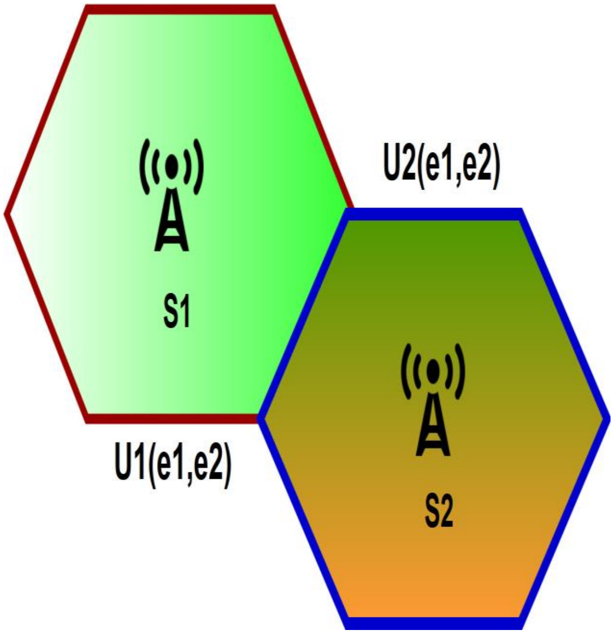

Figure 1: *Diagram of the two towers is used separately*

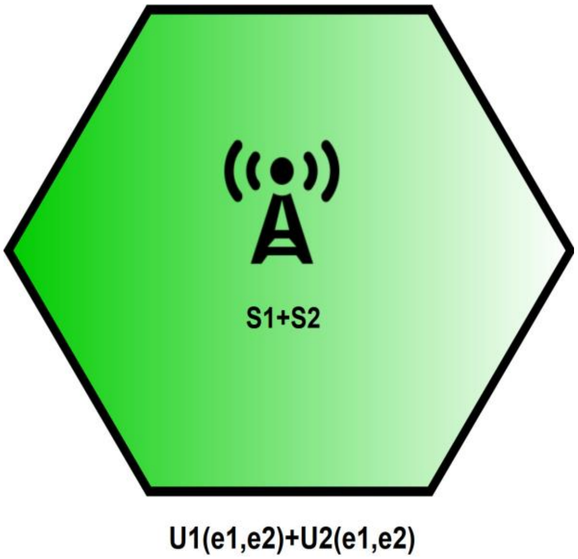

Figure 2: *Diagram of the two towers is used together*

Two mobile towers S_1 And S_2 Were considered and they can be used in two different strategic ways. In this work an equal number of users are served by the individual mobile tower is considered. The interference in this work is reduced by using Successive Interference Cancellation is explained in section IV. Two mobile towers are considered separately as shown in figure 1, the combine of two towers are used in a combine fashion as shown in figure 2. The frequencies allocated to a specific mobile tower are proportional to that frequency component, also the total frequency is reduced as the frequencies are allocated to a mobile tower. The utility function of the mobile tower S_1 Is given by $U_1(e_1,e_2)$= e_1(1-(e_1+e_2))…………………………..(1) The utility function of the mobile tower S_2 Is given by$U_2(e_2,e_1)$= e_2(1-(e_1+e_2))…………………………...(2)

Where e_1 And e_2 Are the normalized frequency components allocated to mobile tower S_1 And S_2 Respectively, so

$0\leq e_1 \leq 1,\ 0\leq e_2 \leq 1$

So, to obtain an optimum value of mobile towers S_1 And S_2 derivative the utility function is done with respect to e_1 And e_2 is equated to zero.

So,$\frac{dU_1}{de_1}=\frac{d}{de_1}e_1$(1-(e_1+e_2))=0……………………………..…(3)

So, from equation 3 best response of e_1 Is given by

$e_1^*=BR_1(e_2)=\frac{1-e_2}{2}$……..…………………..….(4)

Now,$\frac{dU_2}{de_2}=\frac{d}{de_2}e_2$(1-($e_1+e_2$))=0……………………………(5)

So, from equation 4 best response of e_2 is given by

$e_2^*=BR_2(e_1)=\frac{1-e_1}{2}$……………………………………………(6)

From (4) if the optimum value of e_2 is replaced then the equation becomes

$e_1^*=\frac{1}{2}-\frac{1}{2}(\frac{1-e_1^*}{2})$…………………………………………….….(7)

By solving equation (7)

$e_1^*=\frac{1}{3}$……………………………………………………….….(8)

From (6) by replacing the value of e_1 it can be shown that

$e_2^*=\frac{1-e_1^*}{2}=\frac{1-1/3}{2}=\frac{1}{3}$……………………………………………(9)

So, the payoff of a mobile towerS_1 is given by

$U_1^*(e_1^*,e_2^*)=U_1(\frac{1}{3},\frac{1}{3})=\frac{1}{9}$………………………………………(10)

Similarly, the payoff of a mobile tower S_2 is given by

$U_2^*(e_2^*, e_1^*)=U_2(\frac{1}{3}, \frac{1}{3})=\frac{1}{9}$…………………………………………(11)

Now the joint payoff of the combined tower as shown in figure 2 is given by

$U_1(e_1,e_2)+ U_2(e_2, e_1)$

$= e_1(1-(e_1 + e_2))+ e_2(1-(e_1 + e_2))$…………………………..(12)

$(e_1+e_2)(1-(e_1+e_2))$

Now by considering $(e_1+e_2)=e_t$ and joint utility function $U_1(e_1,e_2)+ U_2(e_2, e_1)=U_t$ then equation (12) reduces to

$U_t(e_t)= e_t(1-e_t)$……………………………………………(13)

So, to obtain an optimum value of joint towers derivative is performed with respect to e_t

$\frac{dU_t}{de_t}=\frac{d}{de_t}(e_t(1-e_t)) = 0$……………………………………..(14)

So, the best response e_t can be solved from equation (14) and is given by

$e_t^*=\frac{1}{2}$…………………………………………………..(15)

Now, as $e_t^*=e_1+e_2=\frac{1}{2}$,so $e_1= e_2 = \frac{1}{4}$………………………..(16)

Now from (1) the optimum utility function of the mobile tower S_1 is given by

$U_1(e_1,e_2)= U_1(\frac{1}{4},\frac{1}{4})= \frac{1}{4}(1-(\frac{1}{4}+\frac{1}{4}))=\frac{1}{8}$…………………………(17)

Now from (2) the optimum utility function of the mobile tower S_2 is given by

$U_2(e_2, e_1)= U_2(\frac{1}{4},\frac{1}{4}))= \frac{1}{4}(1-(\frac{1}{4}+\frac{1}{4}))=\frac{1}{8}$………………………..(18)

Now by comparing equation (10) and (11) with equation (17), it can be concluded that as $\frac{1}{8} > \frac{1}{9}$ So, the unused frequency spectrum is larger when the two towers acted combine way. Then a greater number of users can be served when the two towers acted combine way, but the unutilized spectrum is much less when the two towers acted in a separate fashion thus when a user needs broad spectrum it's easily allocated in this topology.

Now to find out the Nash Equilibrium (NE) of mobile tower S_1 game table is created and the frequency allocation performance is compared with the optimum value as derived in equation (8), (9), (10), (11)

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TABLE I: Game theoretic table for mobile tower S_1

e_1 (Hz)	e_2 (Hz)	U_1	e_u(Hz)	Comment
1/3	1/3	1/9	1-(1/3+1/3) =1/3	Equal distribution of used and unused spectrum.
1/5	1/5	0	1-(1/5+1/5) =3/5	Not equal distribution
3/10	1/5	3/20	1-(3/10+1/5) =1/2	Not equal distribution
2/10	7/10	2/100	1-(2/10+7/10) =1/10	Not equal distribution

Where e_u Denotes normalized unutilized spectrum.

So, from the table I, it can be concluded that when the frequency allocated to mobile tower S_1 In the ratio of (1/3,1/3) think it's the best option for mobile tower S_1.Hence (1/3,1/3) is the Nash Equilibrium (NE) for the above-mentioned mobile tower. Similarly, the Nash Equilibrium (NE) for the mobile tower S_2 is same as the mobile tower S_1 as both are symmetric in nature and is given by (1/3,1/3).

Now to find out the Nash Equilibrium (NE) of the mobile tower when they are combined is give game table is created and frequency allocation performance is compared with the optimum value as derived in equation (15), (16), (17).

TABLE II: Game theoretic table for combine mobile tower

e_1 (Hz)	e_2 (Hz)	$(e_1 + e_2)$ (Hz)	U_1 or U_2	e_u (Hz)	Comment
1/3	1/3	2/3	1/9	1-2/3 =1/3	Not equal distribution
1/4	1/4	1/2	1/8	1-1/2 =1/2	Equal distribution of used and unused spectrum.
3/10	1/5	1/2	3/20	1-1/2 =1/2	Not equal distribution of utilized
2/10	7/10	9/10	2/100	1-9/10 =1/10	Not equal distribution

It can be concluded from the table II that when the frequency allocated is (1/4,1/4) then an equal amount of frequencies are allocated to both the towers to serve an equal number of users. Hence (1/4,1/4) is the Nash Equilibrium (NE) of the above network system.

The probability density of mobile tower S_1 is given by

$P(S_1)=e^{-\lambda_1}{\lambda_1}^{k_1}/k_1!$..(19)

Equation (19) is commonly known as Poisson distribution [12].

Where λ_1 denotes the power allocated to mobile tower S_1,k_1 is the number of users under the service of above-mentioned mobile tower.

Then the overall utility function of the mobile tower S_1 is given by

$P (S_1U_1)=(e^{-\lambda_1}{\lambda_1}^{k_1}/k_1!) * U_1$…………………………...(20)

where U_1 is the utility function of mobile tothe wer S_1 as explained in equation (1)

The probability density of mobile tower S_2 is given by

$P(S_2)=e^{-\lambda_2}{\lambda_2}^{k_2} /k_2!$..(21)

Where λ_2 denotes the power allocated to mobile tower S_2,k_2 is the number of users under the service of above-mentioned mobile tower.

Then the probability utility function of the mobile tower S_2 is given by

$P(S_2U_2)=(e^{-\lambda_2}{\lambda_2}^{k_2} /k_2!)* U_2$………………………(22)

where U_2 is the utility function of mthe obile tower S_2 as explained in equation (2)

The probability utility function of the empty spectrum is given by

$P(E)=1-[P (S_1U_1)+ P(S_2U_2)]$

$=1-[(e^{-\lambda_1}{\lambda_1}^{k_1}/k_1!) * U_1+(e^{-\lambda_2}{\lambda_2}^{k_2}/k_2!) * U_2]$------(23)

The threshold of detection γ is described in Appendix A. If the power measured in a frequency component is above γ,then it's known as initialized spectrum and if power measured below γ then it's utilized spectrum

5 SUCCESSIVE INTERFERENCE CANCELLATION (SIC) METHOD

The interference is avoided using V-BLAST (Vertical Bell Labs Layered Space Time Architecture) which employs SIC (Successive Interference Cancellation). Impact of each estimated symbol is cancelled using SIC. The number of transmitted antennas is t and the number of receive antennas are r is considered in this work

Hence the receive signal is given by

$\bar{y}=\bar{H}\bar{x}+\bar{n}$……………………………………(24)

Where, $\bar{H}$ is the channel matrix of dimension r×t is given by $\begin{bmatrix} h_{11} & \cdots & h_{1t} \\ \vdots & \ddots & \vdots \\ h_{r1} & \cdots & h_{rt} \end{bmatrix}$ where,h_{ij} =channel equation from j th the receiving antenna to i th transmitting antenna and $\bar{h_t}$ represents t columns of the matrix $\bar{H}$ is given by $\begin{bmatrix} h_{1t} \\ h_{2t} \\ \vdots \\ h_{rt} \end{bmatrix}$, ,$\bar{x}$ is the transmitted vector of dimension t×1 is given by

$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix}$,$\bar{n}$ is the white Gaussian noise matrix of dimension t×1 each element with mean=0 and variance=σ^2 is given by

$$\begin{bmatrix} n_1 \\ \vdots \\ n_t \end{bmatrix}$$

Now the equation (24) is modified as shown below

$\bar{y}=[\bar{h}_1 \quad \ldots \quad \bar{h}_t]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix}+\begin{bmatrix} n_1 \\ \vdots \\ n_t \end{bmatrix}$……………………………(25)

Now the pseudo inverse of the matrix $[\bar{h}_1 \quad \ldots \quad \bar{h}_t]$ is calculated at the receiver and is given by $Q=[\bar{h}_1 \quad \ldots \quad \bar{h}_t]^{+}$where Q is given by $\begin{bmatrix} \bar{q}_1^H \\ \vdots \\ \bar{q}_t^H \end{bmatrix}$, where $\bar{q}_1^H$ is the Hermitian matrix of $\bar{h}_i$ and satisfies the property $\bar{q}_i^H\bar{h}_j=1$ if i=j and $\bar{q}_i^H\bar{h}_j=0$ if i≠j.

So,$\begin{bmatrix} \bar{q}_1^H \\ \vdots \\ \bar{q}_t^H \end{bmatrix}[\bar{h}_1 \quad \ldots \quad \bar{h}_t]$

$=\begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}=I_t$……………………………..(26)

Where I_t is the identity matrix of dimension t×t.

Now to recover symbol x_1 the received vector is multiplied by $\bar{q}_1^H$,So

$\tilde{y}_1=\bar{q}_1^H\bar{y}=\bar{q}_1^H[\bar{h}_1 \quad \ldots \quad \bar{h}_t]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix}+\bar{q}_1^H\bar{n}$

$=\bar{q}_1^H(\bar{h}_1x_1+$……………..$+\bar{h}_tx_t)+\tilde{n}$ where $\tilde{n}=\bar{q}_1^H\bar{n}$ is considered negligible

$=x_1$…………………………………………..……..(27)

As, $\bar{q}_i^H\bar{h}_j=1$ if i=j and $\bar{q}_i^H\bar{h}_j=0$ if i≠j.

Now, as x_1 is known so it's effect can be removed and is given by,

$\widehat{y_2}=\tilde{y}_1-\bar{h}_1x_1$

$=(\bar{h}_1x_1+$……………..$+\bar{h}_tx_t)+\bar{n}-\bar{h}_1x_1$

$=(\bar{h}_2x_2+$……………..$+\bar{h}_tx_t)+\bar{n}$

$=[\bar{h}_2 \quad \ldots \quad \bar{h}_t]\begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_t \end{bmatrix}$……………………………(28)

Noise is neglected due to negligible value.

So, the new channel matrix is given by $[\bar{h}_2 \quad \ldots \quad \bar{h}_t]$

So, to recover symbol x_2 is given by

$\tilde{y}_2=\bar{q}_2^H\widehat{y_2}=\bar{q}_2^H[\bar{h}_2 \quad \ldots \quad \bar{h}_t]\begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_t \end{bmatrix}$

$=x_2$…………………………………………..(29)

As, $\bar{q}_i^H\bar{h}_j=1$ if i=j and $\bar{q}_i^H\bar{h}_j=0$ if i≠j.

Now, the process is repeated and each symbol is detected. The advantage of this scheme is diversity order is progressively increasing as process proceed through the scheme.

$\widehat{y_2}=\bar{h}_tx_t$…………………………………..(30)

The diversity order of $\bar{h}_t$ is given by r, Streams that are decoded later experience progressively higher diversity.

6 RESULTS & DISCUSSION

The utility function equation (1) of mobile tower S_1 is plotted in figure 3. It's displayed that the utility function is maximum in between the normalized frequency component 0.2Hz to 0.3Hz, but from Table I it's already shown that the utility function is at 1/3=0.33. Now the utility function of the unutilized frequency spectrum is 1-1/3=2/3. In percentage it's= (2/3) *100=66.7%.

The utility function equation (2) of mobile tower S_2 is plotted in figure 3. It's displayed that the utility function is maximum in between the normalized frequency component 0.2Hz to 0.3Hz, but from Table I it's already shown that the utility function is at 1/3=0.33. Now the utility function of the unutilized frequency spectrum is 1-1/3=2/3. In percentage it's= (2/3) *100=66.7%.

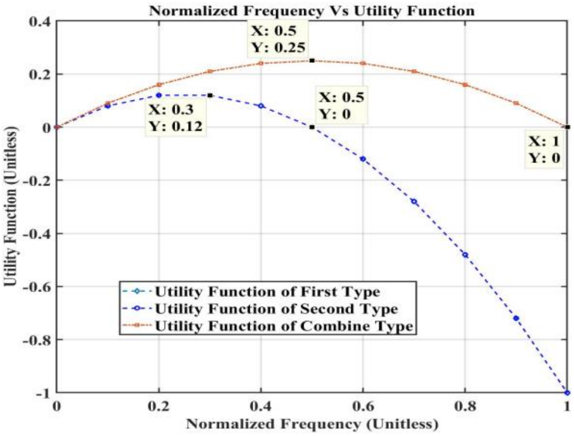

***Figure 3**: Utility Function of Mobile tower*

The utility function equation (12) of combining mobile towerS_1 And S_2 Is plotted in figure 3. It's displayed that the utility function is maximum at the normalized frequency component 0.5Hz, but from Table II it's already shown that the utility function is at 1/8=0.125.From figure utility function is maximum at the value of 1/4 but as the two mobile towers are used in combined fashion so the utility function is given by (1/4) * (1/2) =1/8=0.125.So the unutilized spectrum utility function is given by 1-0.125=7/8. In percentage it's= (7/8) *100=87.50%.

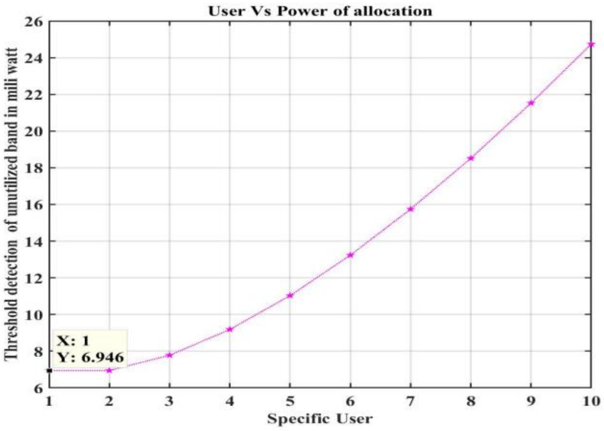

Figure 4: *Threshold Detection of Utility Function of unutilized band*

The Utility functions of the unutilized band as derived in Appendix A is plotted in figure (4). It can be shown that if the power allocated is more 7 milli watts then the utility function of the unutilized band is increased sharply and a greater number of users can be served.

Hence the probability of detection when the signal power detected is more than 7 milli watt as it's shown in figure 5& figure 6.

So, the probability of detection is

P_D=Q(y_1>γ),[13]

Where Q is known Q-function which can be shown as

Q(x)=$\frac{1}{\sqrt{(2\pi)}}\int_x^\infty e^{-\frac{r^2}{2}}$dr [13] and y_1 =$e^{\wedge(y)}$Is given by equation (40) as derived in appendix A. and γ =Threshold value=7 milli watts. So, the probability of false alarm is given by:

P_F=1-P_D.

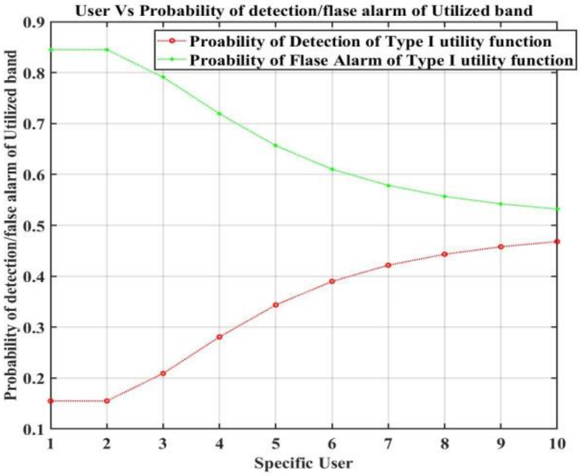

Figure 5: *User Vs Probability of detection & false alarm for two towers used separately*

Figure 5 shows the probability of detection & probability of a false alarm Vs number of users of first type utility function, it can be shown that if the number of users is more than the probability of detection is high as the total power allocated to the mobile tower is distributed equally among the users the interference among the users are less and hence the probability of detection is high.

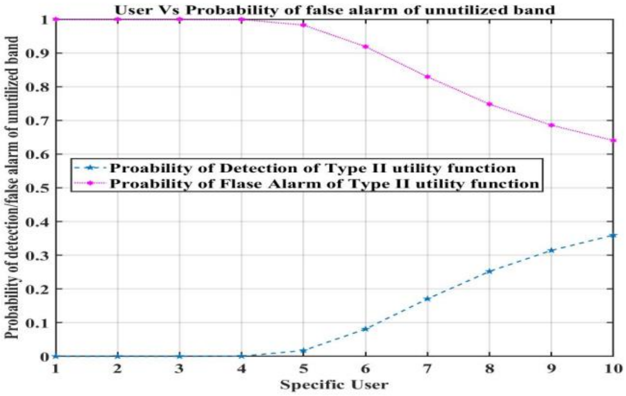

Figure 6: *Probability of Detection/False Alarm Vs Number of Users for two towers are used together*

The probability of detection & probability of a false alarm Vs a number of users of Second type utility function is shown in figure 6. If the number of users is more than probable of false alarm is low as the total power allocated to the mobile tower is distributed equally among the users the interference among the users are less and hence the probability of false alarm is low. Now, by comparing figure 5 & figure 6 it is observed that probability of detection is high in second type of utility function and simultaneously probability of false alarm is low in second type of utility function as two mobile towers are used simultaneously so a greater number of users are served.

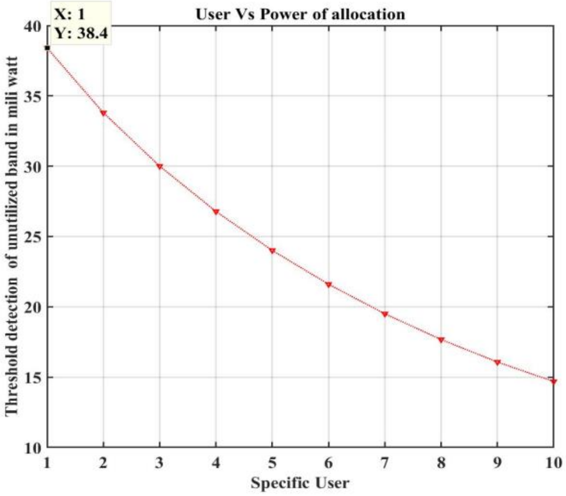

Figure 7: *Threshold Detection of Utility Function*

On the other hand, if the threshold of detection threshold is more (38.4 milliwatt) which is shown in figure (7).In that case more power is allocated to a specific base station, then due to self-interference the utility function is reduced.

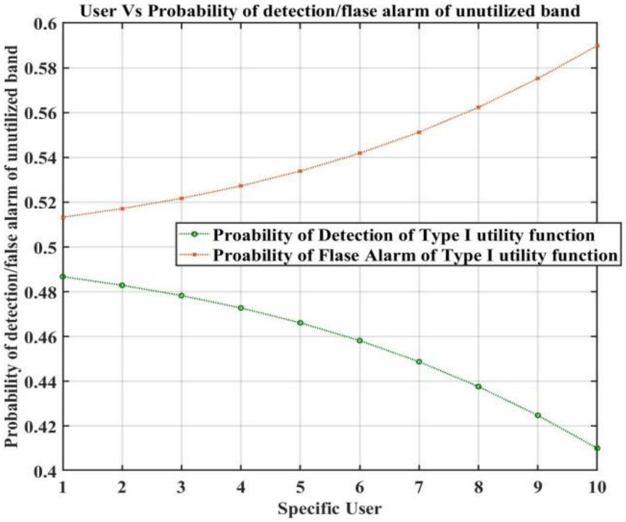

Figure 8: *Probability of Detection/False Alarm Vs Number of Users for two towers is used separately.*

Figure 8 shows the probability of detection & probability of false alarm Vs number of users of first type utility function for threshold value 38.4 milliwatt, it can be shown that if the number of users is more than the probability of detection is low due to self-interference among users .

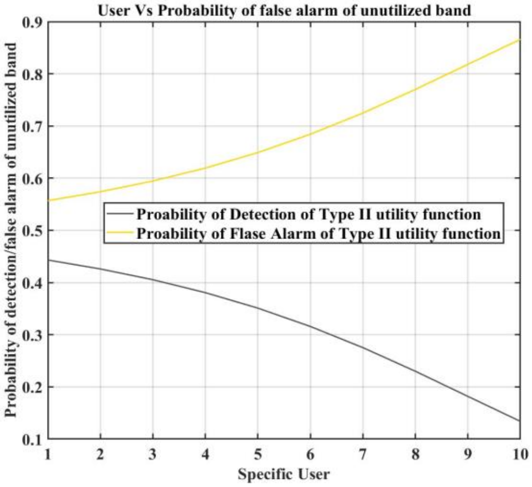

Figure 9: *Probability of Detection/False Alarm Vs Number of Users for two towers are used combined fashion*

The probability of detection & probability of false alarm Vs a number of users of Second type utility function for threshold value 38.4 milli watt is shown in figure 9. If the number of users is more than probability of false alarm is high due to self-interference among users. The probability of detection is low in second type of utility function as two mobile towers are used in a combined fashion.

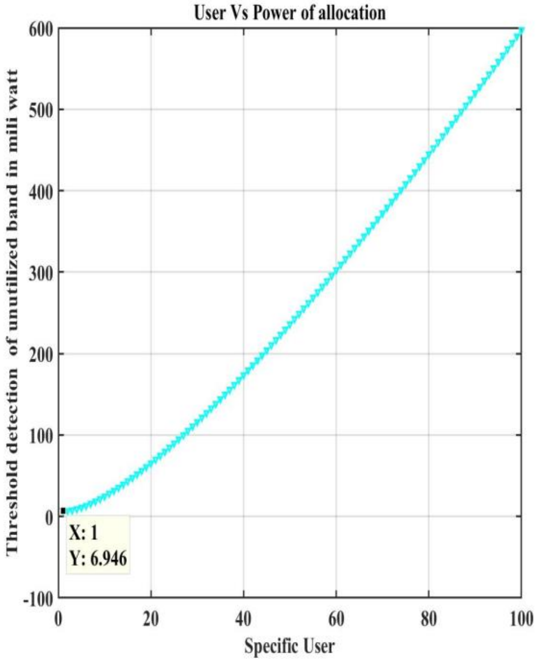

Figure 10: *Threshold Detection of Utility Function*

If the number is more compared with figure (4) and figure (7) then detection threshold is less as no of users are more than previous cases then there will be interference among users if high amount is allocated hence this case the amount of power is allocated=6.946 milli watt.

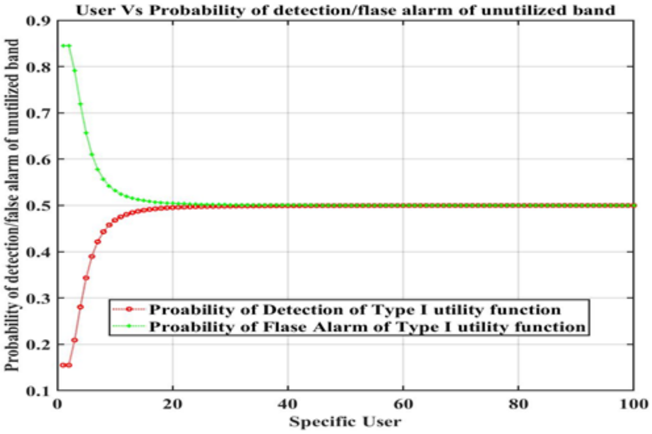

Figure 11: *Probability of Detection/False Alarm Vs Number of Users for two towers is used separately.*

Figure 11 shows the probability of detection & probability of false alarm Vs number of users of first type utility function, it can be shown that if the number of users is more, in that case, all the users may not be allocated frequency so communication will not possible for that user so the probability of detection is less.

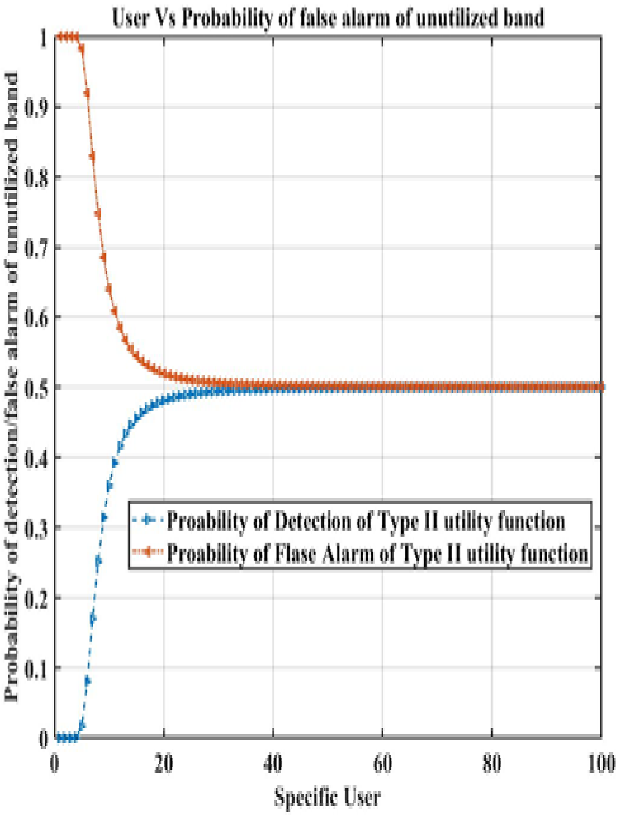

Figure 12: *Probability of Detection/False Alarm Vs Number of Users for two towers are used together*

The probability of detection & probability of false alarm Vs a number of users of Second type utility function is shown in figure 12. If the number of users is more than probable of false alarm is high as the number of users is more, in that case, all the users may not be allocated frequency so communication will not possible.

TABLE III: Comparison Table among different figures

Sl No	Fig No	Parameter Variation	Comment
1	4,7,10	Transmission, Power Variation	More transmit power more threshold.
2	(5,6), (8,9), (11,12)	Utility Function	Better Detection for more frequency allocation.
3	(3,6), (4,7),	Transmission, Power Variation	More transmit power cause interference among users.
4	(3,9), (4,10)	No of user	If the number of users is more frequency may/may not allocate.

7 RELATED WORKS

TABLE IV: Comparison Table with proposed & existing model

Serial No	Feature	Existing Models		Proposed Model
		[2]	[17]	
1	Probability of Detection	×	√	√
2	The probability of False Alarm	×	√	√
3	Interference Cancellation	√	×	√
4	Compressive Sensing	×	√	×

8 CONCLUSION

In this paper frequencies were allocated to the mobile subscribers using game theoretic approach. The tragedy of commons game theoretic approach is a mathematical strategy by which frequencies were allocated among the mobile two towers to serve an equal number of users and the unused spectrum is allocated to serve the users who need extra band for their usage. The game theoretic strategy can be allocated to mobile towers either separately or in combined fashion. The probability of detection is better when the mobile towers are used in a combined fashion provided the power allocated is Less as two mobile towers are used simultaneously so a greater number of users are served, but when power allocated is more the combined topology performance degrades due to self-interference. The probability of detection of unutilized band is only 50% when the number of users is more in both the individual and combined topology as power is distributed among the number usersA better topology can be designed when allocated power is more and no of users are more in future work so that probability of detection of unutilized band can be improved.

9 APPENDIX A

The probability of the first network S_1 Is given by

P(S_1)=$e^{-\lambda_1}{\lambda_1}^{k_1}/k_1$!..(31)

The probability utility function of the network S_1 is given by

P (S_1U_1)=($e^{-\lambda_1}{\lambda_1}^{k_1}/k_1$!) * U_1……………………..(32)

The probability of the second network S_2 is given by

P(S_2)=$e^{-\lambda_2}{\lambda_2}^{k_2}$ /k_2!..(33)

The probability utility function of the network S_2 is given by

P(S_2U_2)=($e^{-\lambda_2}{\lambda_2}^{k_2}$ /k_2!)* U_2………………………(34)

The probability utility function of the empty spectrum is given by

P(E)=1-[P (S_1U_1)+ P(S_2U_2)]

=1-[($e^{-\lambda_1}{\lambda_1}^{k_1}/k_1$!) * U_1+($e^{-\lambda_2}{\lambda_2}^{k_2}$ /k_2!)* U_2]……………..(35)

Using log likelihood ratio test statistics ∧(y) is given by

∧(y)=ln($\frac{\mathrm{P(E)}}{\mathrm{P}\ (S_1U_1)*\mathrm{P}(S_2U_2)}$)………………………………………(36)

Substituting value of P(E) from equation (35) into equation (36)

=ln($\frac{1-[(\ e^{-\lambda_1}{\lambda_1}^{k_1}/k_1!)\ *\ U_1+(\ e^{-\lambda_2}{\lambda_2}^{k_2}\ /k_2!)*\ U_2]}{\left(\frac{e^{-\lambda_1}{\lambda_1}^{k_1}}{k_1!}\right)*\ U_1*(\ e^{-\lambda_2}{\lambda_2}^{k_2}\ /k_2!)*\ U_2}$)………………(37)

So, $\frac{1-[(\ e^{-\lambda_1}{\lambda_1}^{k_1}/k_1!)\ *\ U_1+(\ e^{-\lambda_2}{\lambda_2}^{k_2}\ /k_2!)*\ U_2]}{\left(\frac{e^{-\lambda_1}{\lambda_1}^{k_1}}{k_1!}\right)*\ U_1*(\ e^{-\lambda_2}{\lambda_2}^{k_2}\ /k_2!)*\ U_2}$)=$e^{\wedge(y)}$…………..(38)

Replace U_1=U_2=1/3 in Equation (31)

$$=\frac{k_1!k_2!(9-3*\left[\frac{(e^{-\lambda_1}\lambda_1{}^{k_1}}{k_1!}\right)+\left(e^{-\lambda_2}\lambda_2{}^{k_2}\ /k_2!\right)]}{e^{-(\lambda_1+\lambda_2)}\ \lambda_1{}^{k_1}*\lambda_2{}^{k_2}}\text{……………………..……}(39)$$

For equal amount of high-power allocation and equal number of users in two mobile tower regions

k_1=k_2=k(say) & λ_1=λ_2=λ(say)

Equation (39) reduces to

$$=(k!)^2(e^{2\lambda}\text{-}2e^{\lambda}/\text{k!})\text{……………………………………………}(40)$$

So the threshold amount e^{γ} which is very high.

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