Selection Of Best Alternative For An Automotive Company By Intuitionistic Fuzzy TOPSIS Method

Zulqarnain M., Dayan F.

Abstract: Multi Criteria Decision Making (MCDM) uses different techniques to find a best alternative from multi-alternative and multi-criteria conditions. Classical TOPSIS uses crisp techniques for the linguistic assessments, but due to imprecise and fuzziness nature of the linguistic assessments, there must be some tools to deal this vague information. In this paper, we discuss about Intuitionistic Fuzzy TOPSIS (IF-TOPSIS) and use this method for the selection of best alternative for an automotive company.

Index Terms: MCDM, IF-TOPSIS, Intuitionistic fuzzy positive ideal solution (IFPIS), Intuitionistic fuzzy negative ideal solution (IFNIS), Intuitionistic Fuzzy Sets (IFS), interval valued intuitionistic fuzzy numbers (IVIFN). Multi Attribute Decision Making (MADM).

1 INTRODUCTION

Hwang and Yoon invented the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) in order to solve MCDM Problem with many alternatives [1]. From crisp to fuzzy data, Chen & Hwang remodeled TOPSIS [2]. Furthermore, Chen widened the TOPSIS for Group Decision Making in fuzzy atmosphere [3]. Awasthi et al. used fuzzy TOPSIS for the evaluation and selection of the best location planning for urban distribution centers [4]. Chu [5] and Yong [6] applied fuzzy TOPSIS for choosing plant location with minimum costs and maximum use of resources. The relative closeness (RC) coefficients were obtained as fuzzy numbers and after defuzzification, alternatives were ranked [7]. Wang and Elhang found that their method is much closer to the fuzzy weighted approach presented by Dong and Wong [8]. Mahmoodzadeh et al. incorporating fuzzy AHP and TOPSIS method gave a new procedure for the project selection problem. Improved fuzzy AHP was used to compute the weights of each criterion at first and then TOPSIS algorithm was engaged for ranking the projects to be selected [9]. Yong tao et al. utilized fuzzy TOPSIS for supporting contractors to choose suitable project for bidding with the help of MAGDM. Triangular fuzzy numbers (TFNs) were assigned to each linguistic variable for alternative ratings and criteria weights [10]. Zadeh proposed Type-2 fuzzy sets to encompass uncertainty about the membership function in fuzzy set theory (FST) [11]. Saremi and Montazer, to cope with the decision-making problems having data with large uncertainty, used a Type-2 fuzzy TOPSIS based method. A Study of TOPSIS in Classical, Fuzzy, Intuitionistic Fuzzy and Neutrosophic Environments Interval-valued fuzzy (IVF) set is a special type of Type-2 fuzzy set. Saremi and Montazer applied IVF sets having lower and upper triangular membership functions; they ranked the alternatives [12]. Chen and Tsao broadened IVF-TOPSIS to solve MADM problem [13]. Zulqarnain. M and Saeed. M., proposed and proved the credibility of interval valued fuzzy soft matrix (IVFSM) in decision making also discussed its different properties [14]. They also studied fuzzy soft matrix (FSM) and IVFSM and redefined the product of IVFSM, they used IVFSM and FSM in decision making problem with examples and compare the results and saw that FSM method is more appropriate for decision making [15]. They proposed a new decision making method on IVFSM named as "interval valued fuzzy soft max-min decision making method" with the help of interval valued fuzzy soft max-min decision making function and used this method for decision making [16]. Ashtiani et al. (2009) also established IVF-TOPSIS, in which alternative ratings and weights of the criteria were treated as linguistic variables represented by Triangular Interval Valued Fuzzy Numbers (TIVFN). They used it to solve MCDM problems [17]. By developing an Interval Valued Intuitionistic Fuzzy set (IVIFS) TOPSIS technique, a facility location problem was tackled by Verma et al. [18]. By using Intuitionistic Fuzzy sets, Hung and Chen introduced a novel Fuzzy TOPSIS method involving Entropy Weights for Decision Making [19]. Li and Nan used the technique proposed by Hung and Chen to enhance TOPSIS for solving MADM problems under IFS conditions [20]. Nurandiah and Lazim modelled IFS-TOPSIS to find a solution of a decision problem under IFS conditions [21]. In their work, weights of the decision makers (DMs) were computed; an IF-Decision Matrix is computed according to DMs judgements. The weights of the criteria and Weighted Intuitionistic Fuzzy Decision Matrix were computed. IFPIS and IFNIS were determined. Distances and RC of the alternatives from IFPIS and IFNIS are calculated for ranking the alternatives. Boran utilized the theory of IFS to supply chain management (SCM) [22]. F. Ye extended the idea of TOPSIS method with IVIFN for the selection of virtual enterprise partner [23]. In this paper, we discuss about Intuitionistic Fuzzy TOPSIS (IF-TOPSIS) and use this method for the selection of best alternative for an automotive company.

- ***M. Zulqarnain*** *received the M.S. degree in mathematics from University of Management and Technology, Lahore in 2016. E-mail: ranazulqarnain7777@gmail.com*
- ***Fazal Dayan*** *received the M.S. degree in mathematics from University of Management and Technology, Lahore in 2016. E-mail: fazaldayan1@gmail.com*

2 PRELIMINARIES

Definition 1 [22]
IFS A in a finite set X, can be written as
A = $\{x, \mu_a(x), v_A(x): x \epsilon X\}$
Where $\mu_a(x), v_A(x): X \rightarrow$ [0, 1] are membership and non-membership functions respectively, such that
$0 \leq \mu_a(x) + v_A(x) \leq 1$
Atanassov [24] introduced IFS, as an extension of classical FST. The purpose of IFS was to deal the vagueness.

Intuitionistic Fuzzy TOPSIS Algorithm [25]
Consider a set of "m" Alternatives A = $\{A_1, A_2, A_3,........ A_m\}$ a set of "n" Evaluation Criteria C = $\{C_1, C_2, C_3,........ C_m\}$a set of "$i$" Decision Makers DMs = {DM_1, DM_2,......... DM_i}

Step 1: Selection of Intuitionistic Fuzzy Ratings Scale for Linguistic Variables
Since importance of criteria, DMs and alternative ratings all are in the form of linguistic variables.

Step 2: Determine the weights of the DMs
The Intuitionistic Fuzzy Number (IFN) for rating the k^{th} DM is given below

$$D_k = (\mu_k, v_k, \pi_k)$$

The formula to find the weight of the k^{th} DM is represented by the following equation

$$\lambda_k = \frac{(\mu_k+\pi_k(\frac{\mu_k}{\mu_k+v_k}))}{\sum_{k=1}^{l}(\mu_k+\pi_k(\frac{\mu_k}{\mu_k+v_k}))} : \qquad \sum_{k=1}^{l} \lambda_k = 1$$

Step 3: Aggregated Intuitionistic Fuzzy Decision Matrix (AIFDM) according to the ratings of DMs
The Aggregated Intuitionistic Fuzzy Decision Matrix (AIFDM) is

$$D = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} \begin{bmatrix} \breve{r}_{11} & \breve{r}_{12} & \cdots & \breve{r}_{1n} \\ \breve{r}_{21} & \breve{r}_{22} & \ldots & \breve{r}_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \breve{r}_{m1} & \breve{r}_{m2} & \cdots & \breve{r}_{mn} \end{bmatrix} = [\acute{r}_{ij}]_{m\times n}$$

Where $\acute{r}_{ij}$ can be defined as
$\acute{r}_{ij} = (\mu_{A_i}(x_j), v_{A_i}(x_j), \pi_{A_i}(x_j))$ i = 1,2,3,……,m and j = 1,2,3,…….,n
So the AIFDM can be written as

$$D = \begin{bmatrix} (\mu_{A_1}(x_1), v_{A_1}(x_1), \pi_{A_1}(x_1)) & (\mu_{A_1}(x_2), v_{A_1}(x_2), \pi_{A_1}(x_2)) & \cdots & (\mu_{A_1}(x_n), v_{A_1}(x_n), \pi_{A_1}(x_n)) \\ (\mu_{A_2}(x_1), v_{A_2}(x_1), \pi_{A_2}(x_1)) & (\mu_{A_2}(x_2), v_{A_2}(x_2), \pi_{A_2}(x_2)) & \ldots & (\mu_{A_2}(x_n), v_{A_2}(x_n), \pi_{A_2}(x_n)) \\ \ldots & \ldots & \ldots & \ldots \\ (\mu_{A_m}(x_1), v_{A_m}(x_1), \pi_{A_m}(x_1)) & (\mu_{A_m}(x_2), v_{A_m}(x_2), \pi_{A_m}(x_2)) & \cdots & (\mu_{A_m}(x_n), v_{A_m}(x_n), \pi_{A_m}(x_n)) \end{bmatrix}$$

Let $r_{ij}^{(k)}$ denotes the rating for the i^{th} alternative w.r.t. the j^{th} criterion by the k^{th} DM
$r_{ij}^{(k)} = (\mu_{ij}^{(k)}, v_{ij}^{(k)})$
r_{ij} can be calculated by using Intuitionistic Fuzzy Weighted Averaging Operator (IFWAO) given in below equation.
$r_{ij} = [1-\prod_{k=1}^{l}(1-\mu_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(v_{ij}^{(k)})^{\lambda_k},$
$\prod_{k=1}^{l}(1-\mu_{ij}^{(k)})^{\lambda_k} - \prod_{k=1}^{l}(v_{ij}^{(k)})^{\lambda_k}]$

Step 4: Computation of weights for the criteria
Let weight assigned to the criterion X_j by the k^{th} DM is represented as
$w_j^{(k)} = (\mu_j^{(k)}, v_j^{(k)}, \pi_j^{(k)})$
IFWAO, to compute weights of the criteria is defined as
$w_j = [1-\prod_{k=1}^{l}(1-\mu_j^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(v_j^{(k)})^{\lambda_k},$
$\prod_{k=1}^{l}(1-\mu_j^{(k)})^{\lambda_k} - \prod_{k=1}^{l}(v_j^{(k)})^{\lambda_k}]$

The aggregated weight for the criterion X_j is represented as
$w_j = (\mu_j, v_j, \pi_j)$, j = 1,2,3,...,n
$W = [w_1, w_2, w_3,…,w_n]^{Transpose}$

Step 5: Compute Aggregated Weighted Intuitionistic Fuzzy Decision Matrix (AWIFDM)
After finding the weight matrix and AIFDM the AWIFDM is calculated and is represented as

$$\acute{R} = \begin{bmatrix} \breve{r}_{11} & \breve{r}_{12} & \cdots & \breve{r}_{1n} \\ \breve{r}_{21} & \breve{r}_{22} & \ldots & \breve{r}_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \breve{r}_{m1} & \breve{r}_{m2} & \cdots & \breve{r}_{mn} \end{bmatrix} = [\acute{r}_{ij}]_{m\times n}$$

Where
$\acute{r}_{ij} = (\acute{\mu}_{ij}, \acute{v}_{ij}, \acute{\pi}_{ij}) = (\mu_{A_i.W}(x_j), v_{A_i.W}(x_j), \pi_{A_i.W}(x_j))$
i = 1,2,3,……,m and j = 1,2,3,….,n
Therefore $\acute{R}$ can be written as

$$\acute{R} = \begin{bmatrix} \mu_{A_1.W}(x_1), v_{A_1.W}(x_1), \pi_{A_1.W}(x_1) & \mu_{A_1.W}(x_2), v_{A_1.W}(x_2), \pi_{A_1.W}(x_2) & \cdots & \mu_{A_1.W}(x_n), v_{A_1.W}(x_n), \pi_{A_1.W}(x_n) \\ \mu_{A_2.W}(x_1), v_{A_2.W}(x_1), \pi_{A_2.W}(x_1) & \mu_{A_2.W}(x_2), v_{A_2.W}(x_2), \pi_{A_2.W}(x_2) & \ldots & \mu_{A_2.W}(x_n), v_{A_2.W}(x_n), \pi_{A_2.W}(x_n) \\ \ldots & \vdots & \ddots & \vdots \\ \mu_{A_m.W}(x_1), v_{A_m.W}(x_1), \pi_{A_m.W}(x_1) & \mu_{A_m.W}(x_2), v_{A_m.W}(x_2), \pi_{A_m.W}(x_2) & \ldots & \mu_{A_m.W}(x_n), v_{A_m.W}(x_n), \pi_{A_m.W}(x_n) \end{bmatrix}$$

To find the values of $\mu_{A_i.W}(x_j), v_{A_i.W}(x_j), \pi_{A_i.W}(x_j)$ we used this product rule
$R \otimes W = \{< x, \mu_{A_i.W}(x), v_{A_i.W}(x) > / x \in X\}$
$\mu_{A_i.W}(x) = \mu_{A_i}(x) . \mu_W(x)$
$V_{A_i.W}(x) = v_{A_i}(x) + v_W(x) - v_{A_i}(x) . v_W(x)$
In addition, the hesitation degree in x can be calculated as
$\pi_{A_i.W}(x) = 1 - \mu_{A_i}(x). \mu_W(x) - v_{A_i}(x) - v_W(x) + v_{A_i}(x) . v_W(x)$

Step 6: Determine IFPIS and IFNIS
For finding the IFPIS and IFNIS.
j_1= benefit criteria
j_2=cost criteria
A^* = IFPIS
$\acute{A}$= IFNIS

$A^* = \{(\mu_{A^*W}(x_j), v_{A^*W}(x_j))\}$

$\acute{A} = \{(\mu_{\acute{A}W}(x_j), v_{\acute{A}W}(x_j))\}$
$(\mu_{A^*W}(x_j) = ((\max_i \mu_{A_i.W}(x_j) / j \in j_1), (\min_i \mu_{A_i.W}(x_j) / j \in j_2))$
$(v_{A^*W}(x_j) = ((\min_i v_{A_i.W}(x_j) / j \in j_1), (\max_i v_{A_i.W}(x_j) / j \in j_2))$
$(\mu_{\acute{A}W}(x_j) = ((\min_i \mu_{A_i.W}(x_j) / j \in j_1), (\max_i \mu_{A_i.W}(x_j) / j \in j_2))$
$(v_{\acute{A}W}(x_j) = ((\max_i v_{A_i.W}(x_j) / j \in j_1), (\min_i v_{A_i.W}(x_j) / j \in j_2))$

Step 7: Computation of Separation Measures
To find the separation measures d^* and $\acute{d}$, Normalized

Euclidean Distance is used as

$$d^* = \left(\frac{1}{2n}\sum_{j=1}^{n}[(\mu_{A_i.W}(x_j) - \mu_{A^*W}(x_j))^2 + (v_{A_i.W}(x_j) - v_{A^*W}(x_j))^2, [(\pi_{A_i.W}(x_j) - \pi_{A^*W}(x_j))^2]\right)^{0.5}$$

$$d^- = \left(\frac{1}{2n}\sum_{j=1}^{n}[(\mu_{A_i.W}(x_j) - \mu_{\acute{A}W}(x_j))^2 + (v_{A_i.W}(x_j) - v_{\acute{A}W}(x_j))^2, [(\pi_{A_i.W}(x_j) - \pi_{\acute{A}W}(x_j))^2]\right)^{0.5}$$

Step 8: Computation of Relative Closeness Coefficient (RCC)

The *RCC* of an alternative A^i w. r. t the IFPIS A^* can computed as

$RCC_i = \frac{d^-}{d^- + d_i^*}$: $0 \le RCC_i \le 1$

Step 9: Ranking alternatives

After computation of RCC_i for each alternative A_i, the alternatives are ranked in descending order of RCC_is.

3 APPLICATION OF INTUITIONISTIC FUZZY TOPSIS

PROBLEM SCENARIO

Company: Automotive

Problem: Supplier Selection

Consider three Decision Makers represented by D = {DM_1, DM_2, DM_3}

Five Alternatives m = 5 represented by A = {A_i : i= 1, 2, 3, 4, 5}

Four Evaluation Criteria n = 4 represented by

$$C = \begin{cases} Benefite\ criteria & j_1 = \begin{cases} X_1: & product\ quality \\ X_2: & relationship\ closnes \\ X_3: & delivery\ perfomance \end{cases} \\ cost\ criteria & j_2 = \{X_4: \quad price \end{cases}$$

Solution by Intuitionistic Fuzzy TOPSIS

Intuitionistic fuzzy ratings scale for importance of criteria and DMs is given by the Table 4.19.

Table 1: *Linguistic variables for rating the importance of criteria and decision makers*

LVs	**IFNs**
Very important	(0.90, 0.10)
Important	(0.75, 0.20)
Medium	(0.50, 0.45)
Unimportant	(0.35, 0.60)
Very unimportant	(0.10, 0.90)

The alternative ratings are given in the below table

Table 2a: *Linguistic variables for rating the alternatives*

LVs	**IFNs**
Extremely good (EG)/extremely high (EH) (1.00, 0.00)	(1.00, 0.00)
Very very good (VVG)/very very high (VVH)	(0.90, 0.10)
Very good (VG)/very high (VH)	(0.80, 0.10)
Good (G)/high (H)	(0.70, 0.20)
Medium good (MG)/medium high (MH)	(0.60, 0.30)
Fair (F)/medium (M)	(0.50, 0.40)

Table 2b: *Linguistic variables for rating the alternatives*

LVs	**IFNs**
Medium bad (MB)/medium low (ML	(0.40, 0.50)
Bad (B)/low (L)	(0.25, 0.60)
Very bad (VB)/very low (VL)	(0.10, 0.75)
Very very bad (VVB)/very very low (VVL)	(0.10, 0.90)

Table 3: *The importance and weights of decision makers*

	DM_1	DM_2	DM_3
Linguistic Variables	VI(0.90, 0.10, 0.00) (μ_1, v_1, π_1)	M(0.50,0.45, 0.05) (μ_2, v_2, π_2)	I (0.75, 0.20, 0.05) (μ_3, v_3, π_3)
Weights	λ_{DM_1}= 0.406	λ_{DM_2}= 0.238	λ_{DM_3}= 0.356

Now by using the alternative ratings $r_{ij}^{(k)}$ and the DM weights λ_k, the aggregated intuitionistic fuzzy ratings for the alternatives are calculated below

$$r_{ij} = \lambda_1 r_{ij}^{(1)} + \lambda_2 r_{ij}^{(2)} + \lambda_3 r_{ij}^{(3)} + \ldots\ldots\ldots\ldots + \lambda_l r_{ij}^{(l)}$$

$$r_{ij} = \left(1 - \prod_{k=1}^{l}(1 - \mu_{ij}^{(k)})^{\lambda_k},\ \prod_{k=1}^{l}(v_{ij}^{(k)})^{\lambda_k},\ \prod_{k=1}^{l}(1 - \mu_{ij}^{(k)})^{\lambda_k} - \prod_{k=1}^{l}(v_{ij}^{(k)})^{\lambda_k}\right)$$

Where

i = 1, 2, 3, 4, 5. : j = 1, 2, 3, 4, and i = 3

For i = j = 1 and l= 3

$$r_{11} = \lambda_1 r_{11}^{(1)} + \lambda_2 r_{11}^{(2)} + \lambda_3 r_{11}^{(3)}$$

$$r_{11} = 1 - \prod_{k=1}^{3}(1 - \mu_{11}^{(k)})^{\lambda_k}, \prod_{k=1}^{3}(v_{11}^{(k)})^{\lambda_k}, \prod_{k=1}^{3}(1 - \mu_{11}^{(k)})^{\lambda_k} - \prod_{k=1}^{3}(v_{11}^{(k)})^{\lambda_k})$$

$$r_{11} = \begin{pmatrix} 1 - \left(1 - \mu_{11}^{(1)}\right)^{\lambda_1} \left(1 - \mu_{11}^{(2)}\right)^{\lambda_2} \left(1 - \mu_{11}^{(3)}\right)^{\lambda_3}, \left(v_{11}^{(1)}\right)^{\lambda_1} \left(v_{11}^{(2)}\right)^{\lambda_2} \left(v_{11}^{(3)}\right)^{\lambda_3}, \left(1 - \mu_{11}^{(1)}\right)^{\lambda_1} - \left(v_{11}^{(1)}\right)^{\lambda_1} \\ (1 - \mu_{11}^{(2)})^{\lambda_1} - (v_{11}^{(2)})^{\lambda_2} (1 - \mu_{11}^{(3)})^{\lambda_3} - (v_{11}^{(3)})^{\lambda_3} \end{pmatrix}$$

$$r_{11} = \begin{pmatrix} 1 - (1 - 0.7)^{0.406} (1 - 0.8)^{0.238} (1 - 0.7)^{0.356}, & (0.2)^{0.406}(0.1)^{0.238}(0.2)^{0.356}, \\ (1 - 0.7)^{0.406} (1 - 0.8)^{0.238} (1 - 0.7)^{0.356} - & (0.2)^{0.406}(0.1)^{0.238}(0.2)^{0.356} \end{pmatrix}$$

r_{11}= (0.728, 0.170, 0.102)

For the sake of brevity, remaining 19 r_{ij} s are tabulated in matrix given by the table 5.

Table 4: *Alternative ratings*

Cri.	Alts.	Decision Makers		
		DM_1	DM_2	DM_3
X_1	A_1	$Gr_{11}^{(1)} = (\mu_{11}^{(1)}, v_{11}^{(1)}) = (0.7, 0.2)$	$VGr_{11}^{(2)} = (\mu_{11}^{(2)}, v_{11}^{(2)}) = (0.8, 0.1)$	$Gr_{11}^{(3)} = (\mu_{11}^{(3)}, v_{11}^{(3)}) = (0.7, 0.2)$
	A_2	$Gr_{21}^{(1)} = (\mu_{21}^{(1)}, v_{21}^{(1)}) = (0.6, 0.3)$	$Gr_{21}^{(2)} = (\mu_{21}^{(2)}, v_{21}^{(2)}) = (0.7, 0.2)$	$Gr_{21}^{(3)} = (\mu_{21}^{(3)}, v_{21}^{(3)}) = (0.5, 0.4)$
	A_3	$VVGr_{31}^{(1)} = (\mu_{31}^{(1)}, v_{31}^{(1)}) = (0.9, 0.1)$	$VGr_{31}^{(2)} = (\mu_{31}^{(2)}, v_{31}^{(2)}) = (0.8, 0.1)$	$VGr_{31}^{(3)} = (\mu_{31}^{(3)}, v_{31}^{(3)}) = (0.8, 0.1)$
	A_4	$Mr_{41}^{(1)} = (\mu_{41}^{(1)}, v_{41}^{(1)}) = (0.6, 0.3)$	$Gr_{41}^{(2)} = (\mu_{41}^{(2)}, v_{41}^{(2)}) = (0.7, 0.2)$	$Gr_{41}^{(3)} = (\mu_{41}^{(3)}, v_{41}^{(3)}) = (0.6, 0.3)$
	A_5	$Fr_{51}^{(1)} = (\mu_{51}^{(1)}, v_{51}^{(1)}) = (0.5, 0.4)$	$MGr_{51}^{(2)} = (\mu_{51}^{(2)}, v_{51}^{(2)}) = (0.6, 0.3)$	$MGr_{51}^{(3)} = (\mu_{51}^{(3)}, v_{51}^{(3)}) = (0.6, 0.3)$
X_2	A_1	$Mr_{12}^{(1)} = (\mu_{12}^{(1)}, v_{12}^{(1)}) = (0.6, 0.3)$	$Gr_{12}^{(2)} = (\mu_{12}^{(2)}, v_{12}^{(2)}) = (0.7, 0.2)$	$MGr_{12}^{(3)} = (\mu_{12}^{(3)}, v_{12}^{(3)}) = (0.6, 0.3)$
	A_2	$Fr_{22}^{(1)} = (\mu_{22}^{(1)}, v_{22}^{(1)}) = (0.5, 0.4)$	$MGr_{22}^{(2)} = (\mu_{22}^{(2)}, v_{22}^{(2)}) = (0.6, 0.3)$	$Gr_{22}^{(3)} = (\mu_{22}^{(3)}, v_{22}^{(3)}) = (0.7, 0.2)$
	A_3	$VGr_{32}^{(1)} = (\mu_{32}^{(1)}, v_{32}^{(1)}) = (0.8, 0.1)$	$Gr_{32}^{(2)} = (\mu_{32}^{(2)}, v_{32}^{(2)}) = (0.7, 0.2)$	$VGr_{32}^{(3)} = (\mu_{32}^{(3)}, v_{32}^{(3)}) = (0.8, 0.1)$
	A_4	$Fr_{42}^{(1)} = (\mu_{42}^{(1)}, v_{42}^{(1)}) = (0.5, 0.4)$	$Fr_{42}^{(2)} = (\mu_{42}^{(2)}, v_{42}^{(2)}) = (0.5, 0.4)$	$Mr_{42}^{(3)} = (\mu_{42}^{(3)}, v_{42}^{(3)}) = (0.6, 0.3)$
	A_5	$MBr_{52}^{(1)} = (\mu_{52}^{(1)}, v_{52}^{(1)}) = (0.4, 0.5)$	$Fr_{52}^{(2)} = (\mu_{52}^{(2)}, v_{52}^{(2)}) = (0.5, 0.4)$	$Fr_{52}^{(3)} = (\mu_{52}^{(3)}, v_{52}^{(3)}) = (0.5, 0.4)$
X_3	A_1	$VGr_{13}^{(1)} = (\mu_{13}^{(1)}, v_{13}^{(1)}) = (0.8, 0.1)$	$Gr_{13}^{(2)} = (\mu_{13}^{(2)}, v_{13}^{(2)}) = (0.7, 0.2)$	$VGr_{13}^{(3)} = (\mu_{13}^{(3)}, v_{13}^{(3)}) = (0.8, 0.1)$
	A_2	$Gr_{23}^{(1)} = (\mu_{23}^{(1)}, v_{23}^{(1)}) = (0.7, 0.2)$	$MGr_{23}^{(2)} = (\mu_{23}^{(2)}, v_{23}^{(2)}) = (0.6, 0.3)$	$MGr_{23}^{(3)} = (\mu_{23}^{(3)}, v_{23}^{(3)}) = (0.6, 0.3)$
	A_3	$VGr_{33}^{(1)} = (\mu_{33}^{(1)}, v_{33}^{(1)}) = (0.8, 0.1)$	$VGr_{33}^{(2)} = (\mu_{33}^{(2)}, v_{33}^{(2)}) = (0.8, 0.1)$	$Gr_{33}^{(3)} = (\mu_{33}^{(3)}, v_{33}^{(3)}) = (0.7, 0.2)$
	A_4	$VGr_{43}^{(1)} = (\mu_{43}^{(1)}, v_{43}^{(1)}) = (0.8, 0.1)$	$Gr_{43}^{(2)} = (\mu_{43}^{(2)}, v_{43}^{(2)}) = (0.7, 0.2)$	$Gr_{43}^{(3)} = (\mu_{43}^{(3)}, v_{43}^{(3)}) = (0.7, 0.2)$
	A_5	$Gr_{53}^{(1)} = (\mu_{53}^{(1)}, v_{53}^{(1)}) = (0.7, 0.2)$	$Gr_{53}^{(2)} = (\mu_{53}^{(2)}, v_{53}^{(2)}) = (0.7, 0.2)$	$Mr_{53}^{(3)} = (\mu_{53}^{(3)}, v_{53}^{(3)}) = (0.6, 0.3)$
X_3	A_1	$Gr_{14}^{(1)} = (\mu_{14}^{(1)}, v_{14}^{(1)}) = (0.7, 0.2)$	$Gr_{14}^{(2)} = (\mu_{14}^{(2)}, v_{14}^{(2)}) = (0.7, 0.2)$	$Gr_{14}^{(3)} = (\mu_{14}^{(3)}, v_{14}^{(3)}) = (0.7, 0.2)$
	A_2	$MGr_{24}^{(1)} = (\mu_{24}^{(1)}, v_{24}^{(1)}) = (0.6, 0.3)$	$Fr_{24}^{(2)} = (\mu_{24}^{(2)}, v_{24}^{(2)}) = (0.5, 0.4)$	$MGr_{24}^{(3)} = (\mu_{24}^{(3)}, v_{24}^{(3)}) = (0.6, 0.3)$
	A_3	$VGr_{34}^{(1)} = (\mu_{34}^{(1)}, v_{34}^{(1)}) = (0.8, 0.1)$	$VGr_{34}^{(2)} = (\mu_{34}^{(2)}, v_{34}^{(2)}) = (0.8, 0.1)$	$Gr_{34}^{(3)} = (\mu_{34}^{(3)}, v_{34}^{(3)}) = (0.7, 0.2)$
	A_4	$Gr_{44}^{(1)} = (\mu_{44}^{(1)}, v_{44}^{(1)}) = (0.7, 0.2)$	$MGr_{44}^{(2)} = (\mu_{44}^{(2)}, v_{44}^{(2)}) = (0.6, 0.3)$	$MGr_{44}^{(3)} = (\mu_{44}^{(3)}, v_{44}^{(3)}) = (0.6, 0.3)$
	A_5	$Fr_{54}^{(1)} = (\mu_{54}^{(1)}, v_{54}^{(1)}) = (0.5, 0.4)$	$Mr_{54}^{(2)} = (\mu_{54}^{(2)}, v_{54}^{(2)}) = (0.6, 0.3)$	$Fr_{54}^{(3)} = (\mu_{54}^{(3)}, v_{54}^{(3)}) = (0.5, 0.4)$

Table 5*: Aggregated Intuitionistic Fuzzy Decision Matrix*
D = $[r_{ij}]_{5\times4}$

	X_1	X_2	X_3	X_4
A_1	r_{11}= (0.728, 0.170, 0.102)	r_{12}= (0.626, 0.272, 0.102)	r_{13}= (0.780, 0.118, 0.102)	r_{14}= (0.700, 0.200, 0.100)
A_2	r_{21}= (0.596, 0.302, 0.102)	r_{22}= (0.605, 0.292, 0.103)	r_{23}= (0.644, 0.256, 0.100)	r_{24}= (0.578, 0.321, 0.101)
A_3	r_{31}= (0.051, 0.100, 0.849)	r_{23}= (0.780, 0.118, 0.102)	r_{33}= (0.769, 0.170, 0.061)	r_{34}= (0.769, 0.128, 0.103)
A_4	r_{41}= (0.663, 0.236, 0.101)	r_{24}= (0.538, 0.361, 0.101)	r_{43}= (0.746, 0.151, 0.103)	r_{44}= (0.644, 0.254, 0.102)
A_5	r_{51}= (0.562, 0.337, 0.101)	r_{25}= (0.462, 0.436, 0.100)	r_{53}= (0.526, 0.231, 0.101)	r_{54}= (0.526, 0.374, 0.100)

Step 4: Computation of the weights of the Criteria

The individual weights given by each DM are listed in the Table 6

Table 6: *Weights of Criteria determined by the DMs* $w_j^{(k)}$**=** $(\mu_j^{(k)}, v_j^{(k)}, \pi_j^{(k)})$

Criteria	DM_1	DM_2	DM_3
X_1	VI(0.90,0.10.00) $w_1^{(1)}=(\mu_1^{(1)}, v_1^{(1)}, \pi_1^{(1)})$	VI(0.90,0.10, 0.00) $w_1^{(2)}=(\mu_1^{(2)}, v_1^{(2)}, \pi_1^{(2)})$	I (0.90, 0.10, 0.00) $w_1^{(3)}=(\mu_1^{(3)}, v_1^{(3)}, \pi_1^{(3)})$
X_2	I (0.75, 0.20, 0.05) $w_2^{(1)}=(\mu_2^{(1)}, v_2^{(1)}, \pi_2^{(1)})$	I (0.75, 0.20, 0.05) $w_2^{(2)}=(\mu_2^{(2)}, v_2^{(2)}, \pi_2^{(2)})$	I (0.75, 0.20, 0.05) $w_2^{(3)}=(\mu_2^{(3)}, v_2^{(3)}, \pi_2^{(3)})$
X_3	I (0.75, 0.20, 0.05) $w_3^{(1)}=(\mu_3^{(1)}, v_3^{(1)}, \pi_3^{(1)})$	I (0.75, 0.20, 0.05) $w_3^{(2)}=(\mu_3^{(2)}, v_3^{(2)}, \pi_3^{(2)})$	M(0.50,0.45, 0.05) $w_3^{(2)}=(\mu_3^{(2)}, v_3^{(2)}, \pi_3^{(2)})$
X_4	M(0.50,0.45, 0.05) $w_4^{(1)}=(\mu_4^{(1)}, v_4^{(1)}, \pi_4^{(1)})$	I (0.75, 0.20, 0.05) $w_4^{(2)}=(\mu_4^{(2)}, v_4^{(2)}, \pi_4^{(2)})$	M(0.50,0.45, 0.05) $w_4^{(3)}=(\mu_4^{(3)}, v_4^{(3)}, \pi_4^{(3)})$

By using the table 6, the aggregated criteria weights are computed by using the given operator

$w_j = \lambda_1 w_j^{(1)} + \lambda_2 w_j^{(2)} + \lambda_3 w_j^{(3)} + \ldots\ldots\ldots\ldots + \lambda_l w_j^{(l)}$

$w_j = (1 - \prod_{k=1}^{l}(1-\mu_j^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(v_j^{(k)})^{\lambda_k},$
$\prod_{k=1}^{l}(1-\mu_j^{(k)})^{\lambda_k} - \prod_{k=1}^{l}(v_j^{(k)})^{\lambda_k})$

Where i = 1, 2, 3, 4, 5. : j = 1, 2, 3, 4. And l = 3

For j = 1 and l= 3

$w_1 = \lambda_1 w_1^{(1)} + \lambda_2 w_1^{(2)} + \lambda_3 w_1^{(3)}$

$w_1 =$
$\left(1 - \prod_{k=1}^{3}(1-\mu_1^{(k)})^{\lambda_k}, \quad \prod_{k=1}^{3}(v_1^{(k)})^{\lambda_k}, \quad \prod_{k=1}^{3}(1-\mu_1^{(k)})^{\lambda_k} - \prod_{k=1}^{3}(v_1^{(k)})^{\lambda_k}\right)$

$=(1 -$
$\left(1-\mu_1^{(1)}\right)^{\lambda_1}\left(1-\mu_1^{(2)}\right)^{\lambda_2}\left(1-\mu_1^{(3)}\right)^{\lambda_3}, \left(v_1^{(1)}\right)^{\lambda_1}\left(v_1^{(2)}\right)^{\lambda_2}\left(v_1^{(3)}\right)^{\lambda_3},$
$\left(1-\mu_1^{(1)}\right)^{\lambda_1} - \left(v_1^{(1)}\right)^{\lambda_1}, (1-\mu_1^{(2)})^{\lambda_2} - (v_1^{(2)})^{\lambda_2}\,(1-\mu_1^{(3)})^{\lambda_3} -$
$(v_1^{(3)})^{\lambda_3})$

$$w_1 = (1 - (1-0.9)^{0.406}(1-0.9)^{0.238}(1-0.75)^{0.356}, (0.1)^{0.406}(0.1)^{0.238}(0.2)^{0.356}, (1-0.9)^{0.406}(1-0.9)^{0.238}(1-0.75)^{0.356} - (0.1)^{0.406}(0.1)^{0.238}(0.2)^{0.356})$$

w_1= (0.861, 0.128, 0.011)

In this way, the remaining 3 w_1 S are calculated and are tabulated in the following matrix

w_2= (0.750, 0.200, 0.050)
w_3= (0.680, 0.267, 0.053)
w_4= (0.576, 0.371, 0.053)

Therefore

$$W_{\{X_1,X_2,X_3,X_4\}} = \begin{bmatrix}(0.861, 0.128, 0.011)\\(0.750, 0.200, 0.050)\\(0.680, 0.267, 0.053)\\(0.576, 0.371, 0.053)\end{bmatrix}^{Transpose}$$

Step 5: Construction of AWIFDM

After finding the weights of the criteria and the alternative ratings the aggregated weighted Intuitionistic fuzzy ratings for the alternatives are calculated.

$r'_{ij}=(\mu'_{ij}, v'_{ij}, \pi'_{ij})$ = ($\mu_{A_i}(x).\ \mu_W(x)$,$v_{A_i}(x) + v_W(x)$- $v_{A_i}(x).\ v_W(x)$,1-$\mu_{A_i}(x).\ \mu_W(x)$ - $v_{A_i}(x)$- $v_W(x)$ + $v_{A_i}(x).\ v_W(x)$)

For i = j = 1

$r'_{11}=(\mu'_{11}, v'_{11}, \pi'_{11})$ = ($\mu_{A_1}(x).\ \mu_W(x)$, $v_{A_1}(x) + v_W(x)$- $v_{A_1}(x).\ v_W(x)$, 1-$\mu_{A_1}(x).\ \mu_W(x)$- $v_{A_1}(x)$ - $v_W(x)$+ $v_{A_1}(x).\ v_W(x)$)

r'_{11}= (0.728×0.861, 0.170+0.128-0.170×0.128, 1- $\mu'_{11} - v'_{11}$)

r'_{11}= (0.627, 0.276, 1-0.627 - 0.276)

r'_{11}= (0.627, 0.276, 0.097)

Similarly we find

r'_{12}= (0.470, 0.418, 0.112)
r'_{13}= (0.530, 0.353, 0.117)
r'_{14}= (0.403, 0.497, 0.100)
r'_{21}= (0.513, 0.391, 0.096)
r'_{22}= (0.454, 0.434, 0.112)
r'_{23}= (0.438, 0.453, 0.109)
r'_{24}= (0.333, 0.573, 0.094)
r'_{31}= (0.731, 0.215, 0.054)
r'_{32}= (0.585, 0.294, 0.121)
r'_{33}= (0.523, 0.361, 0.116)
r'_{34}= (0.443, 0.452, 0.105)
r'_{41}= (0.571, 0.334, 0.095)
r'_{42}= (0.404, 0.489, 0.107)
r'_{43}= (0.507, 0.378, 0.115)
r'_{44}= (0.371, 0.531, 0.098)
r'_{51}= (0.484, 0.422, 0.094)
r'_{52}= (0.347, 0.550, 0.103)
r'_{53}= (0.454, 0.436, 0.110)
r'_{54}= (0.303, 0.606, 0.091)

All the above values are given in the table 7 as AWIFDM

Table 7: *Aggregated Weighted Intuitionistic Fuzzy Decision Matrix*

	X_1	X_2	X_3	X_4
A_1	r'_{11}= (0.627, 0.276, 0.097)	r'_{12}= (0.470, 0.418, 0.112)	r'_{13}= (0.530, 0.353, 0.117)	r'_{14}= (0.403, 0.497, 0.100)
A_2	r'_{21}= (0.513, 0.391, 0.096)	r'_{22}= (0.454, 0.434, 0.112)	r'_{23}= (0.438, 0.453, 0.109)	r'_{24}= (0.333, 0.573, 0.094)
A_3	r'_{31}= (0.731, 0.215, 0.054)	r'_{32}= (0.585, 0.294, 0.121)	r'_{33}= (0.523, 0.361, 0.116)	r'_{34}= (0.443, 0.452, 0.105)
A_4	r'_{41}= (0.571, 0.334, 0.095)	r'_{42}= (0.404, 0.489, 0.107)	r'_{43}= (0.507, 0.378, 0.115)	r'_{44}= (0.371, 0.531, 0.098)
A_5	r'_{51}= (0.484, 0.422, 0.094)	r'_{52}= (0.347, 0.550, 0.103)	r'_{53}= (0.454, 0.436, 0.110)	r'_{54}= (0.303, 0.606, 0.091)

Step 6: Computation of IFPIS and IFNIS

Since Product Quality, Relationship closeness and Delivery Performance are benefit criteria that is why they are in the set j_1 = {X_1, X_2, X_3} whereas Price being the cost criteria therefore it is in the set j_2 = {X_4}

The IFPIS is calculated as

A^+ = {(0.731, 0.215, 0.054), (0.585, 0.294, 0.121), (0.530, 0.353, 0.117), (0.303, 0.606, 0.091)}

The IFNIS is calculated as

A^- = {(0.484, 0.422, 0.094), (0.347, 0.550, 0.103), (0.438, 0.453, 0.109), (0.443, 0.452, 0.105)}

Step 7: Computation of Separation Measures

By using Normalized Euclidean Distance Measure the negative and positive separation measures d^* and d' respectively are calculated as follows

$$d_i^+ = \left(\frac{1}{2n}\sum_{j=1}^{n}\left[\left(\mu_{A_i.W}(x_j) - \mu_{A'W}(x_j)\right)^2 + \left(v_{A_i.W}(x_j) - v_{A'W}(x_j)\right)^2 + \left(\pi_{A_i.W}(x_j) - \pi_{A'W}(x_j)\right)^2\right]\right)^{0.5}$$

For i=1 and n=4

$$d_1^+ = \left(\frac{1}{2(4)}\sum_{j=1}^{4}\left[\left(\mu_{A_1.W}(x_j) - \mu_{A'W}(x_j)\right)^2 + \left(v_{A_1.W}(x_j) - v_{A'W}(x_j)\right)^2 + \left(\pi_{A_1.W}(x_j) - \pi_{A'W}(x_j)\right)^2\right]\right)^{0.5}$$

$$d_1^+ = \left(\frac{1}{8}\sum_{j=1}^{4}\begin{bmatrix}\left(\mu_{A_1.W}(x_1) - \mu_{A'W}(x_1)\right)^2 + \left(v_{A_1.W}(x_1) - v_{A'W}(x_1)\right)^2 + \left(\pi_{A_1.W}(x_1) - \pi_{A'W}(x_1)\right)^2 + \\ \left(\mu_{A_1.W}(x_2) - \mu_{A'W}(x_2)\right)^2 + \left(v_{A_1.W}(x_2) - v_{A'W}(x_2)\right)^2 + \left(\pi_{A_1.W}(x_2) - \pi_{A'W}(x_2)\right)^2 + \\ \left(\mu_{A_1.W}(x_3) - \mu_{A'W}(x_3)\right)^2 + \left(v_{A_1.W}(x_3) - v_{A'W}(x_3)\right)^2 + \left(\pi_{A_1.W}(x_3) - \pi_{A'W}(x_3)\right)^2 + \\ \left(\mu_{A_1.W}(x_4) - \mu_{A'W}(x_4)\right)^2 + \left(v_{A_1.W}(x_4) - v_{A'W}(x_4)\right)^2 + \left(\pi_{A_1.W}(x_4) - \pi_{A'W}(x_4)\right)^2\end{bmatrix}\right)^{0.5}$$

$$d_1^+ = \left(\frac{1}{8}\sum_{j=1}^{4}\begin{bmatrix}(0.627-0.731)^2+(0.276-0.215)^2+(0.097-0.054)^2+\\(0.470-0.585)^2+(0.418-0.294)^2+(0.112-0.121)^2+\\(0.530-0.530)^2+(0.353-0.353)^2+(0.117-0.117)^2+\\(0.403-0.303)^2+(0.497-0.606)^2+(0.100-0.091)^2\end{bmatrix}\right)^{0.5}$$

d_1^+ = $[0.125(0.0164+0.0287+0.0220)]^{0.5}$
d_1^+ = 0.0916 ≈ 0.092
For the sake of brevity without finding all values, the remaining values are given in the
table 8.

Table 8: *Separation measures and the RCC for each Alternative.*

Alternatives	d^*	$\dot{d}$
A_1	0.092	0.110
A_2	0.131	0.082
A_3	0.074	0.175
A_4	0.124	0.075
A_5	0.174	0.074

Step 8: Computation of Relative Closeness Coefficient (RCC)
The relative closeness coefficients are calculated as follows

$RCC_i = \frac{\dot{d}_i}{\dot{d}_i + d_i^*}$: i = 1,2,3,4,5.
$RCC_1 = \frac{0.110}{0.110+0.092}$ = 0.5446
$RCC_2 = \frac{0.082}{0.082+0.131}$ = 0.385
$\boldsymbol{RCC_3} = \frac{\mathbf{0.175}}{\mathbf{0.175+0.074}}$ **= 0.703**
$RCC_4 = \frac{0.075}{0.075+0.124}$ = 0.377
$RCC_5 = \frac{0.074}{0.074+0.174}$ = 0.0298

Step 9: Ranking Alternatives
From above calculations the RCC are ranked as follows
$\boldsymbol{RCC_3} > RCC_1 > RCC_2 > RCC_4 > RCC_5 \Longrightarrow A_3 > A_1 > A_2 > A_4 > A_5$
Hence $\mathbf{A_3}$ is the best alternative.

4 Conclusion

First of all, in this paper we discuss about IFS with some definitions and study about IF-TOPSIS to deal those problems which have uncertainty. Secondly, we discuss about IF-TOPSIS algorithm and finally, we apply this method for decision making and used this paper for the selection of best supplier for automotive company and conclude that A_3 is best alternative for automotive company.

References

[1] C. Hwang and K. Yoon, Multiple Attribute Decision Making: Methods and Applications, A State of the Art Survey, vol. 1. 1981.

[2] S.J. J. Chen and C.-L. Hwang, Fuzzy Multiple Attribute Decision Making Methods, vol. 375. 1992.

[3] C.T. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment," Fuzzy Sets Syst., vol. 114, no. 1, pp. 1–9, 2000.

[4] A. Awasthi, S.S. Chauhan, and S.K. Goyal, "A multi-criteria decision making approach for location planning for urban distribution centers under uncertainty," Math. Comput. Model., vol. 53, no. 1–2, pp. 98–109, 2011.

[5] T.C. Chu, "Selecting plant location via a fuzzy TOPSIS approach," Int. J. Adv. Manuf. Technol., vol. 20, no. 11, pp. 859–864, 2002.

[6] D. Yong, "Plant location selection based on fuzzy TOPSIS," Int. J. Adv. Manuf. Technol., vol. 28, no. 7–8, pp. 839–844, 2006.

[7] Y.M. Wang and T.M.S. Elhag, "Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment," Expert Syst. Appl., vol. 31, no. 2, pp. 309–319, 2006.

[8] W.M. Dong and F.S. Wong, "Fuzzy Weighted Averages and Implementation of the Extension Principle," vol. 21, pp. 183–199, 1987.

[9] S. Mahmoodzadeh, J. Shahrabi, M. Pariazar, and M. S. Zaeri, "Project selection by using fuzzy AHP and TOPSIS technique," Int. J. Soc. Manag. Econ. Bus. Eng., vol. 1, no. 3, pp. 324–329, 2007.

[10] T. Yong-tao, L. Shen, C. Langston, and Y. Liu, "Construction project selection using fuzzy TOPSIS approach," J. Model. Manag., vol. 5, no. 3, pp. 302–315, 2010.

[11] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning-I," Inf. Sci. (Ny)., vol. 8, no. 3, pp. 199–249, 1975.

[12] H. Q. Saremi and G. A. Montazer, "An application of type-2 fuzzy notions in website structures selection: Utilizing extended TOPSIS method," WSEAS Trans. Comput., vol. 7, no. 1, pp. 8–15, 2008.

[13] T. Y. Chen and C. Y. Tsao, "The interval-valued fuzzy TOPSIS method and experimental analysis," Fuzzy Sets Syst., vol. 159, no. 11, pp. 1410–1428, 2008.

[14] Zulqarnain M., and Saeed M., "An application of Interval valued fuzzy soft matrix (IVFSM) in decision making, "Sci. Int. (Lahore), vol. 28, no. 3, 2261-2264, 2016.

[15] Zulqarnain, M., and Saeed. M., "Comparison between fuzzy soft matrix (FSM) and interval valued fuzzy soft matrix (IVFSM) in decision making, "Sci. Int. (Lahore), vol. 28, no. 5, pp. 4277-4283, 2016.

[16] Zulqarnain, M., and Saeed, M., “A New Decision Making Method on Interval Valued Fuzzy Soft Matrix (IVFSM), “British Journal of Mathematics & Computer Science, vol. 20, no. 5, pp. 1-17, 2017.

[17] B. Ashtiani, F. Haghighirad, A. Makui, and G. Ali Montazer, “Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets,” Appl. Soft Comput. J., vol. 9, no. 2, pp. 457–461, 2009.

[18] A.K. Verma, R. Verma, and N. C. Mahanti, “Facility Location Selection: An Interval Valued Intuitionistic Fuzzy TOPSIS Approach,” Journal of Modern Mathematics and Statistics, vol. 4, no. 2. pp. 68–72, 2010.

[19] C.C. Hung and L. H. Chen, “A Fuzzy TOPSIS Decision Making Model with Entropy Weight under Intuitionistic Fuzzy Environment,” Proc. Int. MultiConference Eng. Comput. Sci., vol. I, pp. 18–21, 2009.

[20] D.F. Li and J.X. Nan, “Extension of the TOPSIS for Multi-Attribute Group Decision Making under Atanassov IFS Environments,” Int. J. Fuzzy Syst. Appl., vol. 1, no. 4, pp. 47–61, 2011.

[21] Z. Nurandiah and A. Lazim, “A TOPSIS Method with intuitionistic fuzzy group decision making,” Proc. 10th Int. Annu. Symp. UMTAS, no. July, pp. 256–65, 2011.

[22] F.E. Boran, S. Genç, M. Kurt, and D. Akay, “A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method,” Expert Syst. Appl., vol. 36, no. 8, pp. 11363–11368, 2009.

[23] F. Ye, “An extended TOPSIS method with interval-valued intuitionistic fuzzy numbers for virtual enterprise partner selection,” Expert Syst. Appl., vol. 37, no. 10, pp. 7050–7055, 2010.

[24] K.T. Atanassov, “Intuitionistic fuzzy sets,” Fuzzy Sets Syst., vol. 20, no. 1, pp. 87–96, Aug. 1986.

[25] F.E. Boran, S. Genç, M. Kurt, and D. Akay, “A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method,” Expert Syst. Appl., vol. 36, no. 8, pp. 11363–11368, 2009.