Mathematical Model For Television Commercial Allocation Problem

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Abstract: Commercial advertising on television is the main source of revenue for TV stations in Ghana. A key problem faced by the TV stations in Ghana is how to accept and televise the advertisements orders by an advertisers on a specified advertisement break in order to maximize revenue. The problem is complicated by show structure, limited time inventory, different rating points for different target audience groups and competition avoidance. The problem is formulated as mixed integer linear programming model and solved using one of the biggest TV stations in Ghana. From results of our mathematical model, commercial break after these assignments decreases by 27 percent as compared to the existing real life prime time commercial break plan however the total revenue gained from this assignment increases by 11 percent. The results demonstrate that the proposed mathematical model is flexible and capable of obtaining high-quality assignment for optimal scheduling television commercials.

Keywords: Advertisement allocation problem, scheduling, mixed integer linear programming, advertisement, advertising slots

1. INTRODUCTION

Commercial advertising on television is the main source of revenue for TV channels in Ghana. Television stations in Ghana provide TV programs to the public free of charge. By this way, they attract large variety of viewers who become somehow ready to watch commercial adverts. Advertisers want to buy time periods, which is called advertising breaks and located within the TV programs, to persuade the audiences (viewers) to purchase or take some action upon their products or services.TV stations have to schedule programs interspersed with adverts or commercials. It is important to place adverts in such a way that the combination of adverts would have the largest exposure as possible so as to maximize the total returns from the adverts. One of the major problems faced by TV channel planners is to allocate the advertising time to advertisers in order to maximize revenue. The problem is complex due to the limitation of time slots for advertising. In fact, different broadcasters view the problem in different ways. Some advertising requesters require the commercial to be seen by a given number of people, and the broadcaster will schedule the commercial to achieve this goal. Others want to have the broadcast seen by a certain demographic group or certain socio-economic classes of people. Some customers may want their commercials to be at the start/end of a given advertising break and others may want the commercial to be associated with a given TV program. This makes finding an optimal schedules computationally intensive problem that cannot usually be solved by hand. For this reason, naive approaches to scheduling lead to wasted resources and disenchanted audiences when ads fail to reach the interested consumers efficiently and must be aired repeatedly in order to meet impression targets. In this paper we address this interesting mathematical problem by first formulating the problem as mixed linear integer programming model and secondly implementing our model to real-life data set of one of the biggest TV station in Ghana. The rest of this paper is organized as follows. In Section 2 an overview of previous researches in this field is given. Section 3 contains a brief description of the problem. In Section 4 the notation and mathematical model is introduced. Results of numerical experiments and their comparison solution in a traditional way is given in Section 5. In Section 6 we give several concluding remarks and areas of future research.

2. LITERATURE REVIEW

Optimization techniques have been successfully employed in solving various decision problems. However, only a few studies exist that addresses the optimization of the advertising allocation problem on television industry. The research studies dealing specifically with TV breaks advertising allocation problem is sparse. Brown [1] described various hardships in allocating the advertising time to advertisers manually, and he developed an algorithm for exchange of advertisements of different lengths between advertising breaks to gain space for other advertisements. Mihiotis and Tsakiris [2] studied the problem of finding the best possible combination of placements of an advertisement (which channel, when, and how often) with the objective of the highest rating and subject to the limitation of the advertising budget. The problem was modeled as an integer program and a heuristic algorithm was designed for finding a good solution. Bollapragada et al. [3][4] considered the commercial scheduling problem of a single advertiser. The problem was formulated as an integer program and solved sequentially for each advertiser. Bollapragada et al.[5]then studied the problem of scheduling commercials over a specific period so that the airing of the same commercials are spread as evenly as possible and formulated this problem in the form of the integer programming models. They employed a branch-and-bound solution approach for obtaining the solution and also developed a heuristic approach for the multiple-air campaign problem. Jones [6] presented the advertising allocation problem as an example of designing incompletely specified combinatorial auctions in which hundreds of advertisers can submit combinatorial bids for allocating their commercials in the advertising slots. The problem was formulated as an integer programming model, and employed heuristics based on constraint programming to find a set of feasible solutions for their mathematical model. Based on his work, Zhang [7] studied the problem of selling the advertising time to advertisers. He proposed a two-step hierarchical method to find solutions for the problem. His approach starts with selecting advertisers and assigning them to TV programs and ends with allocating the advertising time to the selected advertisers in a program. The problem corresponding to the first step was solved using column generation method. Benoist et al.[8] studied a particular challenge faced by French satellite television. In their work, advertisements are sold as packages, rather than as individual spots which is called the TV-break packing problem. Various solution methodologies have been proposed for this problem. Brusco[9] also used the branch and bound method for solving the smaller scale of this problem and the simulated annealing heuristic approach for the real size problems. The developed heuristic approach has shown a significant improvement in terms of computation time and solution quality. Kimms and Muller-Bungart [10] described a planning problem at a broadcasting company where advertisers place orders for advertisements and their airdates are not fixed by the advertisers. The TV channel has to decide simultaneously which orders to accept or to reject and when advertisements from accepted orders should be broadcasted. They formulated the problem as a mathematical model and presented several heuristics to find solutions for the problem. Pereira et al. [11] developed a decision support system to plan the best assignment for the weekly promotion space of a major Portuguese TV station. The aim of this heuristic-based scheduling software system was to maximize the total viewing for each product within its target audience while fulfilling a set of constraints defined by advertisers. Wuang et al. [12] presented an ant colony optimization (ACO) heuristic for establishing a mechanism for solving the problem of scheduling television advertisements by considering customer requirements, relevant laws and regulations, and the need to fill all available advertising time. In the proposed approach, the scheduling mechanism and ACO heuristics are developed separately, allowing user to vary parameters of ACO heuristic and flexibly adjust the scheduling criterion. Also, Mao et al. [13] proposed an ACO algorithm to optimize the sum of products of revenue and advertisers' credit information in TV advertising and evaluated it using real data in the Japanese TV advertising market.

3. PROBLEM DESCRIPTION

The problem addressed here is how TV stations in countries like Ghana should select commercials to telecast from a set of reservation in order to maximize their revenues. Additionally, TV Networks will save both time and money when they assign the advertisements to the advertisement slots through an optimization model. The model will reduce the time needed to assign the ads to the spots, and bring away the possible errors caused by manual appointments. TV stations will also make a lightning effort to their customers' demands about their ads, and briskly revise the customer reservations. The selection commercials procedure has to consider various factors, such as the length of the commercial video, the target audience of advertisers, inventory of available time per advertisement break, competing advertisers which do not want their advertisements to be telecasted in the same advertisement break, and relationship between TV network and advertisers. TV networks unveil their weekly program schedule and tempt the advertisers and media buyers to buy airtime in bulk for the entire week. The advertisers send requests to be telecasted on an advertisement break of a specified program. That is, they make reservations for this specific advertisement break. Since an advertiser wants to reach a planned number of audiences with its existing advertising campaign, and all advertisers require this, there are numerous of reservations taken for an advertisement break of a show. Another constraint is limiting time inventory. Times may vary according to the regulations or practice by country but there is a common time lag for the advertisement period called advertisement break which is 450 seconds in Ghana. Generally, the total duration of the taken reservations is longer than the regulated time lag, however the total length of the selected commercials must be within an interval, less than an upper limit and more than a lower limit. Finally, all advertiser clients have the target audience of advertisements that should be considered. There are always specific target audience that advertisers want to live up to. A target audience can be defined in various ways- by household, by geographic market, by a given demographic group, such as high school students or workers between age 20 and 60, or even by product usage or ownership, such as people who have a mobile phones or laptops. The target audience should clearly be set and identified according to the needs of the advertiser and the assignment should be done suitably since advertisers generally prefer to pay the price according the amount of people reached. That is, the more people the TV network can reach by telecasting the request at specific time, the more price they can gain. In fact, in real life there is an extensively known media tool called cost per rating point (CPP) that can evaluate the price the clients have to pay for each rating point against a particular target group for their advertisements reached. The rating point is equal to one percent of all households who have a television set. In most countries the performance of TV networks is measured using these rating points. Therefore, our model is formed by considering all these constraints which is described in the following section in detail.

4. MATHEMATICAL MODEL

Allocation television commercials is a problem of selecting advertisements from a set of reservations that advertisers have specific target groups aimed to reach. Conclusive goal is generally to maximize the expected revenue. Client $i \in I$ informs his requirements for an advertisement $j \in J$ such as target audience group $g \quad (g = 1,2,\ldots,G)$, competing advertisers, duration of advertisements, etc. By considering these constraints, the most profitable order and selection of advertisements are decided to maximize the revenue. Each advertiser i specifies his target audience group set $R(i,j)$ for each advertisement j and to reach all these target groups in this set should be provided in the model. Furthermore, the order of telecasting commercial is important since the first and the last advertisements are assumed to have higher CPPs than the others as usual. Because, the audience level for the first advertisement will be high since the break starts just now and the audience level for the last advertisement will be high since the program watched by audience will start immediately after that. Therefore we have three spots to telecast an advertisement. The advertisement that will be selected to telecast in slot $s = 1$ and $s = 3$ has a higher CPP and as a result these slots are also more expensive. We assumed that all rating points s CPP for each slot s are known deterministically for this problem and are expected rating point $E[g]$ for target audience group g too. Advertiser i should also determine the duration of each advertisement j denoted by t_{ij}Moreover each advertisement break has an upper limit of total telecasting time T_U and a lower limit T_L.

Under these constraints, our decision variable (1) is also which advertisement of which client will be telecasted at which slot order.

$$x_{ij} = \begin{cases} 1, & \text{if advertisement } j \text{ of advertiser i is accepted is telecasted at order } s \\ 0, & \text{otherwise} \end{cases} \qquad (1)$$

The problem is modelled by using mixed integer linear programming. The total revenue (TR) that will be gained from advertisement j of client i is calculated in equation (2)

$$TR_{ij} = \sum_{g \in R(i,j)} E[g].\, t_{ij}.\, CPP_s \qquad (2)$$

Presumably, the advertising schedule is set by a programmer or an intermediary who is interested in maximizing revenue. Therefore, our objective function (3) aims to maximize the total expected revenue gained from telecasted advertisements of clients taking into account the relationship between TV network and advertisers as well paying fixed prices. The contribution of a advertiser to the total expected revenue, if it is selected to telecast, the amount obtained from (2)

$$\text{Max } Z = \sum_{i=I} \sum_{j=J} \sum_{s=S}^{3} x_{ijs} \left(\sum_{g \in R(i,j)} E[g].\, t_{ij}.\, CPP_s \right) + \sum_{i=I} \sum_{j=J} \sum_{s=1}^{3} x_{ijs} \,.\, FP_{ijs} \qquad (3)$$

The above mentioned constraints are mathematically stated as follows:

$$\sum_{i=I} \sum_{j=J} \sum_{s=1}^{3} x_{ijs} \,.\, t_{ij} \le T_U \qquad (4)$$

$$\sum_{i=I} \sum_{j=J} \sum_{s=1}^{3} x_{ijs} \,.\, t_{ij} \ge T_L \qquad (5)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij1} = 1 \qquad (6)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij3} = 1 \qquad (7)$$

$$\sum_{s=1} x_{ijs} \le 1 \qquad \forall\, i, j \qquad (8)$$

Constraint set (4) and (5) provides that the total duration of the telecasted advertisements with the maximum and minimum time availability, respectively. The total duration of an advertisement break must be less than an upper limit and more than a lower limit. Constraint (6) and (7) demonstrates that the first and the last slot of the advertisement break is allocated only one advertisement. Constraint (8) ensures that all reserved advertisements of all advertisers are telecasted only once. That is, the same advertisement should not be telecasted more than once in the same advertisement break. Finally, constraint (9) specifies that advertisements of advertisers competing with each other will not be telecasted in the same advertisement break because some advertisers don't want their advertisements are telecasted with their competing advertisers. The proposed model is implemented in one of the biggest TV networks of Ghana. At first, we have defined a problem set with the staff of the network. As a real life implementation of the proposed models, an advertisement break of a program which will be telecasted on prime-time is specified. To predict the number of impressions for a program. In order to do this, we neglected any sampling issues that may be present in the data and assumed perfect data. We treated the number of interested viewers as fixed with a particular probability of watching the program or not. This probability can be modeled by a binomial distribution, because of the two possible values of the audience that is predicted to watch an advertisement is modeled as ex-ante of random variable X with mean μ and standard deviation σ was modeled by a Gaussian with mean μ (w) and standard deviation σ. While we allow μ to vary from week to week, we keep σ fixed for simplicity. Under the Bayesian framework, we view μ (w) as an unknown parameter which we will represent by a subjective probability distribution. We can think of μ (w) as a measure of the popularity of the show at week while the actual viewership will have an unpredictable fluctuation from week to week due to external factors. The standard deviation σ measures this inherent variability in weekly viewership. Then, we have taken the expected values of ratings as we mentioned before that we assumed all ratings for all target audience groups are known deterministically for this problem. We assume there are 24 advertisers. Some of them have only one advertisement to be telecasted and some of them have more than one advertisement. The duration of advertisements of the advertisers are given in Table 1. i denotes the advertiser and j denotes the advertisements in the following tables. Table 2 gives the specific target audience groups that advertisers want to live up to. There are 84 different target audience groups in Ghana Broadcasting Market and the forecasted ratings of all these target groups that is expected to be gained in the specified advertisement break are known. For instance, advertiser 5 wants to reach the 16th target audience group for advertisement 1. This target group shows the audiences who wants to attend a private university and its expected rating point is E[16th target group] =0.875. Moreover, advertiser 21 wants to reach the 45th target group which denotes the audience who wants buy houses and land and its expected rating point is E[45th target group] = 1.281.

Table 1.*The duration of advertisement of clients*

	Advertisements(j)			
Advertiser No, i	1	2	3	4
1	51	74	15	-
2	32	56	-	-
3	30	-	-	-
4	26	51	30	-
5	16	-	-	-
6	10	45	15	73
7	75	-	-	-
8	18	5	30	-
9	35		-	-
10	5	66	-	-
11	44	-	-	-
12	32	-	-	-
13	25	12	20	39
14	53	-	-	
15	27	-	-	-
16	45	41	28	-
17	39	-	-	-
18	14	28	13	7
19	66	16	-	-
20	15	-	-	-
21	45	18	64	-
22	9	-	-	-
23	52	33	-	-
24	11	-	-	-

Table 2. *Target audience group requests of the clients*

	Advertisements(*j*)			
Advertiser No, *i*	1	2	3	4
1	20	30	15	-
2	15	60	-	-
3	30	-	-	-
4	15	20	30	-
5	15	-	-	-
6	45	15	15	20
7	20	-	-	-
8	15	20	30	-
9	30	-	-	-
10	30	15	-	-
11	60	-	-	-
12	20	-	-	-
13	15	30	20	15
14	30	-	-	
15	15	-	-	-
16	45	20	15	-
17	20	-	-	-
18	15	30	20	20
19	30	20	-	-
20	15	-	-	-
21	45	15	20	-
22	15	-	-	-
23	30	20	-	-
24	30	-	-	-

In addition to this, upper and lower limit of the total telecasting time for a commercial break is specified as 420 and 450 seconds respectively. Finally Client 2 and 7 are competitors as well as clients 10 and 15 are also competitors. Both do not want their advertisements to be telecasted in the same advertisement break, and TV station does not take into account the relationships with these clients to satisfy their constraints.

5. COMPUTIATIONAL RESULTS

We implemented our proposed model for creating an optimal allocation of commercials given a set of orders from advertisers using GAMS software package. The results displayed in Table 3 below gives the assignment of the advertisements to the slots with total commercial break in 445 seconds. In order to assess the performance our model, we compared the results of our model to existing real life commercial break plan of the TV station. From data set, optimal allocation can be found that satisfies the top 22 adversities orders and fills all available commercial break. Moreover, the total duration of the commercial break after these assignments is 445 seconds decreasing by 120 seconds as compared to an existing real life prime time commercial break plan but the total revenue gained from this assignment increases to Gh¢ 28,675.20 as compared to current Gh¢ 25,874.38

Table 3. *Assignment of Advertisements to the Slots*

	Time $slots(s)$		
$(\boldsymbol{i},\boldsymbol{j})$	1	2	3
1,1		1	
1,3		1	
3,1		1	
4,1		1	
4,2		1	
6,2		1	
6,3			1
7,1		1	
9,1		1	
10,2		1	
11,1		1	
12,1		1	
13,4		1	
16,2		1	
16,3		1	
17,1		1	
18,1		1	
18,4	1		
20,1		1	
21,2		1	
23,2		1	
24,1		1	

As a result, advertisements of advertisers 2, 5, 8, 14, 19 and 22 are not selected to be telecasted in any break. Moreover, advertisements of advertiser 2 and advertiser 7 are not assigned to the same advertisement break as well as advertisers 10 and 15 because of competition.

6. CONCLUSION

In this paper, the problem of optimal allocating the advertising time on prime-time to advertisers subject to a number of constraint in such a way as to maximize revenue of a TV station was considered. We formulated and solved the problem in the form of a mixed integer linear programming model. From the total duration of the mathematical model of the commercial break after these assignments decreases by 27 percent as compared to the existing real life prime time commercial break plan but the total revenue gained from this assignment increases by 11 percent. The results demonstrate that the proposed mathematical model is flexible and capable of obtaining high-quality assignment for scheduling optimal television commercials in one of the biggest TV stations in Ghana. In future research we would like to explore how solve the problem using hybridization of exact methods such as the mixed integer linear programing model and heuristics or metaheuristics algorithms.

REFERENCES

[1] Brown A.R. (1969), Selling television time: An optimization problem; Computer Journal 12; 201-206.

[2] Mihiotis A.and Tsakiris I. (2004), A mathematical programming study of advertising allocation problem; Applied Mathematics and Computation 148; 373-379.

[3] Bollapragada S., Cheng H., Philips M., Garbiras M., Scholes M., Gibbs T., Humhreville M. (2002), NBCs optimization systems increase revenues and productivity; Interfaces 32(1); 47-60.

[4] Bollapragada S., Garbiras M. (2004), Scheduling commercials on broadcast television; Operations Research 52(3); 337-345.

[5] Bollapragada S., Bussieck M., Mallik S. (2004), Scheduling commercial videotapes in broadcast television; Operations Research 52(5); 679-689.

[6] Jones J.J. (2000), Incompletely Specified Combinatorial Auction: An Alternative Allocation Mechanism for Business to Business Negotiations; PhD Dissertation, University of Florida, FL.

[7] Zhang X. (2006), Mathematical models for the television advertising allocation problem; International Journal of Operational Research 1(3); 302-322.

[8] Benoist T., Bourreau E., Rottembourg B. (2007), The TV-break packing problem; European Journal of Operational Research 176; 1371-1386.

[9] Brusco M.J. (2008), Scheduling advertising slots for television; Journal of the Operational Research Society 59(10); 1373-1382.

[10] Kimms A., Muller-Bungart M. (2007), Revenue management for broadcasting commercials; International Journal of Revenue Management 1; 28-44.

[11] Pereira P.A., Fontes F.A.C.C., Fontes D.B.M.M. (2007), A decision support system for planning promotion time slots; Operations Research Proceedings; 147-152

[12] Wuang M.S., Yang C.L., Huang R.H., Chuang S.P. (2010), Scheduling of television commercials; IEEE International Conference on Industrial Engineering and Engineering Management (IEEM); Macao, 803-807.

[13] Mao J., Shi J., Wanitwattanakosol J., Watanabe Sh. (2011), An ACO-based algorithm for optimizing the revenue of TV advertisement using credit information; International Journal of Revenue Management 5(2); 109-120.