Applying Piaget's Adaptive Idea In Teaching Mathematics: A Teaching Model

Nguyen Phu Loc

Abstract— Piaget's theory of cognitive constructivism explains the cognitive development process as an adaptive process including assimilation and accommodation. In this paper, we clarify what constructivism and cognitive constructivism are; specially clarify the concepts related to the adaptive process in learning. On that basis, we propose a teaching process on the basis of Piaget's adaptive thesis and conduct experimental teaching as an illustration.

Index Term - accommodation, adaptation, adaptation -based teaching model, ABT model assimilation, constructivism, cognitive constructivism.

1 INTRODUCTION

In the trend of modern teaching, educators have paid attention to promote students' activeness in the teaching process. According to Piaget, the originator of cognitive constructivism, this theory holds that when developing knowledge students undergo a process called adaptation. The thesis of adaptation is based on the basic concepts: cognitive diagrams, assimilation, accommodation, adaptation, cognitive balance. The difference between cognitive and social constructivism is that cognitive constructivism emphasizes exploration and discovery by the learner and social constructivism emphasizes collaborative efforts of groups of learners (Wilhelmsen, Åsmul & Meistad, 1998). To take advantage of Piaget's point of view in teaching mathematics we propose an adaptation – based teaching model and conduct experimental teaching as a case study.

2 THEORETICAL BACKGROUND

2.1. Definition of constructivism

Definition 1: constructivism assumes that learning is a positive process and that people gain knowledge and understanding through a combination of experience and ideas (Cambridge Dictionary)

Definition 2: Constructivist theory is a theory that considers learning to be a positive process, where learners build and construct (internalize) new concepts, ideas and knowledge based on their present and past experiences. Learning is not a passive acquisition of learned knowledge, but a process whereby learners create their own knowledge (A Brief Critical dictionary of Education)

From the above two definitions, we can infer that constructivist learning is associated with the learning process and the acquisition of new knowledge; in other words, in constructivist learning, the journey is as important as the destination.

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2.2. Cognitive constructivism and adaptation

Cognitive constructivism is researched and developed by Jean Piaget. Theory describes how learners develop their cognition. According to Piaget (2013), people cannot be provided with information, in which they immediately understand and use. Instead, learners must build their own knowledge through a cognitive process. The cognitive development explained by Piaget is based on the following basic concepts (see Figure 1):

- *Cognitive schema*: Cognitive schema is a cognitive structure (mental structure) or in other words, it is a category of knowledge that helps us explain and understand things. and phenomena in the objective world. Each human being has a cognitive schema system - this is their own understanding.

- *Assimilation:* when individuals assimilate new information, they apply an existing cognitive schema without changing that schema into a new cognitive object (or action); or use a known concept and apply it to a new one (Piaget, 1953).

- *Accommodation*: Adjust; replace the inherent cognitive schema as a result of receiving new information or new experiences (Piaget, 1953).

The process of including assimilation and accommodation is called adaptation - the process by which subject's cognition changes to meet the requirements of the situation or environment.

- *Balance (equilibrium)*: In the process of cognitive development, there is always an imbalance due to the conflict between the inherent cognitive schema and the new cognitive object (or action); conceptual change occurs when learners are confronted with information that contradicts their conceptualizations (Jonassen, 2006). The state of imbalance can only be resolved and reestablished when a new way of thinking is applied to change the existing cognitive schema. The process of cognitive development, according to Piaget, is essentially the process of reestablishing balance through anabolic and adaptive processes (Piaget, 1953)).

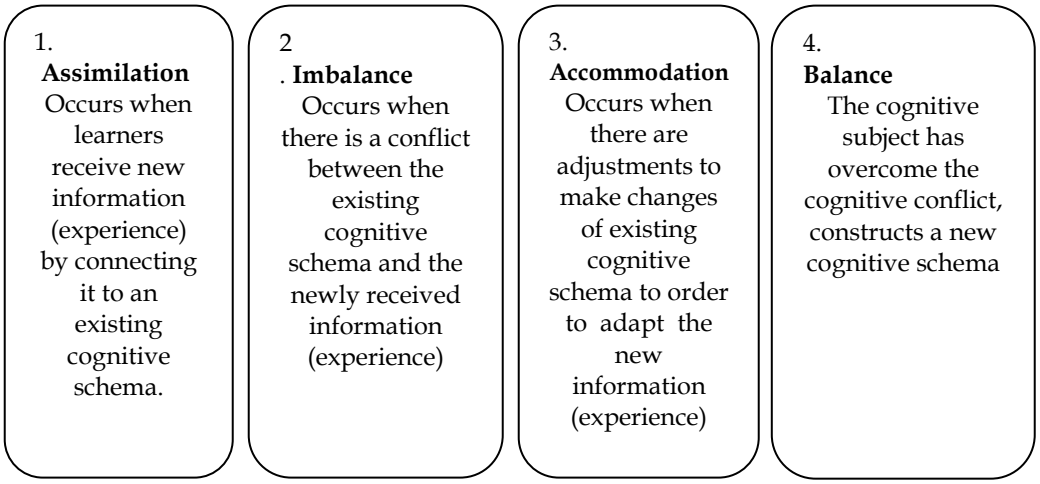

Figure 1: Cognitive development: Piaget's the point of view

The Figure 1 could be explained that cognitive development consists of four basic processes: assimilation, imbalance, accommodation and balance. Assimilation is part of the adaptation, which involves incorporating new information into the existing schema. Accommodation is another part of adaptation, which involves changing the schema to "fit" into the new information; assimilation is essentially a process of restoring some characteristics of the cognitive object, putting it into existing schemas. In accommodation, the subject is forced to change his old schema to suit the new information; adaptation is the process of adapting the subject to the requirements of the environment, by replicating the characteristics of the object into the existing one, thereby transforming the existing schema, creating a new schema, resulting in a state of balance between the subject and the environment. Thus, assimilation does not change perception but only broadens the known, and the accommodation is to change cognitive schema. Balance is the self-equilibrium of the subject after adaptive processes. When a student is exposed to new information that does not match existing knowledge, an imbalance will begin to appear until there is adaptation (assimilation and adaptation) to new information; then there is a rebalance and a newly cognitive schema is developed.

2.3 Adaptation – based teaching model

In order to use cognitive constructivism into teaching, several authors suggested different ideas. According to Claudia J. Stanny [8], elements of cognitive constructivism to design effective learning activities include: activate prior knowledge; create surprise; apply and evaluate the new knowledge; include a closing reflective assignment. Hartle et al (2012) suggested four criteria to identify and assess constructivism as follows: 1) eliciting prior knowledge, 2) creating cognitive dissonance, 3) applying new knowledge with feedback, and 4) reflecting on learning (metacognition). Any given activity or lesson plan can be considered more or less constructivist depending on how many, and to what extent, the four criteria have been incorporated (see Table 1).

Table 1. Field Guide to Constructivist Teaching and Learning (Hartle et al, 2012)

Four Essential Criteria	**Field Marks: Expected methods & learning activities**
1. Eliciting prior knowledge	Demonstration, problems, focused listing, surveys, quizzes, interviews, discussions, concept mapping. Emphasis on eliciting student ideas.
2. Creating cognitive dissonance	Uncover misconceptions, compare lists, discuss missing information, demonstration, and create discomfort. Pose a controversial question, or state/write a surprising or counterintuitive statement
3. Application of new knowledge with feedback	Formative assessments, feedback on new constructs, hypothesis testing, gain of new knowledge. Focus on process of gaining new knowledge, solving problems, design & logic of analysis and presentations.
4. Metacognition (reflection on learning)	Repeat Step 1 and have students reflect on their own learning. Assignments should have students explain variables, processes, or derive conclusions from evidence. Reflective paper, presentation, field report, peer teaching

Based on the adaptive concept of Piaget, we propose a basic teaching model (called adaptation – based teaching, ABT) as follows (see Table 2):

Table 2: A adaptation - based teaching model (ABT model)

Teaching Process	**Teaching activity**
Phase 1: Recalling relevant known knowledge.	Teacher puts forward activities to help students recall known knowledge relevant to new knowledge which will be learned

Phase 2: Inputting information (or an action request) and assimilating	Teacher inputs information (or gives action request); students perform the process of assimilation with inherent cognitive schema.
Phase 3: Creating cognitive conflict.	Teacher creates cognitive imbalance for students by giving situations of cognitive conflict between known and unknown.
Phase 4: Accommodating	Teacher organizes learning activities to help students implement the adaptation process to develop a new cognitive map, reestablish the cognitive balance.
Phase 5: Testing and evaluating	Teacher checks and re-evaluates the "learning" of students
Phase 6: Correcting learning knowledge	Teacher corrects the knowledge they need to learn

3 A PEDAGOGICAL EXPERIMENT

3.1 The purpose of experiment

The tryout is aimed to field test the feasibility and effectiveness of application of the adaptation – based teaching model in teaching a mathematical concept.

3.2. The teaching content

In this case is the concept of directional vector of a straight line in plane (Geometry 10, Geometry 10- Advanced) (see [2], [3]).

3.3. Designing the lesson

Learning objectives

After finishing lesson, the students are able to define the directional vector of a straight line, identify whether or not a vector to be the directional vector of a straight line.

Teaching method

- Using the adaptation – based teaching model.
- Students' learning activities: students worked in groups.

Teaching facilities

- Projector (Projector).
- Learning slips: 2 slips for supporting knowledge construction (1.1, 1.2) and 1 testing slip (1.3).

Designing lesson

Phase 1: Recalling relevant known knowledge

Vector concept; base of the vector; vector length; definition of two vectors in the same direction, conditions for the two vectors to be in the same direction; coordinates of two vectors of the same direction; ways to determine a line.

Using the projector to give students quick answers as follows:

Question 1: Connect a corresponding row of column A and a corresponding row of column B to get a correct statement.

A	B
1. Vector is	Their bases are parallel or coincident
2. The base of a vector is	a) The straight line goes through the starting point and the end point
3. The length of a vector	b) A directed straight line segment
4. Two vector is called parallel if	c) The distance between the starting point and the end point.
5. The necessary and sufficient conditions for the two vectors $\vec{a}$ and $\vec{b} \neq \vec{0}$ to be parallel are	d) There exists a real number k such that $\vec{a} = k\vec{b}$ ($k \in \mathbb{R}$)
6. Vector $\vec{0}$	e) Is parallel to and has the same direction as any vector

Comment:

+ Relevant known knowledge of learners is very important for them to learn new one, is a basic element in teaching according to the approach of constructivism, students are mobilizing old knowledge in phase 1 to give them a ready position in learning, and at the same time old knowledge is also a premise to create new knowledge.

+ Students' previous cognitive schemas on the base of a vector only include: the base of a vector; vectors whose bases are parallel to or coincident with each other are called in the same direction. To develop a cognitive schema of directional vector, it is required that students connect the two cognitive schemas above.

Phase 2: Inputting information (or an action request) and assimilating

Question 2: Observing the following figure 2 and giving comments on the base of vectors with straight lines (a).

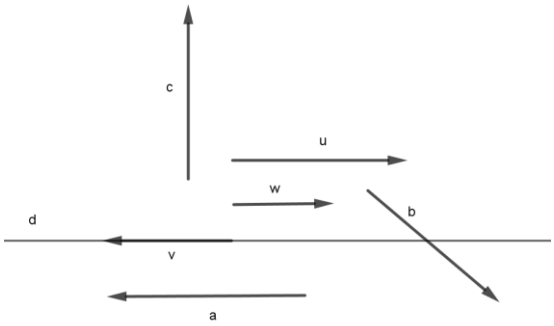

Figure 3: Finding directional vectors

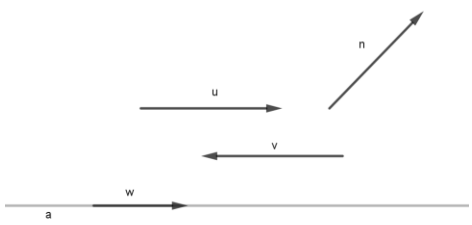

Figure 2: Observing the bases of vectors

The expected answer: the bases of vectors u, v are parallel to the line (a); the base of the vector w coincides with the line (a); the base of a vector n cuts (a).

Phase 3: Creating cognitive conflict

Teacher: In the case of the base of vectors parallel to the straight line (a) or the base of the vector coincides with the line (a) then we say the vectors are called the vectors of the direction of the line (a).

With the statement of the teacher will create students cognitive imbalance. For students, the concept of the directional vector of a straight line is a completely new to them. Cognitive imbalance (cognitive conflict) happens to students by two questions in their minds: what is a directional vector of a straight line? under which conditions a vector is called the directional vector of a straight line?

Teacher asks students to discuss groups of 6 students answer question 3.

Question 3:

Give a vector $\vec{u}$ and point M. Find the conditions of the vector $\vec{u}$ to determine that there is only one straight line (d) passing through M and getting $\vec{u}$ as a directional vector.

Expected answer: The base of $\vec{u}$ is parallel to or coincident with (d).

Comment: The question 3 is used as means of leading to Phase 4.

Phase 4: Accommodating

Question 4: Under which conditions is $\vec{u}$ called a direction vector of the straight line (d)?

Students discuss in groups to answer the above question.

Teacher asks the representatives of student groups to speak.

Teacher corrects students' errors; finally he states definition and give examples and non-examples for students to inculcate concepts.

Comment: In phase 4, students share their opinions in group discussion activities and present their ideas in front of the class and the teacher corrects their misconception. Students will construct a complete and accurate concept. At this time students self-adjust, change perception, and build a complete new cognitive schema. Through question 4, students can practice and apply new cognitive schemas to solve problems.

Question 5: ***Find the directional vector s of the line (d) in the following figure 3:***

Teacher: How many vectors are directional vectors of a straight line?

Expected answer: a line has an infinite number of directional vectors.

Teacher: ask students to comment on the directions of the directional vectors of the straight line (d).

Expected answer: The vectors that are directional vectors of the line (d) are in the same direction.

Teacher: Given a vector and a point M, how many lines can we draw through the point M and get the vector as a directional vector?

Expected answer: unique. A straight line is completely defined if it passes a given point and gets any directional vector.

Students answer questions 3, 4 and 5 above on Learning slip 1.2

Comment:

In phase 4, students state their opinions in groups. After discussing in groups, they present and discuss in front of class. With the support of teacher, students construct a complete and accurate concept. At this time students self-adjust, change perception, and build a complete new cognitive schema- schema of directional vector of a straight line. Through question 4, students can practice and apply new cognitive schemas to solve problems.

Phase 5: Testing and evaluating

Students answer the following 7 test questions in Learning Slip 1.3

Comment:

+ Phase 5, students continue to be consolidated, there are adjustments (if the new cognitive schema is not accurate) to help students accurate knowledge, engrave knowledge effectively.

Phase 6: Correcting learning knowledge

After Phase 5, the teacher identifies errors of students, and gives them corrections in order to help them construct new knowledge exactly.

Implementing the lesson and discussing

Student object

Grade 10A11 students (taught according to the basic program) of Thanh Dong High School, Tan Hiep District, Kien Giang Province; Number of students: 39

Teaching time: February, Week 26 of Semester 2, academic year 2018 – 2019; starting from 13:35 to 13:55 (20 minutes)

Implementing the lesson: N. T. A. H. (Thanh Dong High School, Kien Giang Province – Vietnam) implemented the lesson after she learned how the adaptation - based learning worked

Class organization:

In Phase 1, 2, 3, 4: The class was organized into 6 groups: 3 groups of 6; 3 groups of 7: all students did the process of assimilation and accommodation.

In Phase 5: The class was divided into 18 groups 2 and 1 group of 3 in order to evaluate what they learned in a better way. (see Table 3)

Table 3: The organization of teaching process

Phases of teaching	*Learning slip*	*Group*	Students' activities
Phase1, 2	Slip 1.1	3 groups of 6 and 3 groups of 7	Doing exercises in group, writing answers in learning slip
Phase 3, 4	Slip 1.2	3 groups of	Doing

		6 and 3 groups of 7	exercises in group, writing answers in learning slip
Phase 5 :	Slip 1.3	18 groups of 2 and 1 group of 3	Doing exercises in group, writing answers in learning slip

3.3 Students' learning results

Students' results are shown in the Learning slip 1.1

Table 4: Recording student test results in learning slip 1.1

Questions	Students' the result of answer
Question 1:	All of groups gave right answers Column A: 1, 2, 3, 4, 5, 6 — Column B: a, b, c, d, e, f
Question 2:	Group 1, 2, 4, 5 The bases of $\vec{v}, \vec{u}$ are parallel to the straight line (a); the base of w coincides with (a); the base of n cuts the straight line (a). Group 3. - The base of $\vec{v}$ is parallel to (a). - The base of $\vec{u}$ is parallel to (a). - The base of w coincides with (a). - The base of $\vec{n}$ cuts (a). **Group 6.** - The bases of $\vec{u}, \vec{v}$ are parallel to (a). - The base of $\vec{w}$ coincides with (a). - The base of $\vec{n}$ is not parallel to (a).

Table 5. Statistics of the results of answering the questions in Learning Slip 1.1,

Learning Slip 1.1		True answer	False answer
Question 1	The number of group	6 (100%)	0 (0%)
Question 2	The number of group	5 (83.3%)	1 (16.7%)

Comment:

From Table 4 and Table 5, we have some comments as follows:

Question 1: all groups answer correctly because this is the content of knowledge they have been studying very carefully in chapter 1 - geometry 10. Moreover, in this question, the teacher allows students to connect the corresponding columns, so finding results is easier than asking students to re-state concepts.

Question 2: Most groups complete the answer well, except group 6 is mistaken for a vector that is not parallel to the straight line (a). Because this group consists of only 1 good student, 3 average students, 2 weak students. Students find the base of vectors parallel to the line (a), the base of vectors parallel to the line (a). They found that the base of the line did not coincide, not parallel to the line (a); they could not remember the relative position of the two lines in the plane.

Students' results are shown in the learning slip 1.2

in Learning slip 1.2

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	he slip and did es in group
Find the condition of vector u such that there exists only one straight line (d) passing through M and getting u to be a direction vector	Group 1, 2, 3, 4, 5. $u \neq 0$ Group 6: The base of $\vec{u}$ is parallel to or coincident with (d).
Question 4: Under which	Group 1, 2, 3, 4, 5 stated very well. u is called a directional
	Group 1, 2, 3, 4, 5, 6 The directional vectors of (d) are: u ; v; w ; a

Table 7. Statistics of the results of answering the questions in Learning Slip 1.2

Learning Slip 1.2		True answer	False answer
Question 3	The number of group	5 (83.3%)	1 (16.7%)
Question 4	The number of group	5 (83.3%)	1 (16.7%)
Question 5	The number of group	6 (100%)	0 (0%)

Comment:

Table 6 and Table 7 show that:

Question 3: Most groups had correct answer, except group 6

who got confused when reading the question. Students misunderstood that they were asked to find the condition of a vector so that this vector is the directional vector of the line (d).

Question 4: There are 5 groups that answered correctly because students have just built up for them a schema of new concept through the questions above. However, there is one group writing that the directional vector of a line must be a vector other than the vector – zero.

Question 5: The groups answered correctly because the students have just built up the schema of new concept successfully.

The results of the student's test are shown in the Learning slip 1.3

Table 8: Recording student s' works in Learning slip 1.3

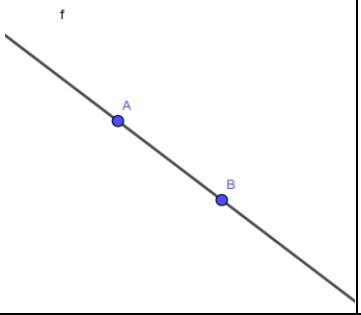
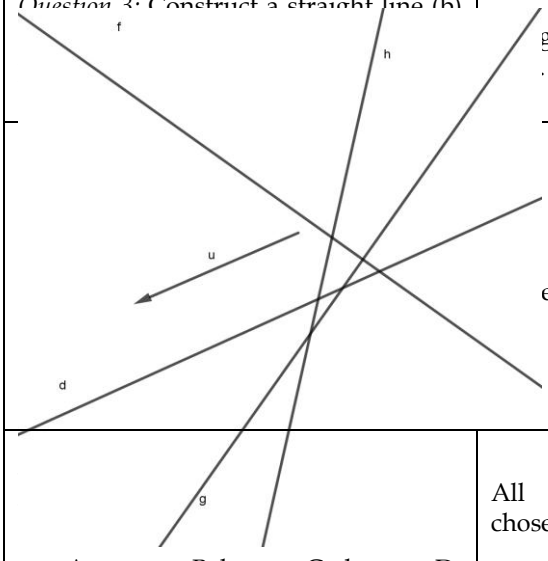
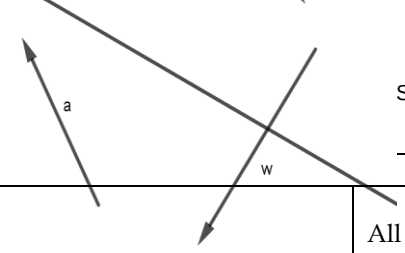
Teacher's Activities	**Students' activities and Results**
Teacher asked students for doing exercises in the Learning slip 1.3	Doing exercises
Question 1: Find direction vectors of the straight line (d)	All group answered: $a; n; u$
Question 2: Draw 2 directional vectors of the following straight line (f). 	5 groups drew $AB; BA$. 10 groups drew AB and a vector $u \neq 0$ whose base is parallel to (f). 4 groups drew 2 directional vectors whose bases are parallel to (f).
Question 3: Construct a straight line (h)  A. g B. h C. d D.	groups did . of group e D (true) All of group chose C (true)
 A. $\overrightarrow{a}$ B. $\overrightarrow{b}$ C. $\overrightarrow{v}$ D. $\overrightarrow{w}$	All of group chose B (true)
Question 7: How many directional vectors does a straight line have? A. 1 B. 2 C. 3 D. infinite	18 groups chose D (true). 1 group chose B (wrong)

Table 9. Statistics of students' answers in Learning slip1.3

Learning slip 1.3		**True answer**	**False answer**
Question 1	The number of group	19 (100%)	0 (0%)
Question 2	The number of group	19 (100%)	0 (0%)
Question 3	The number of group	19 (100%)	0 (0%)
Question 4, 5, 6	The number of group	19 (100%)	0 (0%)
Question 7	The number of group	18 (94.74%)	1 (5.26%)

Comment:

Question 1: All groups answered correctly because they have just built a conceptual schema for their concept of the vector of the line. This question only needs to apply the knowledge they have just constructed.

Question 2: All 19 groups knew how to apply the definition of the directional vector, drawing two directional vectors of the line (f). However, only 5 groups answered correctly as expectation of the teacher, they deduced that (f) passes through two points A and B, (f) has directional vectors that are vectors other than vectors – zero, the starting point and end point are two points they pass through.

Questions 3, 4, 5, 6: Groups got it right.

Question 7: 18 groups chose D (true). However, there was a group choosing B, these students were confused with question 2 (asked to draw 2 vectors indicating the directional vector), so the student chose the answer B (false).

Through Table 8 and the statistics table 9, we realized that teaching according to constructivist approach to the concept of directional vector of straight lines, most of students understood the lesson, knew how to create knowledge for themselves. However, some students still made mistakes, so teacher implemented to help students to adjust their schema.

CONCLUSION

In the process of cognitive development, building new cognitive schema, students often have mistakes or misperceptions. To help students learn new knowledge and skills effectively, if applying ABT in teaching, teachers need to fully implement six phases: the first four phases (from phase 1 to phase 4) are to help students learn new knowledge or

develop new schema; phases 5 and 6 are to detect and correct students' mistakes.

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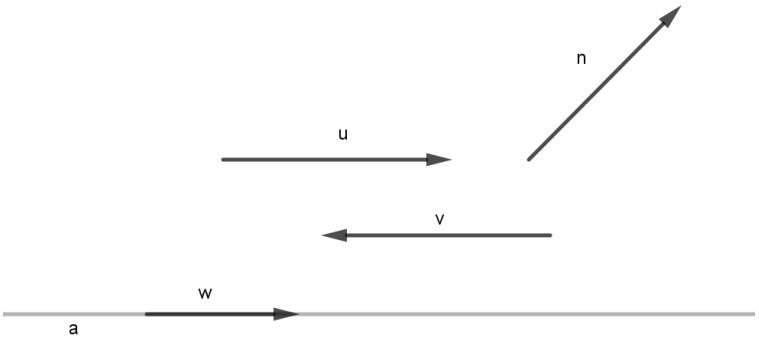
APPENDIX

Learning slip 1.1

Câu hỏi 1: Nối một dòng tương ứng của cột A và một dòng tương ứng của cột B để được một mệnh đề đúng.

A	B
1. Vectơ là	a). Giá của chúng song song hoặc trùng nhau.
2. Giá của vectơ là	a) Đường thẳng đi qua điểm đầu và điểm cuối của vectơ.
3. Độ dài của vectơ là	b) Là đoạn thẳng có hướng.
4. Hai vectơ được gọi là cùng phương nếu	c) Khoảng cách giữa điểm đầu và điểm cuối của vectơ đó.
5. Điều kiện cần và đủ để hai vectơ $\vec{a}$ cùng phương $\vec{b} \neq \vec{0}$	d) Là có một số k để $\vec{a} = k\vec{b}$ $(k \in \mathbb{R})$
6. Vectơ $\vec{0}$	e) Cùng phương cùng hướng với mọi vectơ.

Câu hỏi 2: Nhận xét về giá của các vectơ $\vec{w}, \vec{v}, \vec{u}, \vec{n}$ với đường thẳng (a).



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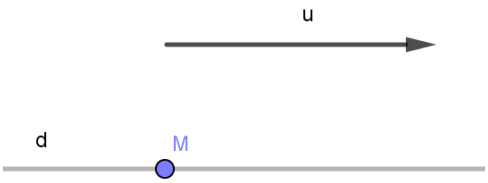
Learning slip 1.2

Câu hỏi 3: Cho một vectơ $\vec{u}$ và một điểm M.



Tìm điều kiện của vectơ $\vec{u}$ để xác định được duy nhất một đường thẳng (d) đi qua M và có vectơ chỉ phương $\vec{u}$.

Câu hỏi 4: vectơ $\vec{u}$ được gọi là vectơ chỉ phương của đường thẳng (d) khi nào?

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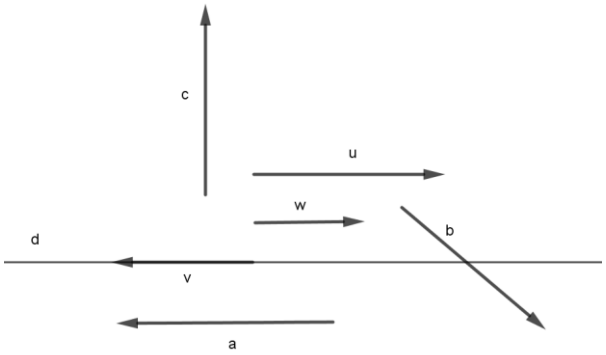
Câu hỏi 5: Tìm các vectơ chỉ phương của đường thẳng (d) trong hình vẽ sau:



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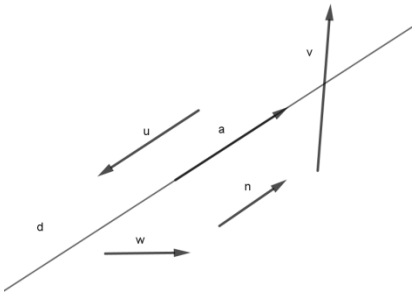
Learning slip1.3

Câu hỏi 1: Tìm các vectơ chỉ phương của đường thẳng (d)



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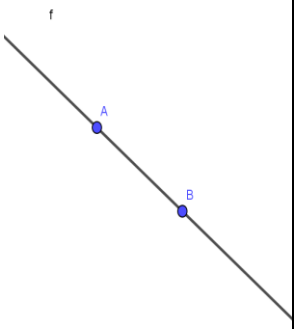
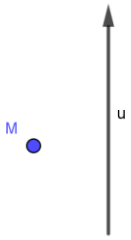
Câu hỏi 2: Vẽ hai vectơ chỉ phương của đường thẳng



(f)

Câu hỏi 3: Dựng đường thẳng (b) biết (b) đi qua M và có vectơ chỉ phương $\vec{u}$ như hình vẽ.



Câu hỏi 4: Vectơ chỉ phương của đường thẳng (a) là:

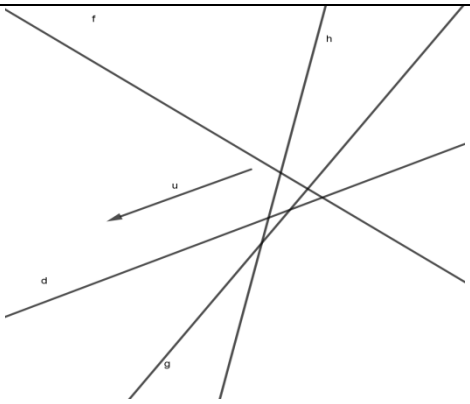
A. Vectơ – không

B. Vectơ khác vectơ – không.

C. Vectơ khác vectơ – không, và có giá cắt (a).

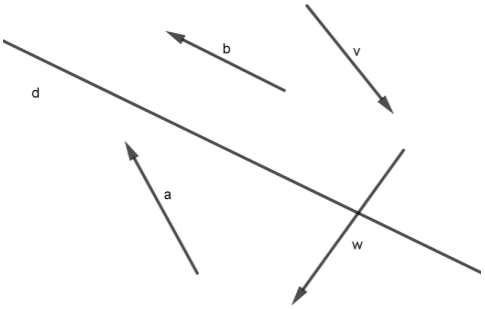
D. Vectơ khác vectơ – không và có giá song song hoặc trùng với (a).

Câu hỏi 5: Vectơ $\vec{u}$ là vectơ chỉ phương của đường thẳng nào trong hình vẽ sau:



A. g B. h C. d D. f

Câu hỏi 6: Vectơ chỉ phương của đường thẳng (d) là



A. $\vec{a}$ B. $\vec{b}$ C. $\vec{v}$ D. $\vec{w}$

Câu hỏi 7: Một đường thẳng có bao nhiêu vectơ chỉ phương?

A. 1 B. 2 C. 3 D. Vô số