Coloring Of Fuzzy Magic Graphs

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Abstract: In this paper, coloring of fuzzy magic graphs such as cycle, path, star are discussed. Also, we find the chromatic number of fuzzy magic graphs.

Index Terms: Fuzzy graph, Fuzzy magic graph, Fuzzy coloring, Fuzzy chromatic number, α-cut, Fuzzy magic cycle, Fuzzy magic path.

1 INTRODUCTION

In the present technological world, Graphs constitute one of the major sources of information that can effectively elucidate the relation between two objects. The significance of graphs in the current scenario can be seen in practically varying backgrounds such as craft scheduling, computer network and also in automatic channel allocation. The ambiguity in most of the world wide problems formulates a void with which it is hard to examine its consequences for analysis either in its description or in its relationship among the factors. This structures the need for the introduction of fuzzy graph theory. In reality, fuzzy graph colouring outlines the central core problem in the fuzzy graph theory and the striking and most prominent fact about it occur with its practical application in the field of combinatorial optimization such as traffic light control and in examinations. Almost every graph has two kinds of coloring, for example, vertex colouring and edge colouring.

DEFINITION

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The \( \alpha \)-cut of fuzzy magic graph defined as \( G_\alpha=(V_\alpha, E_\alpha) \) where \( V_\alpha=\{v\in V/\sigma\geq\alpha\} \) and \( E_\alpha=\{e\in E/\mu\geq\alpha\} \).

THEOREM

If G is a fuzzy magic path of n vertices, \( \chi'_{fm}(G) = 2 \).

Proof

Let G be a fuzzy magic graph such that \( G^* \) is a fuzzy magic path of n vertices. Clearly every edge of G is not strong. Since \( G^* \) is a fuzzy magic graph. So we can give color R and W alternatively to the edges of G. This is a proper fuzzy edge coloring.

Hence \( \chi'_{fm}(G) = 2 \).

ILLUSTRATION

\[ \chi'_{fm}(P_4) = 2 \]

THEOREM

If G is a fuzzy magic cycle of length n, then \( \chi'_{fm}(G) = 3 \), if n is odd.

Proof

Let G be a fuzzy magic cycle graph of odd length and \( e_1, e_2, \ldots, e_{2m+1} \) are edges of G. Now give color 1 to \( e_1, e_3, \ldots, e_{2m+1} \) and 2 to \( e_2, e_4, \ldots, e_{2m} \). All edges of G are colored except \( e_{2m+1} \). This coloring is a proper fuzzy magic edge coloring. Since \( e_{2m+1} \) is strong adjacent to \( e_1 \) which is to 1 and \( e_{2m} \) which is colored as 2, we cannot give color 1 or 2 to \( e_{2m+1} \). So assign another color 3 to \( e_{2m+1} \). Thus \( \chi'_{fm}(G) = 3 \).

ILLUSTRATION

\[ \chi'_{fm}(C_5) = 3 \]

THEOREM

If the fuzzy magic graph G=(\( \sigma, \mu \)) is odd irregular fuzzy magic
graph then each vertex has distinct color.

**Proof:**
Let G=(σ, μ) be a fuzzy magic graph. If d(v)≠k for all v∈V, Since G is a fuzzy magic graph.
Therefore, each vertex has distinct degree, then G is irregular fuzzy magic graph.
Let V = {v₁, v₂, ..., vₙ} be a vertex set of G.
To prove: All the vertices have distinct color.

\[
\chi_{fm}(G) = \chi_{fm}(G^c)
\]

Hence the Proof.

**THEOREM**
If G is a fuzzy magic cycle graph, then
\[
\chi_{fm}(G) ≠ \chi_{fm}(G^c)
\]

**Proof:**
Let G be a fuzzy magic cycle graph. Then all edges of G are not strong edge. Now construct a complement of a fuzzy magic graph G^c: (σ^c, μ^c) such that each vertex is adjacent to every other vertex. Clearly G^c is a complete graph.[The fuzzy magic edge chromatic number of a fuzzy magic complete graph on n vertices is n+1, if n is odd ].

Hence \[
\chi_{fm}(G) ≠ \chi_{fm}(G^c)
\]

**ILLUSTRATION**
\[
\chi_{fm}(C_5) = 3 \quad \chi_{fm}(C_5^c) = 6
\]
\[
\therefore \chi_{fm}(C_5) ≠ \chi_{fm}(C_5^c)
\]

Remark: The above is not true for n=3.

**ILLUSTRATION**

\[
\chi_{fm}(K_{1,3}) = 3
\]

**2. CHROMATIC NUMBER OF MIDDLE FUZZY MAGIC GRAPHS**
Let Cₙ be a fuzzy magic cycle graph of length n ≥ 3 vertices. Then \[
\chi_{fm}(M(C_n)) = 4.
\]

**Proof**
Let Cₙ be a fuzzy magic cycle of length n. Assume that v₁,v₂,...,vₙ are vertices and e₁,e₂,...,eₙ are edges of Cₙ. Since Cₙ is a fuzzy magic cycle, each vertex is adjacent to vᵢ₊₁ and vᵢ₋₁ (1≤ i ≤ n, v₀ = vₙ and vₙ₊₁ = v₁). By the definition of middle fuzzy magic graph, each edge in M(Cₙ) is strong. The vertices of M(Cₙ) are v₁,v₂,...,vₙ, e₁,e₂,...,eₙ and two vertices in M(Cₙ) are adjacent if any one of the following condition is satisfied.

The two vertices are in E, and they are adjacent. One of the vertices u is in V and another vertex e is in E and u lies on e. Now we have to determine the fuzzy magic chromatic number M(Cₙ) which is the minimum number of colors to color all the vertices of M(Cₙ) such that two adjacent vertices will receive different colors.

To find \[
\chi_{fm}(M(C_n))
\]
It is enough to find the minimum number of fuzzy magic independent sets whose union is $V \cup E$ and intersection is empty. Since the vertex set $\{v_1, v_2, \ldots, v_n\}$ of $C_n$ from a fuzzy independent set of $M(C_n)$, assign color 1 to all vertices of this set. The remaining uncolored vertices of $M(C_n)$ are $e_1, e_2, \ldots, e_n$.

If $n$ is odd, $E$ can be partitioned into three fuzzy magic independent sets. They are $\{e_1, e_3, \ldots, e_{n-2}\}$, $\{e_2, e_4, \ldots, e_{n-1}\}$ and $\{e_n\}$.

Hence $\chi_{fm}(M(C_n)) = 4$

**THEOREM**
Let $S_{1,n}$ be a fuzzy magic star graph of length $n \geq 3$ vertices. Then $\chi_{fm}(M(S_{1,n})) = 4$.

### 3. COLORING FUNCTION OF FUZZY MAGIC GRAPH

**DEFINITION**
Given a fuzzy magic graph $G=(V, \sigma, \mu)$, its chromatic number is a fuzzy magic number $\chi(G)=\{(X_\alpha, \alpha)\}$ where $X_\alpha$ is the chromatic number of $G_\alpha$ and the values are the different membership value of vertex and edge of graph $G$. We find all graph $G_\alpha$ which is a crisp graph for all $\alpha$. Then find minimum number of color needed to color the graph $G_\alpha$. In such a way, we find the fuzzy magic chromatic number which is a fuzzy magic number is calculated by its $\alpha$-cut.

**ILLUSTRATION**
Let us take a fuzzy magic graph $G=(V, \sigma, \mu)$ where $V$ has five vertices and membership value of those vertices are $\sigma=(0.08, 0.10, 0.07, 0.09, 0.06)$. The graph consists of 5 edges $e_1, e_2, e_3, e_4, e_5$ and membership values are $0.1 \ 0.02 \ 0.03, 0.04, 0.05$.
REFERENCES