Robustness Of I-PD, PD-PI And PI-PD Controllers Used With Second-Order Processes

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Abstract: Robustness is one of the requirements used in controllers and compensators design. The designs presented in the previous papers did not consider the robustness of the controller or compensator. Therefore, the objective of this paper is to investigate the robustness of I-PD, PD-PI and PI-PD controllers used to control second-order processes against uncertainty in the process parameters. A variation of ± 20% in process parameters is considered through simulation to study its effect on the system performance parameters using the tuned controllers. With I-PD controller controlling an underdamped second order process, the variation in process natural frequency and damping ratio has almost no effect on the maximum percentage overshoot, maximum percentage undershoot, settling time and the phase margin of the control system. The variation of the process natural frequency produced a maximum change of 3.67% in the system gain margin. The damping ratio change does not affect the gain margin of the system. With PD-PD controller controlling the underdamped second order process, the variation in process natural frequency and damping ratio has almost no effect on the maximum percentage overshoot, maximum percentage undershoot and gain margin of the control system. The variation of the process natural frequency produced a maximum change of 5.2% in the system settling time and 3.67% in the phase margin. The variation of the process damping ratio produced a maximum change of 0.05% in the system settling time and has no effect on the system phase margin.

Index Terms: Second order processes; I-PD, PD-PI and PI-PD controllers; uncertainty in process parameters; controller robustness; control system performance.

1 INTRODUCTION
Processes are subject to uncertainty in their parameters during operation. Therefore, it is worth to investigate the effectiveness of the used controllers or compensators with such certainty. Hu, Chang, Yeh and Kwatny (2000) used the H∞ approximate I/O linearization formulation and μ-synthesis to design a nonlinear controller for an aircraft longitudinal flight control problem and address tracking, regulation and robustness issues [1]. Gong and Yao (2001) generalized a neural network adaptive robust control design to synthesize performance oriented control laws for a class of nonlinear systems in semi-strict feedback forms through the incorporation of backstepping design techniques [2]. Lee and Na (2002) designed a robust controller for a nuclear power control system. They used the Kharitonov and edge theorem in the determination of the controller which was simpler than that obtained by the H∞ [3]. Arvanitis, Syrkos, Stellas and Sigrimis (2003) analyzed PDF controllers designed and tuned to control integrator plus dead time processes in terms of robustness. They performed the robustness analysis in terms of structured parametric uncertainty description [4]. Lhommeau, Hardouin, Cottenceau and Laulin (2004) discussed the existence and the computation of a robust controller set for uncertain systems described by parametric models with unknown parameters assumed to vary between known bounds [5].

Dechanupaprittha, Hongesombut, Watanabe, Mitani and Ngammroo (2005) proposed the design of robust superconducting magnetic energy storage controller in a multimachine power system by using hybrid tabu search and evolutionary programming. The objective function of the optimization problem considered the disturbance attenuation performance and robust stability index [6]. Chin, Lau, Low and Seet (2006) proposed a robust PID controller based on actuated dynamics and an unactuated dynamics shown to be global ...... bounded by the Sordalen lemma giving the necessary sufficient condition to guarantee the global asymptotic stability of the URV system [7]. Vagja and Tzes (2007) designed a robust PID controller coupled into a Feedforward compensator for set point regulation of an electrostatic micromechanical actuator. They tuned the PID controller using the LMI-approach for robustness against the switching nature of the linearized system dynamics [8]. Fiorentini and Bolender (2008) described the design of a nonlinear robust/adaptive controller for an air-breathing hypersonic vehicle model. They adapted a nonlinear sequential loop-closure approach to design a dynamic state-feedback control for stable tracking of velocity and altitude reference trajectories [9]. Labibi, Marquez and Chen (2009) presented a scheme to design decentralized robust PI controllers for uncertain LTI multi-variable systems. They obtained sufficient conditions for closed-loop stability of multi-variable systems and robust performance of the overall system [10]. Matusu, Vanekova, Porkop and Bakosova (2010) presented a possible approach to design simple PI robust controllers and demonstrate their applicability during control of a laboratory model with uncertain parameters through PLC [11]. Kada and Ghazzawi (2011) described the structures and design of a robust PID controller for higher order systems. They presented a design scheme combining deadbeat response, robust control and model reduction techniques to enhance the performance and robustness of the PID controller [12]. Surjan (2012) applied the genetic algorithm for the design of the structure specified optimal robust controllers. The parameters of the chosen controller were obtained by solving
2. PROCESS AND I-PD CONTROLLER
The process considered in this analysis has the transfer function, \(G_p(s)\):

\[
G_p(s) = \frac{\omega_{np}^2}{(s^2 + 2\zeta_p \omega_{np}s + \omega_{np}^2)}
\]  

(1)

Where:
- \(\omega_{np}\) = process natural frequency  = 10 rad/s.
- \(\zeta_p\) = process damping ratio  = 0.05

I-PD Controller Tuning:
The I-PD-controller was tuned by the author to control this second order process [17]. The tuning parameters and the system performance measures are:

- \(K_{pc} = 1.7523\)
- \(K_i = 5.3314\)
- \(K_d = 0.1113\)
- \(OS_{max} = US_{max} = \text{zero}\)
- \(T_s = 1.46\) s
- \(GM = 11.1\) dB
- \(PM = 80.8\) degrees

Process Uncertainty:
Due to the change in the operating conditions during operation, the process is subjected to parametric changes. It is assumed that this change be be as large as ± 20% of the assigned process parameters.

I-PD Controller Robustness
The control system is robust when it has acceptable changes in its performance due to model to model changes or inaccuracy [18]. On the other hand Lee and Na add the stability requirement to the robustness definition besides the plants having uncertainty [3]. Toscano adds that the controller has to be able to stabilize the control system for all the operating conditions [19]. In this work, the robustness of the controller and hence of the whole control system is assessed as follows:
- A nominal process parameters are identified.
- The controller is tuned for those process parameters.
- A variation of the process parameters is assumed within a certain range.
- Using the same controller parameters, the step response of the system using the new process parameters is drawn and the control system performance is evaluated through the maximum percentage overshoot, maximum percentage undershoot and settling time.
- The variation in process parameters is increased and the procedure is repeated.

Application of the above procedure results in the following:
- The maximum percentage overshoot and undershoot did not change from its zero level.
- The settling time almost did not change from its nominal level.
- The phase margin almost did not change from its nominal level.
- The change in the gain margin increases as the change in the process natural frequency increases.
- The change in the gain margin and phase margin increases as the change in the process damping ratio increases.

Fig.1 shows the variation in the settling time against the variation in the process parameters.

Fig.1 Effect of process parameters change of system settling time.

- Fig.2 shows the variation in the gain margin against the variation in the process parameters.

Fig.2 Effect of process parameters change of system gain margin.
Fig. 3 shows the variation in the phase margin against the variation in the process parameters.

3. PD-PI CONTROLLER

The author presented an approach to tune a PD-PI controller when used with a second order highly oscillating process [20]. The tuned controller parameters and the system performance measures are:

\[
\begin{align*}
K_{pc1} &= 33.2092 \\
K_i &= 34.9363 \\
K_d &= 43.119 \\
\text{Maximum percentage overshoot:} & \quad \text{zero} \\
\text{Maximum percentage undershoot:} & \quad \text{zero} \\
\text{Settling time:} & \quad \text{zero} \\
\text{Gain margin:} & \quad \text{infinity} \\
\text{Phase margin:} & \quad 90^\circ
\end{align*}
\]

The robustness investigation procedure is applied on the resulting control system for process variation in the range ±20% from the nominal values. The results are as follows:

- The maximum percentage undershoot, undershoot and gain margin do not change.
- The settling time increases as the change in the process natural frequency increases.
- The phase margin increases as the change in the process natural frequency increases.
- The damping ratio of the process almost has no effect on the settling time of the closed-loop control system.
- Fig. 4 shows the effect of the natural frequency change on the system settling time.

4. PI-PD CONTROLLER

The author presented a tuning approach for a PI-PD controller when used with a second order highly oscillating process [21]. The controller parameters and the system performance measures are:

\[
\begin{align*}
K_{pc2} &= 10 \\
K_{pc2} &= 1 \\
K_i &= 15 \\
K_d &= 0.9994 \\
\text{Gain margin:} & \quad \text{infinity} \\
\text{Phase margin:} & \quad 87.1^\circ
\end{align*}
\]

The robustness investigation procedure is applied on the resulting control system for process variation in the range ±20% from the nominal values. The results are as follows:

- The maximum percentage undershoot, undershoot and gain margin do not change.
- The settling time increases as the change in the process natural frequency increases.
- The phase margin increases as the change in the process natural frequency increases.
- The damping ratio of the process almost has no effect on the settling time of the closed-loop control system.
- Fig. 5 shows the effect of the natural frequency change on the system settling time.

Fig. 3 Effect of process parameters change of system phase margin.

Fig. 4 Effect of process parameters change of system settling time.

Fig. 5 Effect of process parameters change on system settling time.
- Fig.6 shows the effect of the natural frequency change on the system settling time.

**Fig.5** Effect of process parameters change on system settling time.

5. **CONCLUSIONS**

- Variation in second-order process parameters within ± 20% was considered.
- Tuned I-PD, PD-PI and PI-PD controllers are robust since they controlled the second order process for set-point change maintaining acceptable performance and stable control system for the range of parameters change.
- With I-PD controller, a change of 20% in process natural frequency resulted in an increase in the gain margin by 33% of the nominal value.
- ith PD-PI controller, a change of -20% in process natural frequency resulted in an increase in the settling time by 56.5% of the nominal value (<0.4 s settling time).
- With PI-PD controller, a change of 20% in process natural frequency resulted in a change of the system settling time of 5.2% of the nominal value.
- With PI-PD controller, a change of 20% in process natural frequency resulted in a change of the system settling time of 5.2% of the nominal value.
- The closed-loop control system is more sensitive to the variations in the process natural frequency than the variations in its damping ratio.

**REFERENCES**


BIOGRAPHY

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