

# A Mathematical Model To Predict The Quantity Of Defective Bottles In An Automated Bottle Washer Using Factorial Design Technique

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**ABSTRACT:** Improper wash of reused bottles in beer and soft drink production plants can result in high cost of production and potentially dangerous health hazard to customers. It is shown that when machines are used over a period of time, they are not able to perform at their best, for which there are many root causes. This paper developed a mathematical model to predict and monitor the amount of Dirt Defect and Caustic Residue Defect in a washed bottle in a beverage bottling company. In this work, a 23 factorial design is applied to investigate the effect of Pressure, Temperature and Caustic soda concentration, on two washing responses namely: bottles with Caustic Residue Defect and bottles with Dirt Defect. A model was generated for the washing responses to the washing variables. The generated models can be used to aid the quality improvement and reliability in the washed bottles for the beverage companies in Ghana.

**KEYWORDS:** Factorial Design, Caustic Residue Defect, Dirt Defect, Automated Bottle Washer, Experimental Design

## 1 INTRODUCTION

The production of soft drinks in Ghana is 6.5 million cubic meters per year, of which 70% is sold in returnable glass bottles, 20% in plastic bottles and 10% in aluminum cans [7]. The advent of light and disposable containers such as aluminum cans and polyethylene terephthalate (PET) bottles for beverage packaging has not at all phased-out the use of recycled glass bottles. The plastic water and soft drink bottles are sold with the intention of single use, then recycling, they can be safely reused if cleaned and handled properly. The reuse of bottles has been discussed as having possible health risks. There are two main concerns, that is the potential for the presence and growth of bacteria in these bottles, but with proper cleaning and handling, this risk can be minimized. Another health concern sometimes mentioned around the reuse of plastic bottles, but which current research shows to be unfounded, is that the plastic may breakdown and release 'chemicals' into the water [3]. Refilling water bottles can result in contamination of the water with bacteria and fungi that can grow in damp or partially full bottles once they have been opened. These organisms generally come from the air, your hands and mouth, or anything that comes in contact with the mouth of the bottle. With time and in warm conditions, bacteria can multiply to harmful levels, but safe handling and proper cleaning can help prevent this from happening [3]. Reusable water bottles should be thoroughly cleaned, rinsed and dried between uses. Dishwashing soap and hot water are acceptable for cleaning your water bottle. The risks of bacterial and fungal growth are higher if you use the bottle with a drink that contains sugars or other foods. Immediately drain, rinse, and wash the water bottle after using it with sports drinks or juices. [3] For this reason one specific source of worry regarding quality in soft drink production is the bottle washing operation. In a soft drink company, bottle washing can pose a major sanitation problem or a major health risk for consumers as a result it is important to maintain maximum process efficiency in the bottle washing operation. Bottle washing is performed by washing machines that operate in different cycles: pre-rinse, pre-wash, caustic wash and final rinse. The substances used are water, detergent, chlorine and a solution of caustic soda (NaOH) [2]. According to [8],

caustic soda (excluding solutions containing not more than 5% caustic soda) is designated as a deleterious substance under Japanese laws, and is a strongly corrosive substance. Persons who handle caustic soda should learn about related laws and regulations (such as the Poisonous and Deleterious Substance Control Law), its properties, and precautions on handling; and should observe them to ensure safety. Determination of important factors and estimation of the effects of these factors on the response variables is important in any experiment [4], [5]. Some of these factors are background variables sometimes called a noise variable in an experiment are not considered as factors but potentially can affect the quality of washed bottles. Proper variation of the factors that have the greatest effect on variation of the wash quality is essential in order that a thoroughly cleaned bottle is produced prior to filling. In Bottle washing a high quality is considered when the exiting bottles do not carryover the solution or/and when bottles are totally free of dirt and debris [1]. While industries in developed countries adopt modern and efficient methods to control factors in bottle washers, most breweries and soft drink companies in Ghana still use old washing machines where values of input factors are manually manipulated. Most industrial bottle washers in developed countries of late use a CNC controller to adjust washing solution levels and parameters such as heat, pressure, applied load and flow rate in a pre-programmed manner. Other industrial parts washers are controlled or programmed through a computer-based interface. In situations where many variables are manually operated, maintaining fixed and accurate variables in the washing process is very crucial to meeting quality standards and satisfying customer needs. Incorrect variable values are important in a bottle washer due to their influence on caustic soda carry over, and bottles that still have dirt and debris after washing [2]. The aim of this paper is to investigate the caustic bath and rinsing stages of an automated bottle washer in a bottling company in Ghana. A factorial design of experiments approach will be used to find the cause of caustic soda carry over and bottles that still have dirt after washing. Optimum values for the critical factors would be established for best performance.

## 2 MATERIALS AND METHODS

### 2.1 Experimental Setup

The experimental set up as shown in a schematic diagram of Fig. 1 works on the following principle. The empty glass bottles (indicated by circles) are delivered on the conveyer belt and automatically uncased at point 2. Two persons visually inspect the bottles to remove objects from them before they are conveyed into the bottle washer. The bottles enter the washer in rows of twenty five. Each row of bottles goes through a series of cleaning process for 20 minutes before leaving the washer.

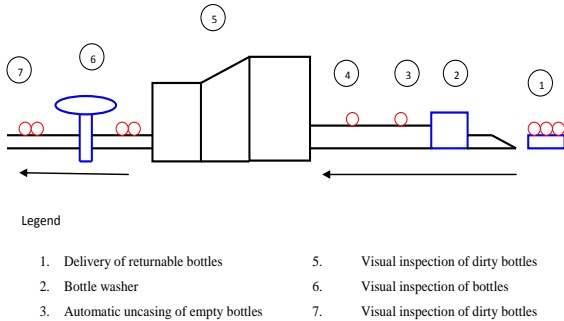


Fig. 1: Schematic Diagram of Experimental Set Up

The electronic scanner fixed on the bed of the conveyer then scans the exiting bottles and rejects those with caustic carry over from the conveyer. The scanner rejects by a bib and a red light on any trace of caustic in the bottle, and that bottle is picked. Bottles cleaned from caustic carry over proceeds on the conveyer for Dirt Defect and debris inspection. This inspection is done by two people. Counting and recording of defective bottles are done manually.

### 2.2 Selection of Washing Parameters

The coca cola manual recommends F-8420, a high caustic liquid product for bottle washing. It contains organic as well as inorganic sequestrants for control of hard water and scales build up in bottles. Concentration for each percentage caustic soda required is 15.0 litres per every 1,000 litres capacity. The plant under study varies their caustic concentration levels from 1.5% - 2.5% at temperatures between 70 oC to 80°C. The pressure used by the plant varies between 0.62 N/mm<sup>2</sup> and 0.76 N/mm<sup>2</sup>. Hence a factorial design is applied in investigating the effect of washing conditions on cleaning parameters for three variables, namely: Pressure (N/mm<sup>2</sup>), Temperature (°C) and Caustic soda concentration (%). The design matrix is as shown in Fig. 2, which shows the variables in ranges of two points such that the points take on high (+) and low (-) values for each variable. The possible number of associations for the three variables in two ranges is 2<sup>3</sup>. The eight points on the corners of the cube depict this. Based on the above information, 1.5% Sodium hydroxide (NaOH) is used for the low value and 3% for the high. The temperature levels selected for the caustic solution in the soaking and rinsing compartments were 60oC as low value and 80oC as high value see Fig 2. The low and high values for pressure in the rinsing compartment were chosen as 0.62 N/mm<sup>2</sup> and 0.76 N/mm<sup>2</sup> respectively.

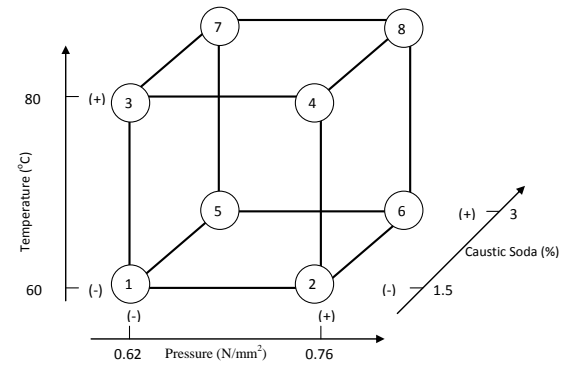


Fig. 2: Washing Conditions in Design Matrix Form

### 2.3 Measurement of Response Variables

When these bottles exit from the washer, they are then transported through a conveyor to be inspected through two light scanners to determine bottles with dirt defect or debris. From this stage, the bottles go through an electric device that scans and rejects those containing caustic residue defects. The bottles are now considered clean, free from dirt defect, debris and caustic residue defect, and are passed on for filling. As any quantity of dirt defect or caustic residue defect in a bottle is highly significant in a soft drink company, the number of defective bottles was considered rather than amount of dirt defect or caustic residue defect in a bottle. For each set of wash, five thousand (5000) bottles are washed and inspected to reject the defective bottles. Each experiment is replicated at each set of washing condition. The measurable quantities of defective bottles based on the two responses are recorded. The averages of the two runs are computed for each set of conditions. Table 1 shows the recorded values for defective bottles.

Table 1: The Measured Values for the Washing Responses

Order		Washing Variables			Number of Defective Bottles	
Std Order	Run Order	Pressure N/mm <sup>2</sup>	Temperature °C	Caustic Concentration%	Caustic Residue	Dirt
2	1	0.76	60	1.5	900	65
14	2	0.76	60	3.0	1230	78
4	3	0.76	80	1.5	1050	50
3	4	0.62	80	1.5	870	48
1	5	0.62	60	1.5	830	42
16	6	0.76	80	3.0	1360	74
11	7	0.62	80	1.5	880	47
7	8	0.62	80	3.0	1600	74
6	9	0.76	60	3.0	1280	81
8	10	0.76	80	3.0	1440	70
5	11	0.62	60	3.0	1320	63
9	12	0.62	60	1.5	770	47
12	13	0.76	80	1.5	1050	50
10	14	0.76	60	1.5	940	63
15	15	0.62	80	3.0	1550	64
13	16	0.62	60	3.0	1340	55

The computed averages are used as the responses for each condition in calculating the main effects and the interaction effects for each washing response. The major goal of these experiments is to determine which of the responses are influenced by the washing parameters and to generate a model that may be used to predict the number of defective bottles based on Caustic Residue Defect and Dirt Defect.

### 3 ANALYSIS AND DISCUSSION OF RESULTS

#### 3.1 Computation of the Effects

A 2<sup>3</sup> factorial design is shown in Fig. 3 and Fig. 4 with the eight runs forming the corners of the cube. This design allows three main effects to be estimated (p, t, c) along with three two-factor interactions (pt, pc, tc) and a three-factor interaction (ptc). The main effect of each of the process variables reflects the changes of the respective responses as the process variables change from a low to a high level. By appropriate analysis of the cubes in Fig. 3 and Fig. 4, the effects of the factors can be estimated.

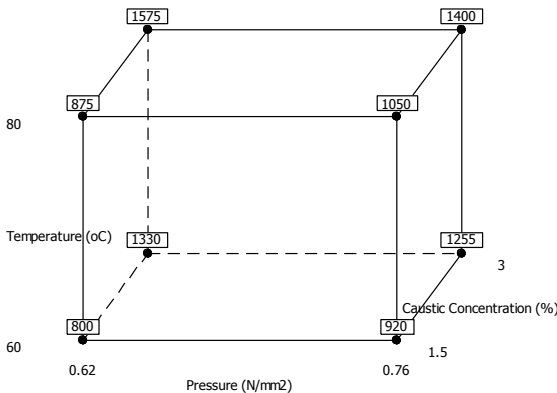


Fig. 3: Cube Plot (Mean Data for Caustic Residue Defect)

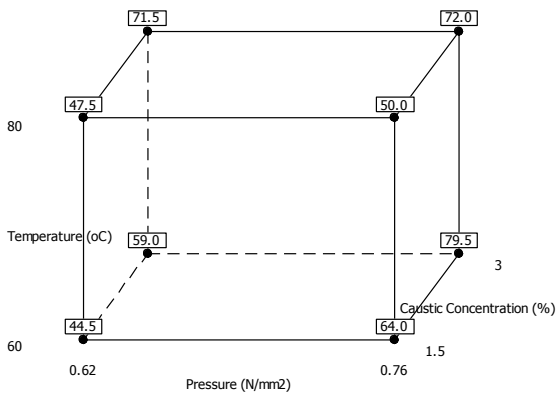


Fig. 4: Cube Plot (Mean Data for Dirt Defect)

The average of the four measures is the main effect of the factor (variable) and two or more of the variables may jointly influence the responses. These joint influences are referred to as interactions. The main effects can be estimated with the lower case letters (1), p, c, tc, t, tc, pt, ptc representing the total of all n replicates at each of the eight runs in the design. To calculate main effects, Minitab subtracts the mean response at the low or first level of the factor from the mean response at the high or second level of the factor.

Hence, the three main effects (pressure, p, temperature, t and caustic soda, c), the two-factor effect is a measure of the interactions of any two variables and the three-factor interaction effect, pressure, temperature and percentage caustic concentration were estimated using the mean of the runs. Table 2 summarizes the findings:

Table 2: Estimated Effects and Coefficients

Term	Caustic Residue Defect			Dirt Defect		
	Effect	Coef	P	Effect	Coef	P
m		1150.63	0.00		61.00	0.00
p	11.25	5.62	0.51	10.75	5.38	0.00
t	148.75	74.37	0.00	-1.50	-0.75	0.35
c	478.75	239.38	0.00	19.00	9.50	0.00
p*t	-11.25	-5.63	0.51	-9.25	-4.63	0.00
p*c	-136.25	-68.13	0.00	-0.25	-0.13	0.87
t*c	46.25	23.13	0.02	4.00	2.00	0.03
p*t*c	-38.75	-19.37	0.05	-0.75	-0.38	0.63
Caustic Residue Defect			Dirt Defect			
S = 32.6917			S = 3			
R-Sq = 99.23%;			R-Sq = 96.99%			
R-Sq(pred) = 96.90%			R-Sq(pred) = 87.98%			
PRESS = 34200			PRESS = 288			
m = Constant						
p = Pressure (N/mm <sup>2</sup> )						
t = Temperature (°C)						
c = Caustic Conc.(%)						

To fit the full model, which includes the three main effects, three two-way interactions, and one three-way interaction, the values in the P column of the Estimated Effects and Coefficients table can be used to determine which of the effects are significant.

#### 3.2 Identification of Important Effects

The objective is to select factors that have large effects on the responses by creating a factorial design and collected the response data to fit a model. The response data collected is used to generate graphs to evaluate the effects. The use of the output from fitting a mathematical model, and also the use of the two graphical methods to help see which factors are important for minimizing the defective bottles in the washing process. To generate two effects plots that will help you determine which effects are active, the Normal and Pareto were selected at a confident level of 0.05. The results obtained are presented in Fig. 5 and Fig. 6 respectively.

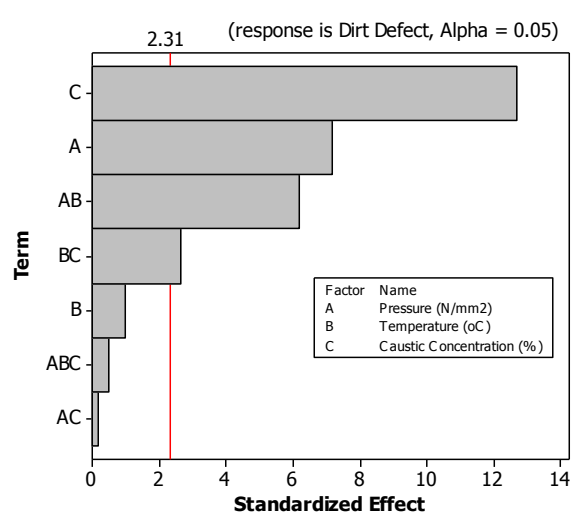
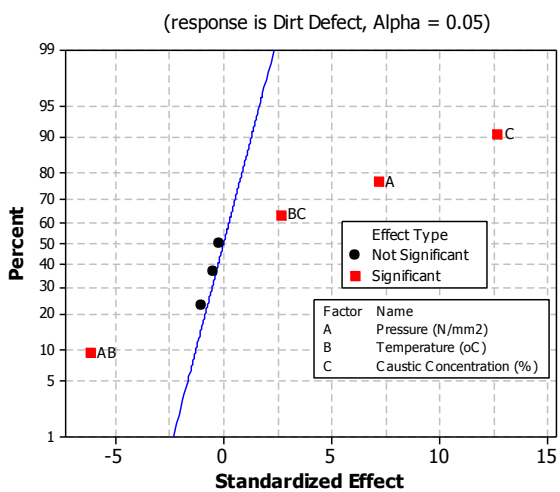
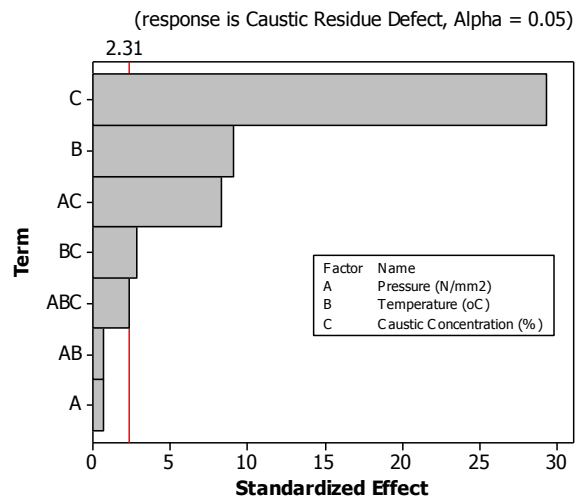
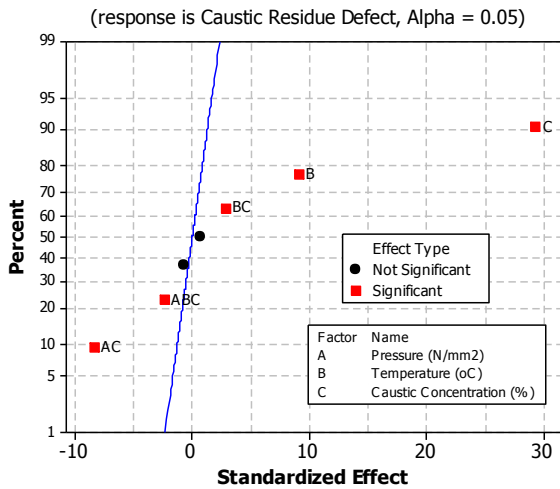


Fig. 5: The normal probability plot

Fig. 6: Pareto chart of the effects

Active effects are effects that are significant or important. In the normal plot of the effects, points that do not fit the line well usually signal active effects. Active effects are larger and further from the fitted line than inactive effects. Inactive effects tend to be smaller and centered on zero, the mean of all the effects. Here, the effects of Temperature, Caustic Concentration, Temperature/Caustic interaction, Pressure/Caustic interaction and Pressure/Temperature/Caustic Concentration interaction are significant for Caustic Residue Defect response while Pressure, Caustic Concentration, Pressure/Temperature interaction and Pressure/Caustic Concentration are significant for the Dirt Defect response using  $\alpha = 0.05$ .

A Pareto chart of the effects is another useful tool that you can use to help determine which effects are active. The Pareto chart uses the same normal plot to determine the significance of effects. Similarly, the effects of Temperature, Caustic Concentration, Temperature/Caustic interaction, Pressure/Caustic interaction and Pressure/Temperature/Caustic Concentration interaction are significant for Caustic Residue Defect response while Pressure, Caustic Concentration, Pressure/Temperature interaction and Pressure/Caustic Concentration are significant for the Dirt Defect response using  $\alpha = 0.05$ . It is clear from the two results that, some of the main effects and some of the interaction have significant effect on both the Caustic Residue Defect and the Dirt Defect. Hence a model can then be developed to predict and monitor the quantity of defective bottles for both the washing responses.

### 3.3 Generation and Evaluation of the Model

A full  $2^3$  model consist of three main effects, three two-factor interactions and one three-factor interaction. It is easier to obtain residuals from a 2k design by fitting a regression model to the data. For this experiment, the model is defined as:

$$N_N = \left[ \begin{matrix} \alpha_0 + \alpha_1 p + \alpha_2 t + \alpha_3 c + \alpha_4 pt + \\ \alpha_5 pc + \alpha_6 tc + \alpha_7 ptc \end{matrix} \right] \pm s$$

where  $\alpha_0, \alpha_1, \dots, \alpha_7$  are constants,  $s$  is standard error and  $p, t,$  and  $c$  are pressure, temperature and percentage caustic concentration respectively. Hence the significant effects and interactions were used to develop the empirical model for the two washing responses. Thus, using their coefficient in Table 2, then, the model for the Caustic Residue Defect response and the Dirt Defect response are:

$$N_C = \left[ \begin{matrix} 1150.6 + 74.3t + 239.4c - 68.1pc \\ + 23.1tc - 19.4ptc \pm 32 \end{matrix} \right]$$

and

$$N_D = 61 + 5.4p + 9.5c - 4.63pt + 2.0tc + 3$$

respectively.

The model can be evaluated by generating plots to visualize the effects, evaluate the fit of the reduced model, and also do a residual analysis. A good standard by which to evaluate the model is to look at p-values. The fitted values are the results predicted by the model and the residuals are the actual defective bottles minus the predicted defective bottles. The results obtained for Caustic Residue Defect and the Dirt Defect responses are presented in Fig. 7 and Fig. 8 respectively.

As illustrated in Fig 7 and Fig. 8, the residual error only increased by a small amount and also, the p-value for each term in the model is less than 0.05, indicating that the two models are good candidates for further exploration and validation. Also, the residuals plots were satisfactory, and showed no cause for concern. Therefore, the two models developed are considerably simpler and fit the data almost as well as the model with all terms.

### 3.4 Prediction and Optimization of the Model

Since the caustic residue defect contributes 96% of the total defective bottles, then the model developed for the caustic residue defect can be used to predict the quantity of defective bottles per each batch of wash bottles. Hence the results obtained for the quantity of defective bottles produced for a period of a year were compared with the predicted value. The plant uses caustic soda as both a cleanser and an effective germicide. It is considered more profitable to use caustic solution over other available alkali mainly because caustic solution is cheaper and is less prone to thermal shock. At a solution temperature of 80 oC, it attacks glass and softens its surface. Thus, organic matter is dissolved, grease and oil are emulsified and dirt is removed in suspension. The factory uses a 32% sodium hydroxide stock solution for their bottle washing operations. Bottles stay inside the soaking compartment for 5 minutes before moving on to the rinsing compartment. At the rinsing compartment, the bottles are sprayed inside and outside by a caustic spray jets at 0.62 N/mm<sup>2</sup> pressure. The caustic solution here is 70 oC at the same concentration as the soaking compartment. The causticity of the soaking compartment is monitored twice every hour by the quality control department. To adjust the caustic concentration according to specifications (within the range of 2.0 and 2.5%), fresh caustic solution is added to the compartment. Caustic solutions are totally drained out of the bottle washers only after the bottle washer reaches an operating time of 1,000 hours. This means that the caustic solution in the compartment is reused for a period of 1,000 hours. The rinsing spray jets alignment and pressure, and the temperature at both the caustic bath compartment are also monitored and adjusted according to the response of the output parameters. The bottle washer operates a maximum of six batches each production day and washes five thousand (5000) bottles per batch, equivalent to thirty thousand (30,000) bottles a day. A data was collected for the defective bottles using ten batches in each month as illustrated in Table 3 for this analysis. Also, the total number of bottles washed with the defective bottles for each month was also collected and the results are presented in Table 4.

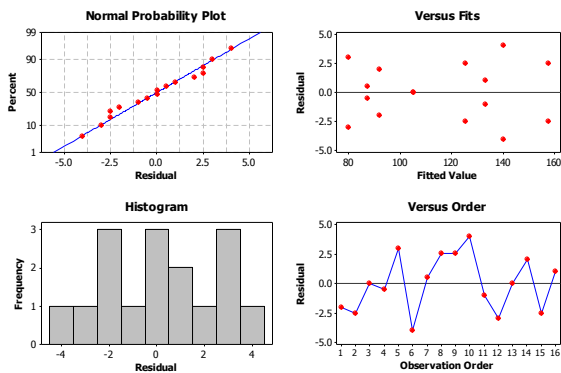


Fig. 7: Residual Plot for Caustic Residue Defect Response

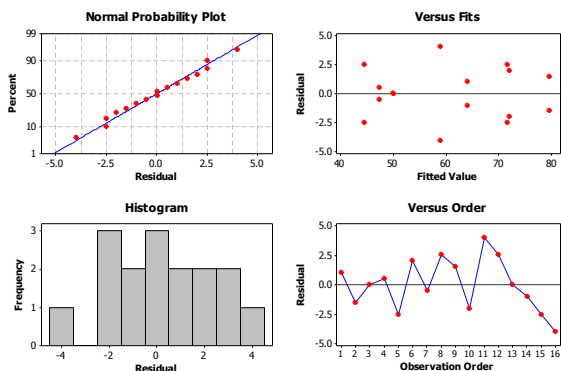


Fig. 8: Residual Plot for Dirt Defect Response

**Table 3: Defective Bottles for a Year**

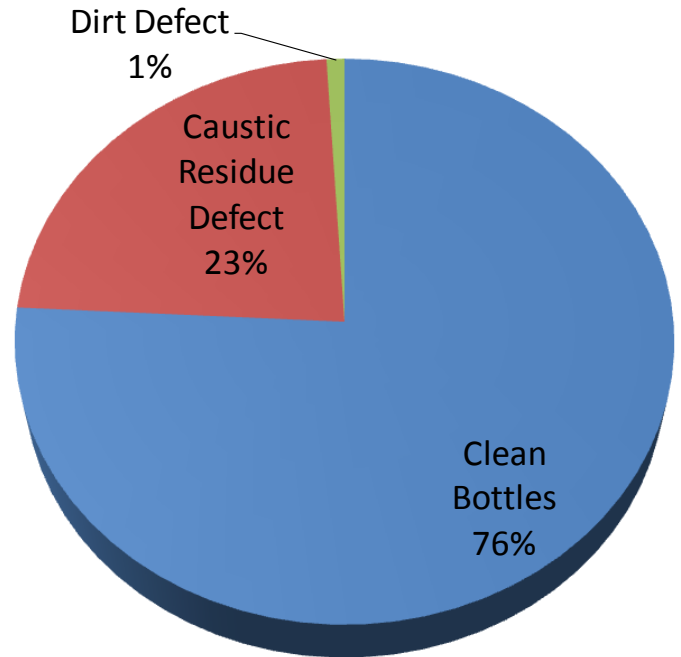
Month	Batch						
	1	2	3	4	5	6	7
JAN.	1261	1242	1260	1263	1257	1247	1265
FEB.	1240	1219	1236	1238	1233	1234	1210
MAR.	1198	1253	1200	1203	1228	1258	1205
APR.	1199	1219	1235	1238	1233	1234	1220
MAY	1249	1207	1223	1225	1221	1221	1228
JUN.	1200	1184	1170	1172	1258	1189	1174
JUL.	1229	1191	1197	1204	1195	1246	1242
AUG.	1234	1206	1202	1205	1200	1201	1207
SEPT.	1230	1213	1231	1234	1229	1218	1236
OCT.	1248	1204	1215	1218	1213	1204	1220
NOV.	1217	1201	1218	1221	1216	1206	1224
DEC.	1228	1211	1229	1232	1227	1216	1234

The results collected were compared with the predicted value at a pressure of 0.62 N/mm<sup>2</sup>, a temperature of 70°C and a caustic concentration of 2.0 and 2.5%. The predicted value obtained is 1230±32. Comparing the two results, it was found out that 88% of data collected falls within the predicted value. Hence, the model obtained for the caustic residue defect can be used to predict the quantity of defective bottles in a bottle washing company.

**Table 4 Total monthly defected bottles for a Year**

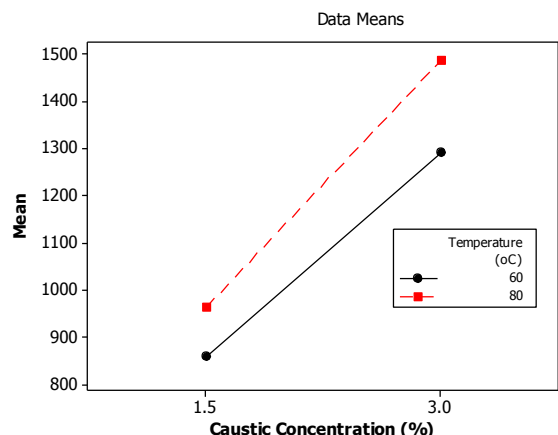
Month	CLEAN BOTTLES	CAUSTIC RESIDUE DEFECT	DIRT DEFECT	TOTAL BOTTLES WASHED
JAN.	590000	202000	8000	800000
FEB.	580800	172400	7600	760800
MAR.	720000	231000	9000	960000
APR.	710000	211400	8600	930000
MAY	582800	171000	7000	760800
JUN.	575200	158400	6600	740200
JUL.	540000	154000	6000	700000
AUG.	570200	163200	6800	740200
SEPT.	446400	148500	5500	600400
OCT.	522000	152200	5800	680000
NOV.	582000	190200	7800	780000
DEC.	610000	201800	8200	820000
AVE.	585783	179675	7242	772700

It has also been detected from the developed model that the caustic residue defect does not depend on the pressure; however, it depends on its interaction with both the temperature and the caustic concentration. To obtain an optimum value for the defective bottles, the pressure should be set at low and high level to determine which give the minimum number of defective bottles. It was found out that the pressure at its low level gives the minimum number defective bottles. However, the data obtained also shows that, for every batch of washed bottles, an average of 23% are rejected based on caustic residue defect while 1% is rejected based on the dirt defect as illustrated in Fig. 9.



**Fig. 9: Percentage distribution of washed bottles**

It is also known from Minitab that, the interaction plots for the significant main effects is used to select parameters values to obtain an optimize value for the responses. The interaction plot obtained for the two main effects, temperature and the caustic concentration is illustrated in Fig 10.



**Fig. 10: Interaction Plots for Caustic Residue Defect**

The significant interaction between temperature and caustic concentration shows up as two lines with sharply differing slopes. The number of defective bottles for 1.5% caustic concentration is less than the number of defective bottles for 3.0% caustic concentration at both temperatures of 60oC and 80oC. However, it seems that the difference in the number of defective bottles between runs using 1.5% caustic concentration and runs using 3.0 % caustic concentration at temperatures of 60oC is much lesser than the difference in the number of defective bottles between runs using 1.5% caustic concentration and runs using 3.0 % caustic concentration at temperatures of 80oC. Hence, in order to get the least number of defective bottles for a bottle washing plant, it is suggested that the temperature should be set at 60oC, while the caustic concentration should be set at 1.5% to give a minimum value of  $811 \pm 32$  defective bottles. This conformed to the results obtained during the factorial design experiment. That is, at 0.62 N/mm<sup>2</sup> pressure, with a temperature of 60oC and 1.5% caustic concentration, the number of defective bottles were between 770 and 830 which indicates a reduction in the number defective bottles by 8%.

#### 4 CONCLUSIONS

This research has investigated pressure, temperature and caustic concentration on the performance of an automated bottle washer in order to: 1. determine the effects of pressure, temperature and caustic concentration on cleaning responses during washing operations, 2. estimate and analyze the performance of the washer and consequently build a prediction model for it. 3. come out with a clear recommendation regarding the most favorable settings that will result in minimum number of defective washed bottles. It is concluded that, the significant effects were obtained for each of the washing defect responses and the caustic residue defect response has the highest significant effects due to the washing parameters. The predicted model obtained for the caustic residue defect is

$$N_c = \left[ \begin{array}{l} 1150.6 + 74.3t + 239.4c - 68.1pc \\ + 23.1tc - 19.4ptc \pm 32 \end{array} \right] \quad \text{which}$$

indicates that 88% of company's results fall within the predicted value. Finally, the most favorable settings of the washing parameters are setting the pressure at 0.62 N/mm<sup>2</sup>, temperature at 60oC and 1.5% caustic concentration which gives a minimum value of  $811 \pm 32$  defective bottles.

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