Analysis Of Students' Strategies In Solving Multiplication Problems

Ahmad Fauzan, Yulyanti Harisman, Arini

Abstract: This study aims to observe how elementary school students in Indonesiasolve multiplication problems. Four cases of multiplication were given to the student. Each case was designed with different goals that allows students to do a repeated addition, multiplication with ten, multiplication with multiple of ten, and multiplication with standard algorithms. The case given was taken from a local instructional theory that had been developed in the previous study. The development process of the local instructional theory has been published in a different place. This research was survey research with data analysis using grounded theory. Each student was given four cases that had been provided for a certain period of time. The results of student answers were collected, and grouped by giving a code based on strategies that students chose. The way of the students' choses on each case would describe detail in this paper.

Index Terms: multiplication, repeated addition, multiplication with ten, multiple of ten, standard algorithms.

1. INTRODUCTION

Multiplication is one of the fundamental areas in elementary school mathematics. Generally, students in the second grade of elementary school learn formal instruction on multiplication after they study addition and subtraction [1-3]. Sum et al in her research stated that understanding of multiplication very well is essential in elementary school mathematics because it influences student's development more on advanced mathematical concepts in multiplicative conceptual fields, for example, rate, rational number, ratio, fraction, dimensional analysis and other functional relations [4].The results of several studies showed that students have difficulties in multiplication. Armanto stated that many misconceptions occur in multiplication by students in class [5]. Students simply follow the standard algorithm multiplication given by the teacher without knowing how it happened [6]. Zhang et al stated that multiplication has become a problem from time to time in the United States. [7]. Fischbein et al also states that students had difficulty when they faced verbal problems not only in multiplication but also in division [8]. Although some students in elementary school understand the introduction to multiplication, many of them fail to comprehend it. Clark et al pointed out some basic problems faced by the students. First, students add instead of multiplying, if they cannot find out the result. Second, many cannot figure it out from one they know. Last, the students who have no big problems with computation get difficulties with the meaning of multiplication [9].Much researchhas been conducted to find the source of students' problems in multiplication. McCrink et al researched 5–7-year-old students who did not take formal schooling in multiplication and division. The result of the research pointed out that the students were able to double a represented numerical amount They did multiplication by using their core estimatednumeralscheme and did not needofficialeducation or knowledge about multiplication to make this calculation [10]. Greer in his research gave problems about multiplication to two female students. The finding of the research is the behavior of each girl was fairly consistent across the written test. Their approaches are representative of the two commonly observed patterns of behavior. The first is to rely on informal building-up methods. The second is how to deal with strategies [11]. Furthermore, Park et al did research to analyze the source of the concept of multiplication considered from two different alternative hypotheses. The first hypotheses proposes that the perception of multiplication runs aground on the thoughtful of frequent addition. The second onesuggests that repeated addition is only a calculation procedureand the understanding of multiplication takes its origins in the representation of correspondence. The research involved 6-year-old elementary students in England. They were not taught about about multiplication in school, yet they were pretested in additive and multiplicative reasoning problems. The conclusion of this study is the schema of correspondence is the source of the concept of multiplication [12].The above studies tend to use the same procedure, in which some multiplication problems are designed and tested on the subject of the study. Different from previous studies,tis research designed problems in the form of local instructional theory (LIT) using a realistic mathematics education (RME) approach. We chose to develop RME-based LIT for teaching multiplication because several researches showed that the LIT's are very powerful to build students' understanding of mathematical concepts [13],[14],[15],[16],[17],[18]. The process of developing the RME-based LIT for teaching multiplication has been described in the previous paper [19]. It was concluded in the study that the RME-based LIT reached the criteria of validity, practicality, and effectiveness..In this paper, we analyze the students' strategies in dealing with multiplication problems. Four cases are selected to represent four learning activities from LIT that have beendeveloped. The first case is to see what strategies used by who have not yet studied multiplication. This case is named with a case of repeated addition. The second case allows students to come up with a strategy of multiplying by ten. This case is designed to be greater than the first case. The third case is designedto allow students to do multiplications with multiple of ten. This case is taken from LIT for learning activities after multiplication with ten. The fourth case is designed to allow students to do multiplication using standard algorithms. Each case was given

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to 30 elementary school students to see the strategy that they chose in solving problems. The strategies chosen by students in each case are explained in detail in this paper.

2 METHOD

2.1. Description of the Study

This study was conducted to see how 30 elementary school students solved multiplication problems. Problems have been designed based on the previous studies that has proved that a Local Instructional Theory (LIT) is valid and practical [19]. Local instructional theory has been developed for two years and contains design problems presented in the form of activities with a realistic mathematics education base. Each problem was designed with different goal that allows students to do a repeated addition, multiplication with ten, multiplication with multiple of ten, and multiplication with standard algorithms. This articletakes the data from what strategy used by each student when he/shesolved multiplicative problems.

2.2. Participant

This research involved one third-grade class, consisting of thirty elementary school students, in Indonesia. Third grade class was chosen because the multiplication starts being taught in this level. They have learned how to add and subtract numbers when they were in second grade.

2.3. Task

The multiplication problem given comes from activities in the local instructional theory based on realistic which were valid and practical [19]. The form of the multiplication problem given represents the learning objectives to be achieved. The description of the problem will be presented in detail in Table 1.

TABLE 1
DESCRIPTION OF MULTIPLICATION PROBLEMS

Case 1: Repeated addition
Kirana likes to read students's storybooks. Every day, Kirana always takes the time to read books in the library. In one day, Kirana read about 8 pages. How many pages of the book did Kirana read after 15 days?
Case 2: Multiplication with ten
Aina helped her mother pour the agar into the cups that her mother had compiled. Cupswere arranged neatly as many as 23 lines. Each row consisted of 15 cups. How many cups are poured by Aina?
Case 3: Multiplication with multiple of ten
Aini's father works as a builder. Aini's father put 57 bricks in each layer of the wall. How many bricks does a wall need to contain 36 Layers?
Case 4: Multiplication with standard algorithms
Danil rides a bicycle to TPQ every day. Danil rides bicycles with 32 paddle every minute. How much did Danil paddle a bicycle in 28 minutes?

2.4. Procedure

This research was survey research. The preliminary stage was to provide the problems in Table 1 to 30 elementary school students. Next, students's work is collected as document to be analyzed.

2.5. Data Analysis

Firstly, the results of students' answers are analyzed using the rubric in Table 2 below.

TABLE 2
RATING SCALES FOR ASSESSING UNDERSTANDING OF THE PROBLEM, CHOOSING AND IMPLEMENTING A SOLUTION STRATEGY AND GETTING THE ANSWER OF MULTIPLICATION PROBLEMS

Understanding the problem	Choosing and implementing a solution strategy	Getting the answer
0, completely misinterprets the Problem	0, no attempt or completely inappropriate strategy	0, no answer, or wrong answer based on an inappropriate solution strategy
1, misinterprets part of the problem	1, makes copying or computational error; partial answer for a problem with multiple answers or answer labeled incorrectly	1, makes copying or computational error; partial answer for a problem with multiple answers or answer labeled incorrectly
2, complete understanding of the problem	2, chooses a strategy that could lead to a correct solution if implemented properly	2, obtains correct solution

Source [20] After the students' work has been been scored, it was marked by a code.. Analysis with coding aims to get the conclusions of what strategies students choose in solving problems presented in Table 1. Coding is done to describe phenomena in certain groups. Some studies that use coding analysis were [20], [21] which researched a category or group of student behavior in solving problems, and [22], [23] thataimedtodescribe the category of teacher professionalism in the learning process.

3 RESULT AND DISCUSSION

In this section, variations in student answers for each case in Table 1 will be presented in four episodes. Episode 1 shows students' mathematical representations case 1.

3.1. Episodes 1: Repeated addition

After coding out of 30 student answers for case number 1, there were two variations of student answers that were represented by Oli and Kelvin

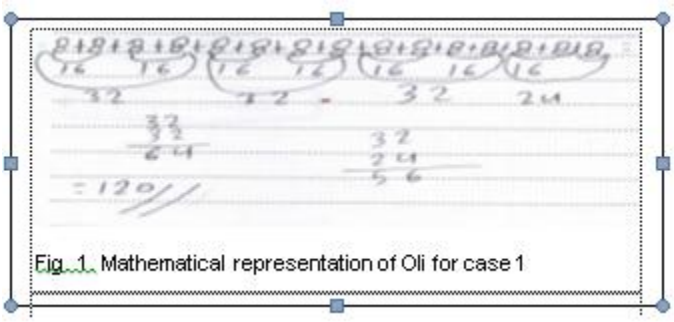

Fig. 1. Mathematical representation of Oli for case 1

Basedon the Fig 1, Oli chosea strategy by adding eight for fifteen times. Oli understood the problem completely. He chose a strategy that could lead him to a correct solution if it was implemented properly, and obtain correct solution.Like Oli, other 28 students also did the same thing, to choose a repeated addition strategy to solve the problem 1.In contrast,

Kelvin completely misinterpreted the problem. He had no attempt or completely applied inappropriate strategy. His answer was wrongbased on an inappropriate solution strategy. Kelvin just added the numbers.

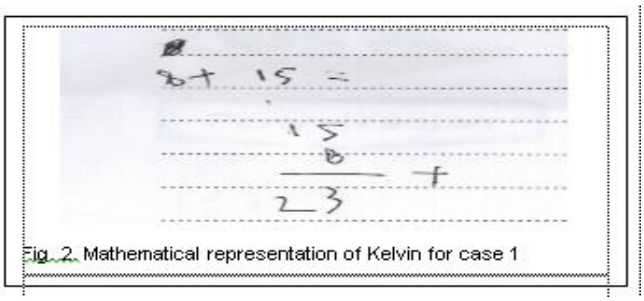

Fig. 2. Mathematical representation of Kelvin for case 1

Kelvin was the only student who chosethis strategy to solve the case number 1.

Episodes 2shows students' mathematical representations for Case 2 that is multiplication with ten.

3.2. Episode 2: Multiplication with ten.

In case number two, the variation of answers will be represented by the answer from Maharani and Ramadhani.

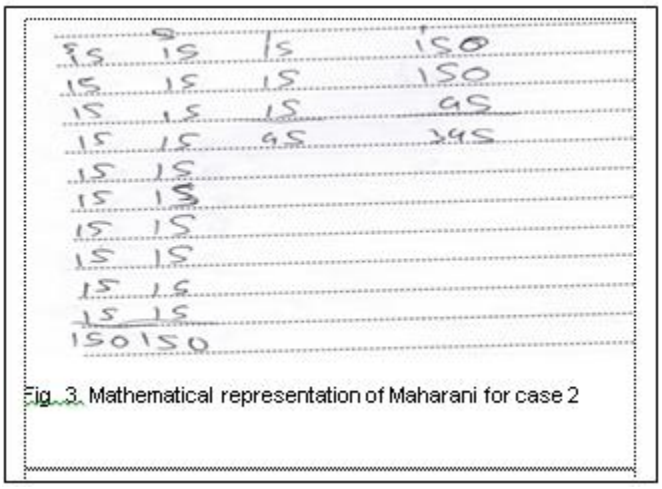

Fig. 3. Mathematical representation of Maharani for case 2

Maharani did the strategy of repeated addition to answer case number two, even though the multiplier number on the problem is quite large. However, Maharani completely resolved the problem. She chose a strategy that could take her to a right solution if implemented properly, and geta right solution. There were three other students who do the same thing with maharani.UnlikeMaharani, Ramadhani chose the strategy shown in Figure 4 below

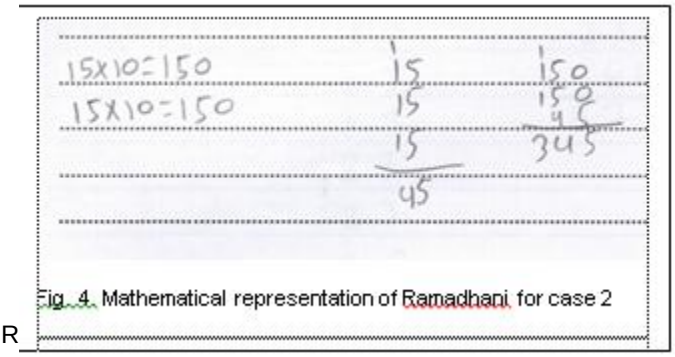

Fig. 4. Mathematical representation of Ramadhani for case 2

R is case. The strategy chosen by Ramadhani is more effective than the strategy chosen by Maharani. There were 27 other students who did the same strategy as Ramadhani did.Ramadhani also chose a strategy that could take her to a right solution if implemented properly, and geta right solution.

3.3. Episode 3: Multiplication with multiple of ten.

Variation of answers for case number 3 will be represented by the answer fromFarel, Mifta , Hasbi, and Zaki

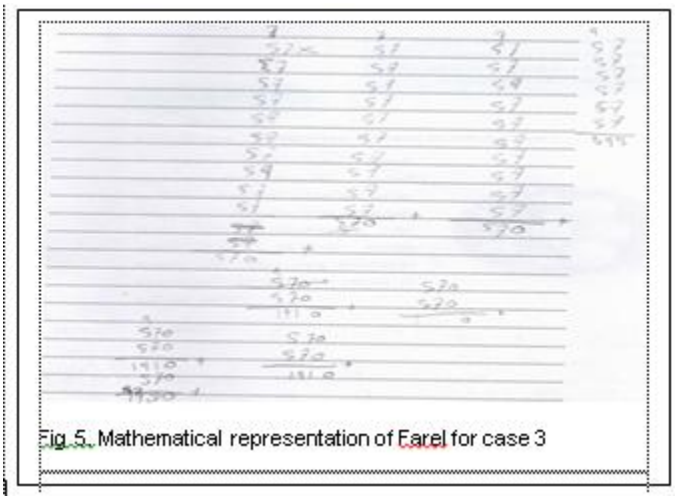

Fig. 5. Mathematical representation of Farel for case 3

Farel,togetherwith three other students, still chose the strategy of repeated additions to solve the case number three. The case number three was taken from multiplication activities with a multiple of ten on the LIT prepared to facilitate students to come up with the idea, because the numberspresented werebigger than the two previous cases. There is nothing wrong with what the students did. They chose a strategy that could take her to a right solution if implemented properly, and geta right solution.. However, this strategy is deemed ineffective if the multiplier number on the problem is large enough and students are expected to have the idea of multiplying by a multiple of tenThen Miftah's answers with 19 other students are presented in Figure 6 below.

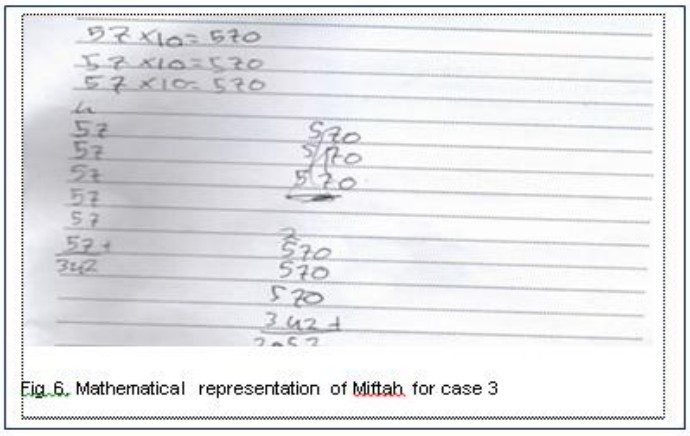

Fig. 6. Mathematical representation of Miftah for case 3

Miftahchose a strategy that could take her to a right solution if implemented properly, and geta right solution.. In This case she still used the strategy of multiplication with ten.Actuallythe students were expected to generate multiplying strategies with a multiple of ten after passing through the previous two casesFurthermore, Zaki'sandHasbi's answers are respectively presented in figures 7 and 8 below.

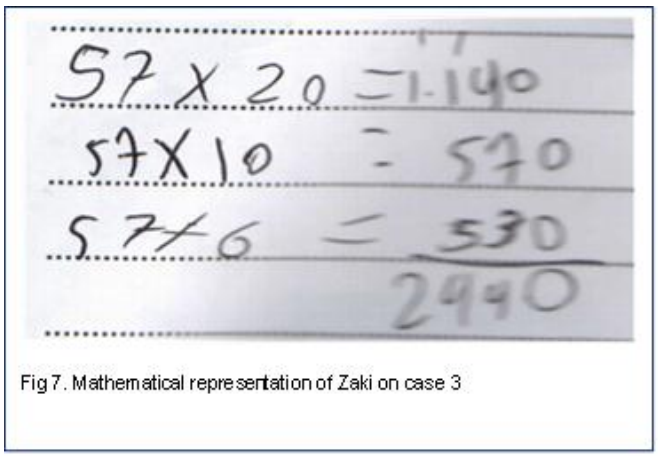

Fig 7. Mathematical representation of Zaki on case 3

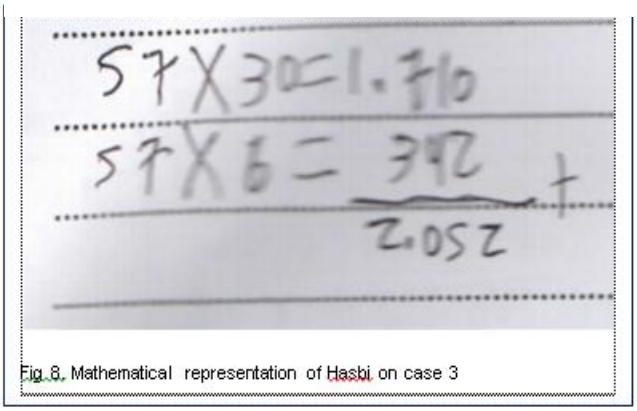

Fig 8. Mathematical representation of Hasbi on case 3

There were two students who had the same answerwith Zaki and four students with Hasbi. Both students seem to use a multiplying strategy with a multiple of ten in solving case number 3.Both of them chose the strategy that could take the to a right solution if implemented properly, and geta right solution. The difference thingbetween them is that Hasbi has reached that 30 is a multiple of ten that best meets the problem, while Zaki still separates between 20 and 10 as a multiple of ten

3.4. Episode 4: Multiplication with standard algorithms

Variation of answers for case number 4 will be represented by the answer fromRafka, Aini, kirana, and Alfino. The answer from Rafka is shown in Figure 9 below.

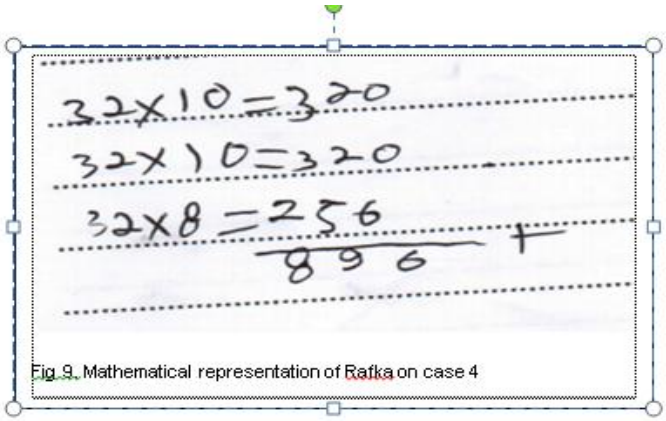

Fig 9. Mathematical representation of Rafka on case 4

Three other students had the same answer with Rafka.The case number 4 is designed in learning activity 4 at LIT to facilitate students to do multiplication using a standard algorithm after passing through the multiplication case using repeatedadditions, multiplying by ten, multiplying by a multiple of ten. However, Rafka still used multiplying with ten strategy to solve that problem. Rafkachose a strategy that could take her to a right solution if implemented properly, and geta right solution.The next the variation of the answersof case number 4 will be represented by Aini's answer which is shown in Figure 10 below

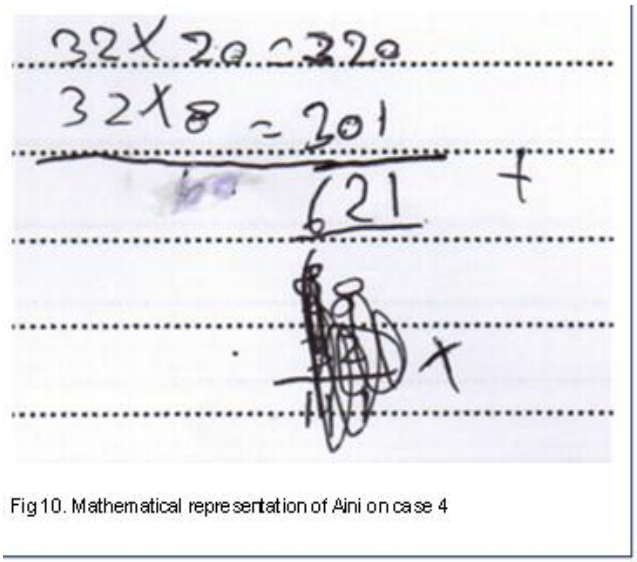

Fig 10. Mathematical representation of Aini on case 4

Three other students had the same answer withAini. Aini chose a multiplying strategy with a multiple of 10 in this case. There is nothing wrong with Aini's answer, but in this activity a strategy using a standard algorithm is expected to be able to be raised by students. Ainichose a strategy that could take her to a right solution if implemented properly, and geta right solution.Furthermore, the next variation of answers will be shown by Kirana and Alvino's answers, respectively, which are presented in Figures 11 and 12 below.

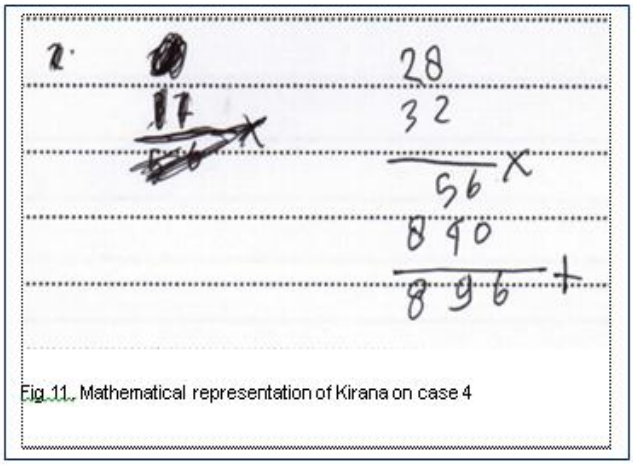

Fig 11. Mathematical representation of Kirana on case 4

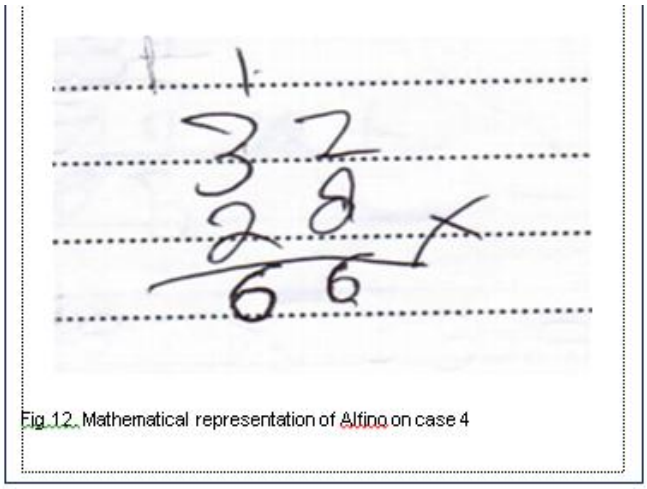

Fig 12. Mathematical representation of Alfino on case 4

There were 8 students who had the same answer withKirana and six students with Alfino. It seems that both of them have already used the multiplication strategy with a standard algorithm. Kirana did it right, but Alfino made a mistake in the multiplication process. Both of them chose a strategy that could take them to a right solution if implemented properly, and geta right solution.The findings of this study are in line with several research findings. Fisrt, the way Kelvin answers the case numbe one is very similar to the characteristics of the students Shohefield revealed it in his research. Schoenfeld's research finding indicatesthat approximately 60% of a hundred students' way when they are given math problems is by reading the problem and answering them in a hurry with careless, without analyzing the problem[24]. The students tend to add the numbers given. Kaur &Blane, Laster, and Schoenfeld also say that if a student relies on mathematics that justimplicates numbers and operations, then manipulation of the numbers in a problem mayensue with little or no thought to the problem itself [25 - 27]. Muir's findings also state that there are three student behaviors in solving problems, namely "naïve", "routine" and "sophisticated". Muir articulatesthat students who have low ability are categorized into naive problem solvers. This category refers to behavior that demonstrates consistency in manipulating numbers and applies one or two strategies to resolve variations in problems. Next, the routine problem solvers are students who are persistent with their adopted strategies, even when they do not work. Last, the sophisticated problem solver are students who can generate their own strategies [20]. Refering to the Muir's theory of behaviour above [20], the way Kelvin responds the case number one belongs to naive behaviour. Students who consistently do repeated additions in each case refer to routine students. Meanwhile, students who are able to do multiplying strategies with ten in case number two, multiplying with principles of ten in case number three, and using lag algorithms in standard in case number four were students called sophisticated students.

4 CONCLUSION

There are three types of students found in this study, namely: students who do not understand the multiplication problem, students who alwaysdothe same way in each case such as always do the strategy of repeated additions for each case, and students who are able to generate more effective strategies and solve each case.This research, however, does not conduct further interviews with students to figure out why they do the same strategy for each case. Therefore, this deficiency can be used as a further research idea by studying the factors that make students understanding in multiplication low or high. .One of the factor can be from a teacher.Harisman [28] said that teachers who are professional in learning will produce students who are sophisticated in their learning achievements and vice versa

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