Influence Of Components With Small Inertia On The Torsional Vibrations Of A Crankshaft

R. Nisarg, Ashish Paygude, G.S. Vijay

Abstract : In a four stroke internal combustion engine significant values of torque are present in several orders of the speed of the engine causing torsional vibrations of the crankshaft. The shaft is designed to have its fundamental torsional mode frequency to be very much higher than the speed of the engine. When a component of insignificant moment of inertia is attached to the crankshaft, it is normally considered that its influence on the dynamics is negligible. It is shown in this work that even with a small variation in the moment of inertia or the speed of the engine can cause significant torsional vibrations under certain conditions.

Index Terms : Crankshaft, Dynamic response, Fourier transform, Torsional vibration

1 INTRODUCTION

One of the important aspects to be considered in designing a crankshaft is the torsional deformations which are manifested as shear stresses in the shaft. These torsional deformations are due to the torque developed by the engine to perform the external work. In a four stroke internal combustion engine the toque is developed only in one stroke and hence there is a significant variation in torque over a cycle. These variations are reduced to a large extent by employing a flywheel and having multi-cylinder engines with different firing order. Still there exist, though small, components of torque at different frequencies and these torques cause torsional vibrations of the shaft.Analyzing for its torsional vibrations is an important step towards a proper design of the crankshaft. Predicting and also methods to reduce torsional vibration is a subject of research. Basavaraj et. al. [1] have presented a review of the vibration analysis of crankshaft. Methods of reducing the torsional vibrations have been discussed by Liming Sun [2]. It is to be noted that though torsional vibrations are harmful and one looks for their reduction, they are also used for the detection of anomalies as pointed out in [3, 4]. Therefore, predicting the torsional deformation and their relations with the engine parameters is very much necessary.

The natural frequency of the torsional vibration of the crankshaft is dependent on the mass moment of inertia of the system [5]. Studies are also presented for non-constant moments of inertia [6] as well as non-linear dynamic system [7].In general a small change in the moment of inertia is not expected to make any significant impact on the torsional vibrations.

In practice several elements are normally mounted on to the crankshaft, like fly wheel, gears, pulleys, etc. Cooling fans are powered through a pulley attached to the crankshaft. In a particular case the cooling fan has to be mounted directly on to the crankshaft. As the moment of inertia of the fan to be attached was very small, less than $1/30^{th}$ of the moment of inertia of all other elements put together, it is natural to consider that the addition of the fan will not affect the dynamics of the crankshaft and in-turn the torsional deformations significantlyIn this work, it is shown that though the moment of inertia of the fan is insignificant compared to the moment of inertia of the system, the deformations during torsional vibrations can be significant under certain conditions. This is investigated using a mathematical model but applied to a practical example.

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2. MATHEMATICAL MODEL AND NATURAL MODES

2.1 Model of the Crankshaft

The system under consideration (crankshaft and elements attached to it) can be mathematically represented by a spring - mass model. Such models are generally used in determining torsional modes of crankshaft [2]. As the natural modes of interest are related to torsion, the spring represents torsional stiffness and the mass should be the moment of inertia. In this case the model has 9 inertias and 8 torsional springs, as shown in Fig. 1.

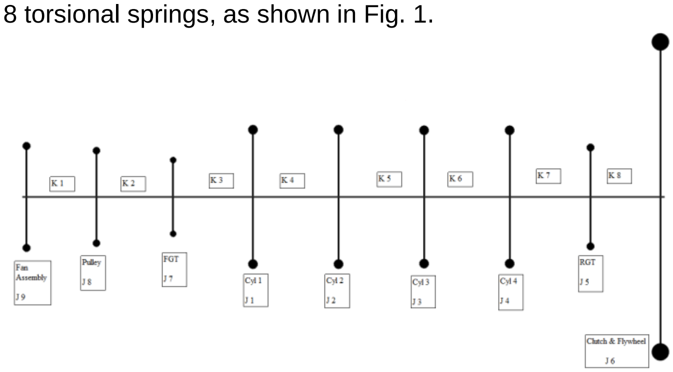

Fig. 1 *Torsional spring-mass system*

All the components that are connected to the crankshaft are considered in the model and maximum replication of the actual arrangement is incorporated. As the shapes are not regular, the moments of inertia are obtained through a CAD software.

J1 to J4 represent the moment of inertia of the moving elements in the four cylinders. Flywheel is included in the model. There are parts of the clutch assembly that are rigidly mounted onto the flywheel which are also incorporated in the model. J6 represents the moment of inertia of the flywheel and a portion of the clutch assembly. J9 represents the moment of inertia of the fan assembly.The torsional stiffness is controlled by the stiffness of the crank throw. Crank throw is a section of the crankshaft as shown in **Error! Reference source not found.**. The stiffness of the crank throw is obtained separately through a finite element model. The model is developed in ANSYS. By applying a known value of torque on a constrained section of the crank throw, the directional displacement is obtained. A typical deformed shape is shown in Fig. 2. The corresponding angular deflection is determined with the known dimensions of the crank throw and subsequently the stiffness is calculated. In a typical case application of a torque of 1 Nm results in an angular deformation of 1.94 $\times 10^{-7}$ rad. And consequently the stiffness of the element is 52 $\times 10^{5}$ Nm/rad.

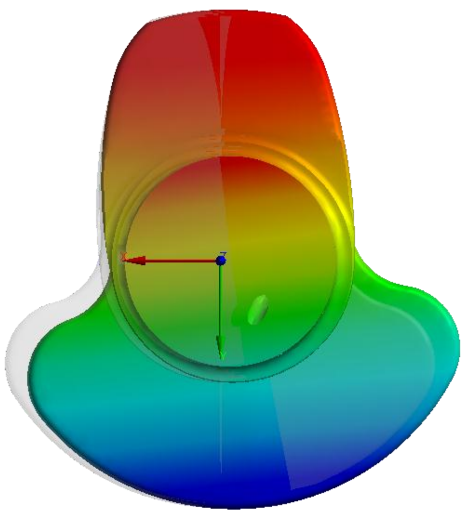

Fig. 2 *Deformation of the crank throw under unit torque*

Similar approach is used to determine the stiffness across all sections of the crankshaft. The shape of the crank throw is similar across the four cylinders. The shape at the extreme ends, at the crank nose and the flywheel section of the crankshaft are different and analysed separately and the stiffness values are obtained.

2.2 Natural Modes

Using the model described previously the natural modes of the system are computed. The model has 9 degrees-of-freedom and hence 9 natural modes. A Matlab Module is used to solve the eigen value problem. As the system is free in the rotational direction, the first mode is a rigid body motion (rotation) of the system, having zero natural frequency. The next 8 modes are elastic modes and they are listed in Table 1.

Table 1 List of modes and their frequencies

Mode	Frequency (Hz)
1	328
2	898
3	1369
4	1710
5	1924

The frequency of the fundamental torsional mode is 350 Hz if the cooling fan is not mounted. As mentioned earlier, the mass moment of inertia of the system without the cooling fan is 1.314 kg m^2. The moment of inertia of the cooling fan is 0.036 kg m^2. Thus the total moment of inertia of the system with the fan mounted is 1.350 kg m^2. One would think that as the increase in moment of inertia is only 2.7 % the expected drop in the frequency of the fundamental torsional mode would be about 1.35 %. Hence the natural frequency of the system with the cooling fan is expected to be about 345 Hz. But it is to be noted that the natural frequency is dropped to 328 Hz when the cooling fan is mounted. This is because in this mode of vibration all elements are not moving in phase, instead a few are out of phase with the rest. Therefore, even if the change in moment of inertia is very small, it can drop the natural frequency more than a proportional drop.The speed of the crank shaft is 2400 rpm which corresponds to 40 Hz. As the speed of the crankshaft is 40.0 Hz, it is natural to expect that the change in the natural frequency from 350 Hz to 328 Hz, is not expected to cause any significant increase in the deformation.

3. DISTURBANCE TORQUE

To determine the response of the system it is essential to have a mathematical model of the forcing function. In this case the torque acting on the crank shaft is the forcing function. The torque can be determined from the known values of the pressure developed in the cylinder.A typical diagram showing the variation of the pressure in the cylinder is shown in Fig.3. As it is a four – stroke engine, the forcing function repeats itself for every 720 degrees of rotation of the crankshaft and hence the variation of the pressure in the cylinder is shown over 720 degrees. This data is obtained from test results of a typical engine of the same family. As these results are confidential the value along Y-axis is an indicative number and presented in some units.

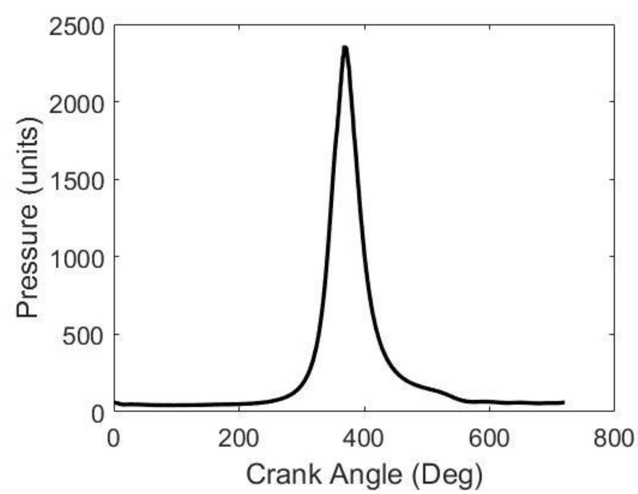

Fig. 3 *Typical Pressures developed in the cylinder*

Based on the dimensions of the cylinder, piston, connecting rod and crankshaft, the pressure developed in the cylinder is converted into torque generated. There are several other components that influence the torque. For example, the rotating inertias of the counterweights induce a torque. A combined effective torque over 720 degrees is obtained and a typical torque is shown in Fig. 4. The torque values given are only indicative, not the exact values.

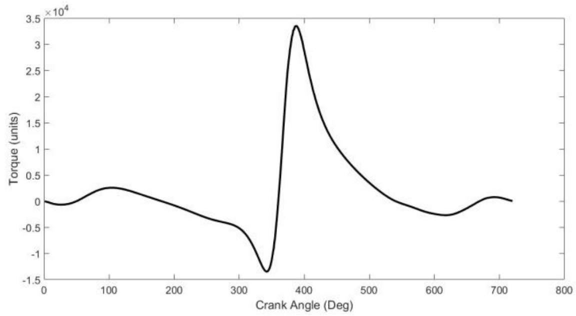

Fig. 4 *Typical Torque generated by a single cylinder*

To understand the behavior of the crankshaft system to the above forcing function, we need to convert the torque into frequency components. This is done through Fourier series. The results are shown in Fig. 5. The results show that the mean torque is about 1700 Units which is the useful torque. The torque in the third order is quite significant, 3300 Units, which is very much higher than the even the mean torque which must be considered for determining the deformations due to torsional vibrations. If any of the natural frequencies is very close to the excitation frequency, given in Fig. , it can result in significant torsional vibrations.

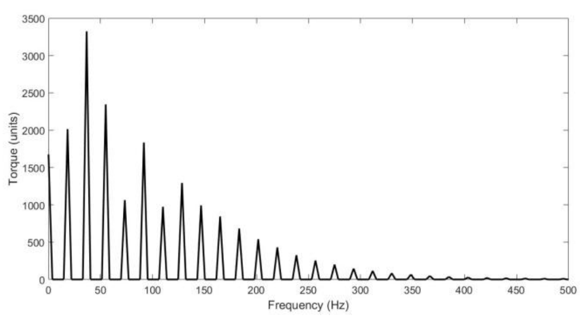

Fig. 5 *Frequency spectrum of the effective torque of a single cylinder*

The frequency spectrum shown in Fig. 5 indicates that there is a lot of variation in the torque output from a single cylinder engine. A combination of several cylinders at different firing orders across different crank angles reduces these variations. The firing order of cylinders is maintained within two revolutions of the crankshaft i.e., 720 degrees. Hence effective torque on the crankshaft of a four-cylinder engine with staggered power stroke with a specific firing order is computed and given in Fig. 6 over 720 degrees. This total torque is the forcing function.

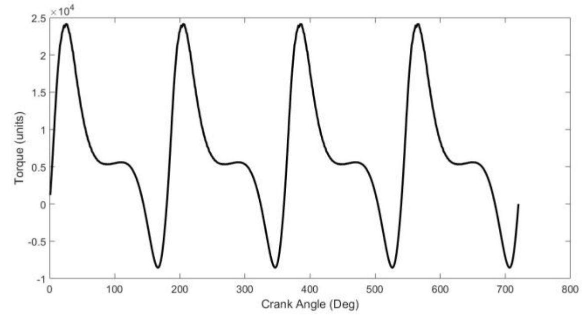

Fig. 6 *Effective torque developed by the engine*

The frequency spectrum of the torque is shown in Fig. 7. It is to be noted that the spectrum in the negative frequencies is not shown and the magnitude of various frequency components given in Fig. 7, except the mean, needs to multiplied by 2.0. The mean torque is about 6430 Units. In a four – cylinder, four – stroke diesel engine, the critical exciting orders of the forcing function are the even orders i.e., second, fourth, sixth, eighth and tenth. The second order disturbance torque is about 8160 Units at 80 Hz. The fourth order disturbance torque is 7600 Units at 160 Hz. The frequency spectrum given in Fig. 7

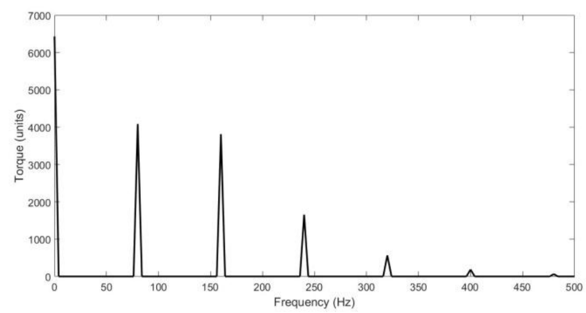

Fig. 7 *Frequency spectrum of the torque developed*

4. TORSIONAL DEFORMATIONS

Having determined the natural modes and knowing the disturbance torque, the torsional deformation can be determined using the modal superposition theorem. A modal damping factor of 0.07 is considered, which is generally used by the engine designers.

4.1 Response Estimation Technique

The differential equation of motion is,

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f(t)\} \quad (1)$$

where M is the mass matrix, K is the stiffness matrix and C is the damping matrix. The forcing function is represented by $f(t)$. Define a matrix, called modal matrix, whose columns are the modal vectors. Here the modal matrix is denoted by $[U]$. Using the transformation

$$\{x(t)\} = [U]\{q(t)\} \quad (2)$$

and pre-multiplying Eq. (1) with the transpose of the modal matrix results in uncoupled differential equations [8]

$$M_1\ddot{q_1} + C_1\dot{q_1} + K_1 q_1 = \overline{f_1} \quad (3)$$

where q is the generalised coordinate and $\overline{f(t)}$ is the generalised force. The suffix stands for the order of the mode. By defining the damping factor suitably, Eq. (3) can be made into a set of uncoupled differential equations. Dividing by the mass we get a set of following equations

$$\ddot{q_1} + 2\zeta_1\omega_1\dot{q} + {\omega_1}^2 q_1 = \overline{f_1}/M_1 \quad (4)$$

where ζ is called modal damping factor. The suffix stands for the mode order. Solving (4) the responses in natural

coordinates are obtained. The responses in physical coordinates are then determined using (2).
The torsional deformations are determined using the method described above. The results are normally discussed for different orders of the speed.

4.2 Deformations without Cooling Fan
The frequency of the fundamental torsional mode is 350 Hz. The speed of the crankshaft corresponds to 40.0 Hz which does not cause resonance with the crankshaft system. The 9th order of this speed corresponds to 360 Hz which is very close to the first natural frequency of the crankshaft. But since the magnitude of excitation is very negligible at the 9th order, there is not much significant deformation at this order. The magnitude of the 8th order of the excitation causes a deformation of 0.125 Units at the crank nose.

4.3 Deformations with Cooling Fan
The frequency of the fundamental torsional mode is 328 Hz. The speed of the crankshaft corresponds to 40.0 Hz which does not cause resonance with the crankshaft system. The 8th order of this speed corresponds to 320 Hz which is very close to the first natural frequency of the crankshaft and causes a deformation of 0.182 Units at the crank nose which is considerably more with a small addition of moment of inertia.Mounting of the fan assembly on the crankshaft reduces its natural frequency. Though this reduction may be very much smaller and the natural frequency is very much away from the speed of the engine it can cause severe torsional deformations under certain conditions like if it is close to the 8th order frequency. But if the natural frequency is reduced further below 320 Hz, the deformations would be less until the natural frequency becomes closer to the 6th order frequency.A small change in the speed also can result in significant deformations. When the speed is increased to 2500 rpm from 2400 rpm (41.6 Hz from 40 Hz), the deformation became 0.218 units, though the disturbing torque is slightly lower. One can see that a small change in the speed can result in significant deformations under certain conditions since the excitation is present in several orders of speed.

5. SUMMARY AND CONCLUSIONS
When a component of small moment of inertia is attached to the crankshaft it is normally considered that its influence on the dynamics is negligible. It is shown in this work that the reduction in the natural frequency will be larger than what is expected from a proportional drop. Even if the reduction in the natural frequency is small, it can result in significant vibrations under certain conditions as the excitation is present at several discrete frequencies and the natural frequency can become close to any one of them. Even a small change in the speed can result in significant deformations.

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