

Evaluation Of Friction Losses In Pipes And Fittings Of Process Engineering Plants

F. W. Ntengwe, M. Chikwa, L. K. Witika

ABSTRACT: The impact of flow rate (Q) on the head loss (h_L) in pipes and fittings was studied on different sizes or diameters (D) of pipes fitted with gate valve, 45 and 90° bends using water as process fluid. Diameters of pipes ranged from 25 to 100 mm while the process fluid flow rates ranged from 0 to 50 m³/h. The Darcy-Weisbach, Hazen-Williams and Poisselli's methods were used to evaluate friction losses. The results showing increasing D of the pipe and decreasing the h_L in the pipe line, gate valve, 45° and 90° elbow, entry and exits to pipes are presented. The results of increasing Q with increasing exponential values of h_L regardless of D of pipe also presented. Therefore, a number of choices can be made between transporting process fluids using small D pipes ($50 > D > 25$ mm) and Reynolds (Re) numbers in the laminar region and large D pipes ($100 > D > 50$ mm) using Re numbers in turbulent regions.

KEYWORDS: Friction-loss, Fittings, Flow-rates, Impact, Process, Engineering, Plants, Fluids-transport

1. INTRODUCTION

Fluid flow in pipes is continuously impacted by the resistance to flow offered by the roughness of pipe at the walls based on the law of similarity [1], [2]. Smooth pipes offer little or negligible resistance to flow while rougher surfaces offer increasing resistance depending on the degree of roughness. Such resistance affects flow rate (Q) and velocity distribution of process fluid in the pipe [3]. The resistance increases for Q values in transition and turbulent regions. Studies elsewhere have shown that high velocities produce high resistances to flow in pipes and hence h_L values for particular type of surface roughness [4]. Darcy-Weisbach, Hazen-Williams, Moody and Fanning showed that for any flow of fluid in a pipe exhibiting some form of roughness; head losses (h_L) due to friction were produced [4]. Head losses due to gate valve, 45° and 90° elbows have also been dealt with by other authors [5]. Other losses include entry and exit losses. Such losses are commonly added to the pipe line in question in order to determine the equivalent length (L_e) of a pipe. In this paper h_L were evaluated in five poly-vinyl-chloride (PVC) pipes with different diameters (D) but same length (L) using water as process fluid at 25°C. A pumping source was used to produce different Q s in each pipe and elevation effects were neglected meaning that the pipe line was horizontal. Each loss due to pipe line, gate valve, 45 and 90° elbows and entry and exit losses were correlated to the flow rate and D of pipe in order to establish the relationships which could be useful to design engineers and plant operators during design and plant operation. Decisions could be made early about the size and type of roughness of the pipe and appropriate Q of the fluid during design or plant operation based on the correlations presented in this paper. The challenge with the delivery of fluids is either non delivery or insufficient delivery to the desired destination. Oftentimes, it is either the insufficient pumping

due to faulty pumps or high friction losses in the delivery system. This may be caused by pipe blockage or increased roughness which may contribute to high friction losses or the system has a high positive delivery head or insufficient net positive suction head at the pump. The purpose of the study was therefore to establish levels of friction losses in different sizes of pipes fitted with fittings; gate valve, 45 and 90° elbows and exit and entry structures in order to determine the head losses that contribute to the increase of equivalent lengths of pipes that increase the delivery heads and hence problems of fluids delivery to desired destinations.

2. CONCEPTS OF FRICTION LOSS

Friction and h_L in pipes, bends and valves present a challenge, as stated earlier, in the transportation of fluids in pipe-lines of process industry. While the transportation of fluids is dependent on the quantity of fluid and capacity of the pump and pipe line, properties of the fluids to be transported are important as these determine whether the fluid will flow with little or high resistance. The choice of any pipe for a particular process fluid is based on the correlation between the flow rate and cross-sectional area of pipe as given in (A1). The D of the pipe can then be determined from (A2) for a particular maximum discharge.

$$A = \frac{\pi d^2}{4} = \frac{Q}{v} \quad (A1)$$

$$D = C' \left\{ \frac{Q}{v} \right\}^{0.5} \quad (A2)$$

The h_L in pipes can be estimated using Darcy-Weisbach or Hazen-Williams formulas [6, 7] as given in (A3) and (A4) for given Q , L and D of pipe.

$$h_L = C'' L \frac{Q^2}{D^5} \quad (A3)$$

$$h_L = \frac{10.77L}{D^{4.865}} \left\{ \frac{Q}{C_{hw}} \right\}^{1.852} \quad (A4)$$

These equations demonstrate that the h_L is directly proportional to the square of Q and inversely proportional to the fifth power of diameter. If the D of pipe is changed, the h_L can be evaluated from (A5) provided that the pipe surface

- F. W. Ntengwe*1, M. Chikwa*1 and L. K. Witika*2
- *1 Copperbelt University, School of Mines and Mineral Sciences, Box 21692, Kitwe, Zambia
- *2 University of Zambia, School of Mines, Box 32379, Lusaka, Zambia

roughness and that viscosity of the fluid is the same. The assumption is that the h_L for the first pipe must be known.

$$h_{L2} = h_{L1} \left\{ \frac{D_1}{D_2} \right\}^5 \quad (A5)$$

In this study the h_L values due to each fitting and pipe line were calculated separately and then correlated to the Q and D of pipe as required. The head losses were then added to determine the equivalent lengths of pipes in order to validate Darcy-Weisbach method and others. If a pump is able to produce the power that can overcome the losses then the delivery problem is solved. However, the deliver is at the expense of friction loss which takes up some energy power resulting in reduced pumping efficiency.

3.0 EXPERIMENTAL

3.1 Process Evaluation

The process evaluation was carried out using water at 25°C with ρ of 998.2 kg/m³ and μ of 1.003x10⁻³ Ns/m². Four sizes of pipes of different internal D s were used as shown in Figure A1 (a), (b), (c) and (d) and Table A1 with D s from 25 to 100 mm. The fittings; Gate valve, 45 and 90° elbows were placed in the pipe line in order to determine their contribution to h_L variation. A single value of Q was allowed into the pipe one at a time and the characteristics of Q ; v , ε/D and Re were determined in order to establish the values of f which were used to evaluate K^1 , h_L and h_{Lf} using Darcy-Weisbach formula [6] as given in (A6) and (A7) respectively.

$$h_L = K^1 v^2 \quad (A6)$$

$$h_{Lf} = K^{11} v^2 \quad (A7)$$

Moody method [6], [4] for determining the friction factor f from relative roughness and Re was more convenient than models developed elsewhere [8], [9], [10], [11]. Friction factor in curved pipes have also been covered elsewhere [12]. Equation (A8) gave the total h_L and Le of pipes. The values of characteristics are shown in Table A1 for pipes P1 to P4 respectively.

$$h_{Ll} = h_{LjG} + h_{Lj45} + h_{Lj90} + h_{LjE} + h_{LjEx} \quad (A8)$$

3.2 Correlations

The values of Q were used to determine the correlations with h_L , entry and exit losses, losses in gate valve, 45 and 90° elbows and the correlation between h_L and D of pipe [6]. This was done in order to establish whether there would be deviations from Darcy-Weisbach and Hazen-Williams and Hagen-Poiselli theories [4], [7].

3.3 Data Analysis

Exploration methods were used to analyze data in order to establish correlations [13], [14], [15], [16]. Excel and SPSS 11.5 were used as tools to analyze, evaluate and interpret the data in order to draw meaningful conclusions.

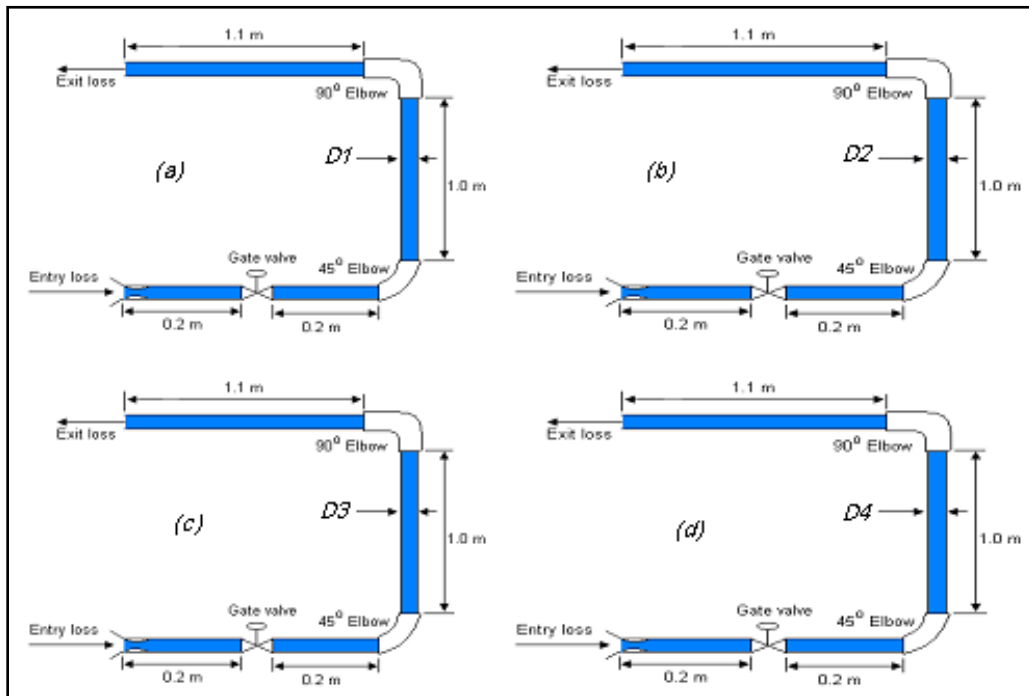


Fig. A1: The top view of the pipe line showing the fittings, entry and exit of the pipelines

Table A1: Characteristics of flow in pipes of different sizes

		P1 = 25 mm pipe				P2 = 50 mm pipe			
Q	v	Re	ϵ/D	f	v	Re	ϵ/D	f	
(m ³ /h)	m/s	-	m ⁻¹	-	m/s	-	m ⁻¹	-	
10	5.50	1.4x10 ⁵	5.9x10 ⁻⁵	0.017	1.37	1.0x10 ⁵	2.95x10 ⁻⁵	0.020	
20	10.9	2.8x10 ⁵	5.9x10 ⁻⁵	0.015	2.74	1.4x10 ⁵	2.95x10 ⁻⁵	0.017	
30	16.4	4.1x10 ⁵	5.9x10 ⁻⁵	0.015	4.12	2.1x10 ⁵	2.95x10 ⁻⁵	0.016	
40	21.9	5.5x10 ⁵	5.9x10 ⁻⁵	0.014	5.48	2.8x10 ⁵	2.95x10 ⁻⁵	0.014	
50	27.4	6.9x10 ⁵	5.9x10 ⁻⁵	0.014	6.85	3.5x10 ⁵	2.95x10 ⁻⁵	0.015	
		P3 = 75 mm pipe				P4 =100 mm pipe			
Q	v	Re	ϵ/D	f	v	Re	ϵ/D	f	
(m ³ /h)	m/s	-	m ⁻¹	-	m/s	-	m ⁻¹	-	
10	0.61	4.6x10 ⁴	1.97x10 ⁻⁵	0.015	0.34	3.40x10 ⁴	1.47x10 ⁻⁵	0.016	
20	1.22	9.2x10 ⁴	1.97x10 ⁻⁵	0.016	0.69	6.97x10 ⁴	1.47x10 ⁻⁵	0.017	
30	1.83	1.4x10 ⁵	1.97x10 ⁻⁵	0.017	1.03	1.04x10 ⁵	1.47x10 ⁻⁵	0.018	
40	2.44	1.8x10 ⁵	1.97x10 ⁻⁵	0.018	1.37	1.38x10 ⁵	1.47x10 ⁻⁵	0.019	
50	3.05	2.3x10 ⁵	1.97x10 ⁻⁵	0.021	1.71	1.73x10 ⁵	1.47x10 ⁻⁵	0.022	

4.0 RESULTS

4.1 Impact of Flow Rate on Friction Losses in Fittings

The placement of a bend, gate valve, 45 and 90° elbow in a pipe line introduces extra friction in addition to normal friction due to the walls of the pipe. A gate valve provides friction to the flow of the fluid in a pipe. The results in Figure A2 show that the loss in Q due to the gate valve was the lowest (5 m) and that due to exit loss was the highest (25 m) at Q of 40 m³/h. Similarly, the entry losses were lower (12.3 m) than exit losses which were higher (24.5 m) at the same flow rate. The h_L due to 45° angle was higher (20.6 m) than that due to 90° angle which was lower (18.4 m) at the same flow rate. However, the observed-general trend was that the h_L increased as the values of Q increased regardless of type of bend and gate valve placed in the pipe line.

4.2 Impact of Flow Rate on Friction Loss in Gate Valve

The h_L due to gate valve increased from 0.3 - 7.3 m, 0.02 - 0.45 m, 0.001 - 0.09 m and 0.001 - 0.03 m for Q values of 10, 20, 30, 40 and 50 m³/h and D of pipes of 25, 50, 75 and 100 mm respectively (Figure A3). However, the gate valve losses were higher in the 25 mm pipe and lowest in the 100 mm pipe. This means that the gate valve h_L decreased as the pipe D increased regardless of the level of Q in the system. It is important to note also that gate valve h_L losses could not be eliminated in anyway whether the pipe was rough or smooth but could only be reduced by increasing the D of the pipe.

4.3 Impact of Flow Rate on Entry Losses

Like gate valve h_L , entry losses are fixed by the size of entry of a pipe. The smaller the D of entry of the pipe the higher the entry losses and the opposite was true. The results in Figure A4 show that the entry losses were highest (0.7 m) for the 25 mm pipe and lowest (0.027 m) for the 100 mm pipe at a Q value of 30 m³/h. Similarly, the trends at other Q values indicated that the smaller the diameter of the pipe was the higher the entry loss. But it was also observed that when Q increased from 0 - 50 m³/h the entry losses increased from 0 - 1.92 m for the 25 mm pipe, 0 - 1.2 m for the 50 mm pipe, 0 - 0.26 m for the 75 mm pipe and 0 - 0.075 m for the 100 mm pipe regardless of the size of pipe line. Therefore, entry losses could only be decreased when the diameter of the pipe increased.

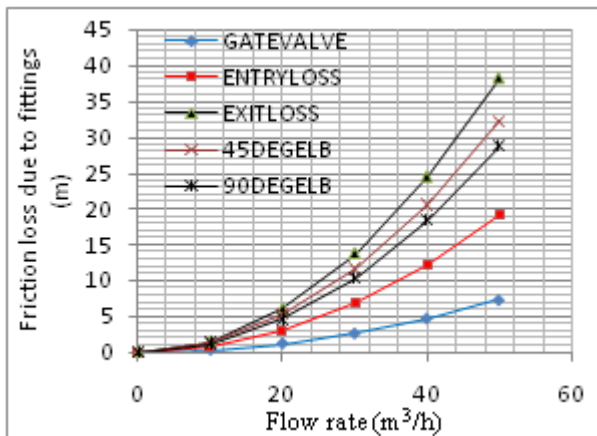


Fig. A2: Impact of flow on the friction loss in fittings

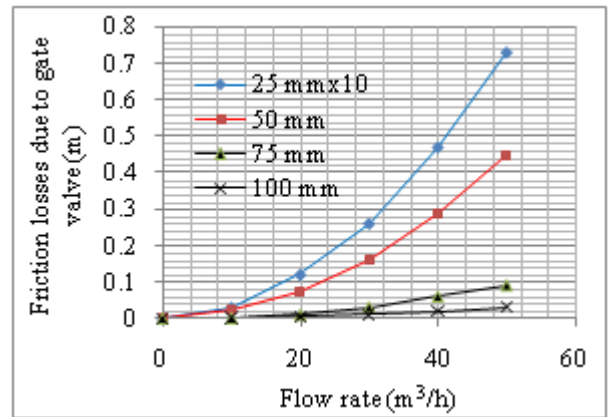


Fig. A3: Impact of flow on friction loss in gate valve

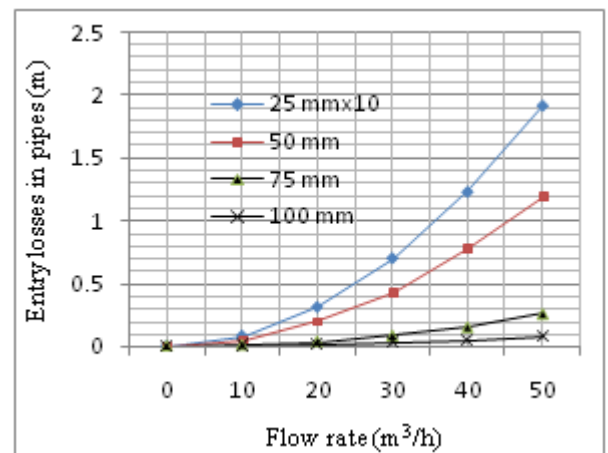


Fig. A4: Impact of flow rate on friction loss in entry of pipe

4.4 Impact of Flow Rate on Friction Loss in 45° Elbow

A 45° elbow placed in the pipe line introduced a loss in fluid flow. The loss depended on the flow velocity, diameter of the pipe and flow rate. When the flow increased from 30 - 50 m³/h, the friction loss in each pipe increased from 1.2 - 3.2 m for the 25 mm pipe, 0.73 - 2.1 m for the 50 mm pipe, 0.2 - 0.43 m for the 75 mm pipe and 0.02 - 0.05 m for the 100 mm pipe (Figure A5). It was also observed that values for the 25 mm pipe were higher than those for the 100 mm pipe meaning that the increase in diameter of the pipe decreased the losses in the 45° elbow. Therefore increasing the D of the pipe reduced the h_L in the 45° elbow but did not remove it completely.

4.5 Impact of Flow Rate on Friction Loss in 90° Elbow

A 90° elbow placed in the pipe line introduced, also, a loss in fluid flow. The loss depended on the flow velocity, D of the pipe and Q similar to that in the 45° elbow. When Q increased from 0 - 50 m³/h, the h_L in each pipe increased from 0 - 2.87 m for the 25 mm pipe, 0 - 1.8 m for the 50 mm pipe, 0 - 0.40 m for the 75 mm pipe and 0 - 0.11 m for the 100 mm pipe (Figure A6). It was also observed that values for the 25 mm pipe were higher than those for the 100 mm pipe meaning that the increase in diameter of the pipe decreased the losses in the 90° elbow. Therefore increasing D of pipe reduced the h_L in the 90° elbow, a concept that

agrees with Darcy-Weisbach, Hazen-Williams and Hagen-Poiseuille principles [4], [7].

4.6 Impact of Flow Rate on Exit Losses

Like in bends and valves, exit losses increased as Q increased (Figure A7). An increase from 0 - 50 m³/h produced an increase in exit losses from 0 - 3.8 m for 25 mm pipe, 0 - 2.4 m for the 50 mm pipe, 0 - 0.5 m for the 75 mm pipe and 0 - 0.15 m for the 100 mm pipe. It could also be observed that the 25 mm pipe produced highest exit losses than those of 50 mm pipe which were higher than those of 75 mm pipe which were also higher than those produced by the 100 mm pipe. As an example, exit loss for the 25 mm pipe at 40 m³/h was 2.5 m which was higher than that of 50 mm pipe (1.5 m) which was higher than that of 75 mm pipe (0.3 m) which was higher than that of 100 mm pipe (0.1 m). Therefore the exit losses were found to depend directly on the Q and indirectly on the size of the pipe. This agrees well with Darcy-Weisbach, Hazen-Williams and Hagen-Poiseuille principles [4].

4.7 Impact of Flow on Total Head Losses in Pipes

According to Darcy, the head loss is directly proportional to the square of velocity and indirectly to the diameter of pipe. The Q values and cross area of pipe were linked to the velocity of fluid in the pipe. An increase in the Q produced an increase in head loss. The results in Figure A8 showed that when Q increased from 40 m³/h, the h_L increased from 0 - 33 m for the

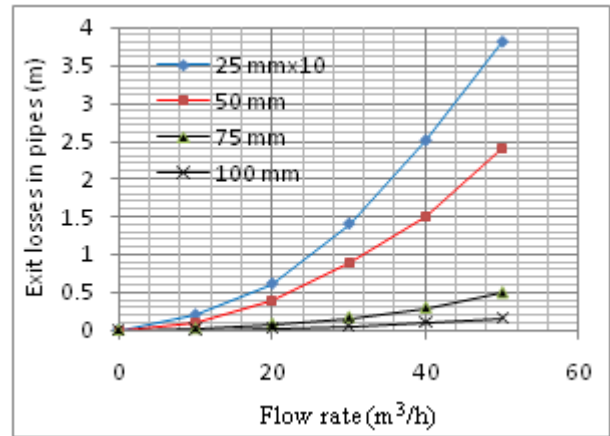


Fig. A7: Impact of flow rate on exit losses in pipes

25 mm pipe, 0 - 1.12 m for the 50 mm pipe, 0 - 0.2 m for the 75 mm pipe and 0 - 0.04 m for the 100 mm pipe. It could also be observed that the 25 mm pipe value was higher (33 m) than that of the 50 mm pipe (1.12 m) which was higher than the one for the 75 mm pipe (0.2 m) which was higher than that for the 100 mm pipe (0.04 m) at 40m³/h flow rate. Therefore a choice of pipe roughness, Q and diameter of pipe was important in attaining the required operating conditions for the chosen pipe and process fluids.

4.8 Impact of diameter of pipe on equivalent length

The equivalent lengths of pipes hence total h_L were found to be inversely proportional to the diameter of pipes. This statement agrees well with Darcy's formula or method [4]. As D increased the equivalent length (L_e) decreased correspondingly regardless of the level of flow rate. In this regard, the 25 mm pipe produced highest L_e and highest h_L followed by the 50 mm pipe. The 75 and 100 mm pipes produced least L_e and h_L for all the levels of flow rates (Q10 - Q50 m³/h) (Figure A9). The total h_L decreased as the D of the pipe increased. As the D of pipe increased from 25 - 125 mm the h_L decreased from 177 - 0 m for the 25 mm pipe, 30 - 0 m for the 50 mm pipe, 1.8 - 0 m for the 75 mm pipe and 0.22 - 0 m for the 100 mm pipe. It is therefore important to understand that the transportation of fluids could be carried out with minimal losses when pipe diameters greater than 75 mm were used.

5. DISCUSSIONS

Every organization requires that the transportation of process fluids be efficient, i.e., that the transporting system must not provide excessive hindrance to the flow of fluid in the pipe. Any fittings, bends and valves placed in the fluids-transport system should also offer little resistance to flow of material in order to maintain high efficiency in transportation and minimize losses due to friction whether the fluid is a single, double or triple phase system. For a single phase system, such as water in this study, it was quite clear that D of pipe, Q of fluid, type of pipe, bend or valve and the type of entry and exit played a key role not only in determining the h_L but also contributed significantly to the L_e of pipe [17], [18]. It was observed further that the flow rate cannot be left out during the design of pipe system because it is linked to the area and hence the diameter of the pipe.

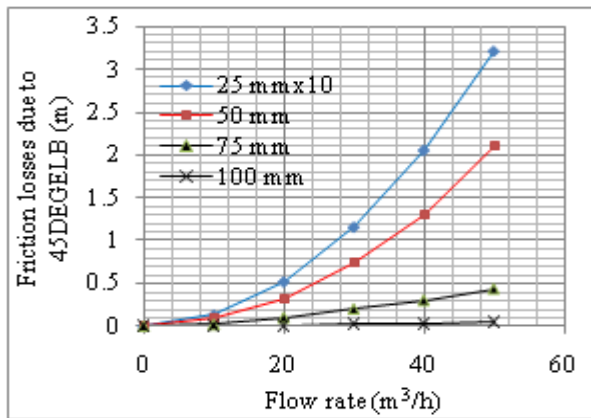


Fig. A5: Impact of flow on friction loss in 45° elbow

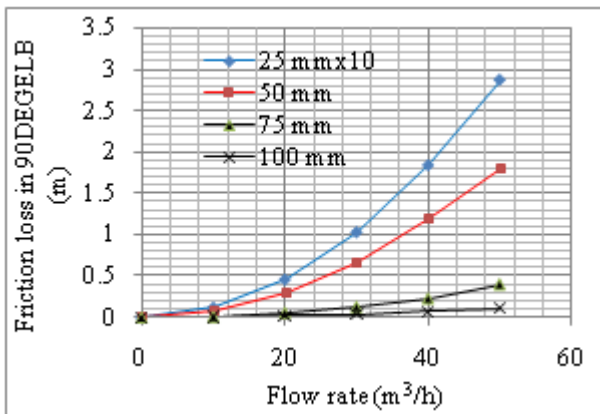


Fig. A6: Impact of flow on friction loss in 90° elbow

It was also observed that different levels of flow of fluid produced different levels of h_L in pipes, gate valve, 45 and 90° elbows. The defect law applies only to systems that are not cylindrical [19]. When Q was doubled the h_L quadrupled [17], [20]. This was confirmed elsewhere [18]. It means therefore that appropriate Q is required to be determined early in the design process so as to avoid high h_L in pipes. The results in this study provide a solution to the choice of the level of Q for particular design of pipe.

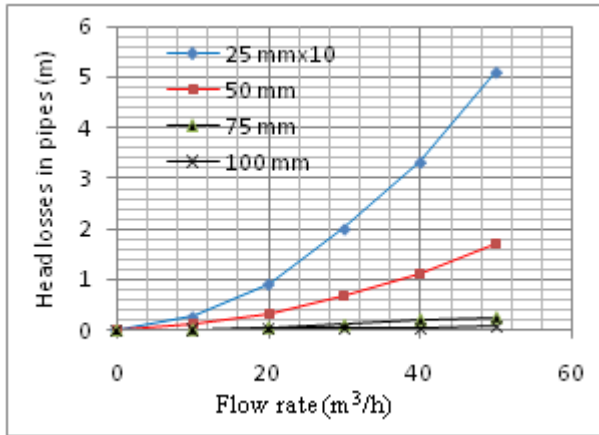


Figure A8: Impact of flow rate on head loss in pipes

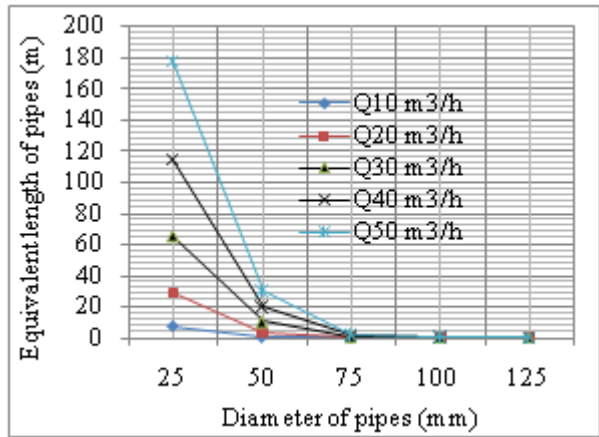


Figure A9: Impact of diameter of pipe on head loss in pipe lines

The other key characteristic is the D of the pipe. It has been shown in this study that when the D of the pipe increased the h_L decreased significantly. The pipes that were greater than 75 mm D produced less h_L than pipes less than 75 mm in D for high Reynolds numbers or turbulent flows. Small D pipes ($75 > D > 0$) would require Q values with low Reynolds numbers, i.e., laminar region. White (1929) [21] reported laminar conditions for Re values up to 8000 meaning that the introduction of a curved pipe or bend slows lowers the velocity of process fluid. Therefore, a choice can be made between one D of pipe and the other in order to obtain the optimum D that would provide minimal h_L in the pipe line, bends and valves.

The type of bend or elbow to be placed in the pipe line in order to develop low h_L could depend on the data produced in this paper. If a comparison between the 45° elbow and 90° elbow is made, it can be observed that the 90° elbow produced high values of h_L than the 45° elbow and therefore the later would be the best choice [20]. Sinnott (2005) [18] showed high values of velocity heads and hence equivalent lengths for the 90° elbow and low values for the 45° elbow thus confirming that the 45° elbow would be the best choice. However, what would dictate is the type of change of direction of pipe than the choice of elbow required in most practical situations. It would therefore be important to make the right choice of elbow for a particular pipe line or trade off with a larger D pipe in order to lower the losses. Entry and exit losses cannot be avoided because there is always an entry and exit point for any process fluid transported through pipe lines. In this study, entry losses were lower than exit losses which agreed with the trend given by Sinnott (2005) [18]. It was also observed that entry and exit losses in pipes were high for small D pipes ($25 < D < 50$ mm) and less for large D pipes ($50 < D < 100$ mm). If one would choose to use small D pipes, one would require operating the transportation system with Re numbers in the laminar region in order to lower the h_L due to entry and exit of pipes. It is also important to note that there are several types of entry geometry to pipes such as inward projecting, square-edged, chamfered and rounded inlets with resistance coefficients (K) of 1, 0.5, 0.25 and 0.04 respectively. This means that the inward projecting inlet would produce higher levels of h_L than the square-edged inlet that was used in this study. The K for the exit of pipe was unity since the pipes released water to a large-space volume.

6. CONCLUSION

The h_L decreased with the increase in D of pipes but increased with the increase in velocity or Q regardless of size of pipe. However, exit and entry losses increased with the increase in Q and increased with the decrease in D of pipe. The 25 mm pipe produced the highest level of h_L than other D_s of pipes. The fittings; gate valve and elbows were found to contribute to the h_L dependent on D of pipe and the Q values. Therefore, engineers and operators can make choices between D of pipe or fitting and Q in order to balance the characteristics for optimum operating conditions.

NOMENCLATURE

- A Area of pipe, $A = \pi d^2 / 4$, m^2
- C^1 Constant, $C^1 = (4/\pi)^{1/2}$
- C^{11} Constant, $C^{11} = 8f / \pi^2 g$
- C_{hw} Hazen-Williams coefficient
- D Diameter of pipe, m
- ϵ/d Pipe roughness, m^{-1}
- ϵ Absolute roughness, $\epsilon = 1.56 \times 10^{-6}$
- f Friction factor
- g Acceleration due to gravity, m/s^2
- h_L Head loss, m
- h_{Lf} Head loss due to fittings, m
- k Constant for type of fitting
- K^1 Constant, $K^1 = fL/2dg$, $s^2 m^{-1}$
- K^{11} Constant, $K^{11} = k/2g$, $s^2 m^{-1}$
- L Length of pipe, m
- Q Fluid flow rate, m^3/h
- Re Reynolds number, $Re = vdp/\mu$

v Velocity of fluid, $v=Q/A$, m/s
 ρ Density of fluid, kg/m³
 μ Fluid viscosity, mNs/m²

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