Process Thinking Of Students In Translating Representation Of Covariational Dynamic Events Problems

Sandie, Purwanto, Subanji, Erry Hidayanto

Abstract: This study examines students' thought processes in translating mathematical representations of covariational problems from graphic to pictorial forms. This research is qualitative research. The research subjects are 3rd-semester students of Mathematics Education at Universitas Negeri Malang. Students solve the problems given and the process of students in solving problems is recorded and verbalize what is thought (think aloud) and interviewed in-depth. The results showed that very few students could solve the covariational problem of dynamic events. The ability to coordinate changes from one variable value to changes in the value of other variables is still not good. Therefore, the ability to translate representations that involve covariational reasoning needs special attention.

Keywords: covariational reasoning, translation, representation, dynamic events problems.

1. INTRODUCTION

Representation is one of the five standard processes aside from problem solving, reasoning, connecting, and communicating in Principles and Standards for school mathematics (NCTM, 2000). Mathematical objects (ideas, concepts, relationships) can only be traced through representation (Duval, 2006), and mathematical activity can only be seen through representation (Gyamfi & Bosse, 2013). Therefore, representation has an important role in learning mathematics (Jinfa Cai & Lester, 2005). Representation as a standard process shows that in learning, students must be able to choose, implement, and translate representations between mathematically to solve problems. Representation helps someone in understanding and interpreting mathematical ideas of various forms (Pape & Tchoshanov, 2001), and the process of changing the form of representation from one form to another is known as translation (Lesh, Post, & Behr, 1987). This opinion is in line with some experts (Fennel & Rowan, 2001; NCTM, 2000; Rahmawati et al., 2017) which states that, if someone can represent a mathematical object in a number of different forms of representation indicates that, someone has a good understanding of mathematical concepts.

To translate the representation of covariational problems of dynamic events, other components of the standard are needed such as problem solving, reasoning, connecting, and communicating. The component that plays a major role in the translation of covariational problem representations is covariational reasoning. Covariational reasoning is needed to support the successful process of translational representation (Yemen-Karpuzcu, Ulusoy, & Işıksal-Bostan, 2015) and solving the problem of dynamic events (Carlson, 1998; Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Thompson & Carlson, 2017a; Thompson, Hatfield, Yoon, Joshua, & Byerley, 2017). Meanwhile, the translation of representation and covariational reasoning is still a big problem in the world of Education. This was revealed through a research report on the difficulties in translating representations (Adu-Gyamfi, Stiff, & Bossé, 2012; Bossé, Adu-Gyamfi, & Chandler, 2014; Bossé, Adu-Gyamfi, & Cheetham, 2011) and covariational reasoning (Carlson, 1998; Carlson et al., 2002; Carlson, Madison, & West, 2015; Moore, 2014; Sandie, Purwanto, Subanji, & Hidayanto, 2019; Thompson & Carlson, 2017a; Thompson et al., 2017). Therefore, the translation of representation that requires covariational reasoning needs to be more thorough.

1. RESEARCH METHODS

This research is a qualitative research (Creswell, 2014). Participants consisted of 25 undergraduate students in the Department of Mathematics Education, Faculty of Mathematics and Natural Sciences, State University of Malang. Students are given covariational problems in the form of graphs that are translated into pictorial forms. Students are asked to verbalize what they think when doing translation (think aloud). After that, students are interviewed to confirm and dig deeper into their thought processes.

The test instrument that has been used is a problem with the translation representation of graphic form into pictorial form. The covariational problem was showed in Figure 1.

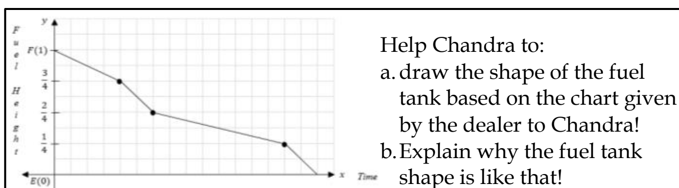

Fig 1 Covariational Problems

The research data obtained will be analyzed with the covariational reasoning framework proposed by Carlson et al., (2002).

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2. RESULT AND DISCUSSION

The results showed that there were students who could answer right and wrong. The results of student work are presented through table 1.

Table 1 shows that only 12% of students answered correctly. There is also the work of students who answer correctly shown through Figure 2, and the work of students who answer incorrectly is shown through Figure 3.

TABLE 1
DATA RESEARCH

No	Answer	Number
1.	Student answers correctly	3
2	Student answers incorrectly	22

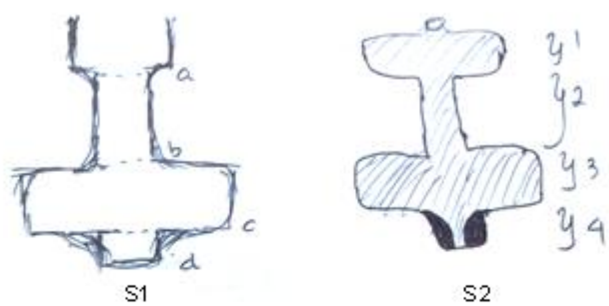

***Fig 1**. the work of students answers correctly*

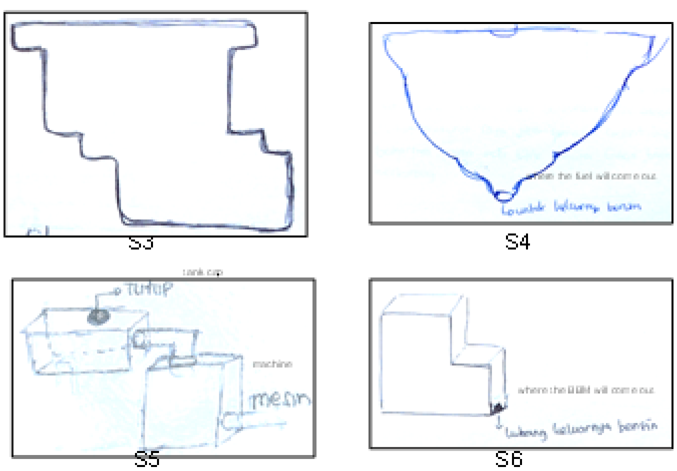

***Fig 3**. the work of students answers incorrectly*

Based on the data shown in table 1 shows that the translation of representations that require covariational reasoning is still a big problem for students. To solve the covariational problem of dynamic events from graphical representations that are translated into pictorial representations (fuel tanks) requires good covariational reasoning abilities. The stages of representation consist of 4 stages namely unpacking the source, coordinating the introduction, constructing the target, and determining equality (Bossé et al., 2014). In the stage of unpacking the source, students must be able to sort out important information that is in the source representation (graph) to be translated into the target representation (pictorial). Therefore, the problem given must be understood first. To understand the problems given, students need good reading skills (Carlson et al., 2015). When students can understand the problem, students are able to sort out important information and eliminate the information that is in the source representation. For students who have difficulty in constructing target representations, students are unable to understand problems that result in students not being able to sort out important information and eliminate information that is not important to the source representation. In the preliminary coordination stage, students coordinate information that has been considered important. The y-ordinate on the graph in the source representation has been translated into tank height in the target representation. The x-axis on the graph has been translated into the width of the tank in the target representation. The value of y on the graph has the same difference, so the tank height also has the same height. The x values on the graph have different values, so the width of the tank also varies. A large x value indicates a large tank width and vice versa. In the preliminary coordination stage, it involves complex mental actions of covariational reasoning (Carlson et al., 2002). When students coordinate the x-axis on the graph as the width of the tank, it is Mental Action 1 (MA1). On the y-ordinate, students know that the point starts from point $(0, F(1))$ moving towards $(0, 0)$. On the x-axis, students know that the point starts from point $(0, 0)$ to $(x, 0)$. This is an activity on Mental Action 2 (MA2). When students determine the value of the height of the fuel tank and the width of the fuel tank, the student has coordinated the value with the size in the coordinate graph. This is an activity on Mental Action 3 (MA3). The value shown in this problem is not indicated by the fixed value. Meanwhile, students only reasoned in determining the width and height of the tank adjusted to the graph.After that the students have coordinated changes in grades on the graph adjusted to the shape of the tank. Students have understood that the graph is a straight line so that the change in values on the x-axis and the y-ordinate is static so that the shape of the tank under certain conditions is straight. This is an activity of Mental Action 4 (MA4). Furthermore, in condition F (1) heading to E (0) has different events so that the tanks in each condition also have different tank widths. This is an activity of Mental Action 5 (MA5) Meanwhile, for students who are unable to properly construct a fuel tank. Students have not been able to do good covariational reasoning. Students have not performed Mental Action perfectly either from MA1 to MA5. Students only describe the shape of the tank with the aim of fulfilling the obligation in answering the test given. Based on research data, shows that each student's covariational reasoning is very lacking (Carlson, 1998; Moore & Paoletti, 2015; Moore, Paoletti, & Musgrave, 2013; Paoletti & Moore, 2017; Thompson, 1994; Thompson & Carlson, 2017b; Thompson, Carlson, & Silverman, 2007). Students have never been given a problem like this. Therefore, training to read needs to be done so that students can understand the problem well. The ability to understand the problem well affects the covariational reasoning competence in translating the representation of covariational problems in dynamic events.

3. CONCLUSION

Covariational reasoning on translational representation supports one another in success in understanding the problem. The five standard processes are a unified whole, where each component is complementary. Covariational reasoning competencies need to be owned by prospective mathematics teachers, keeping in mind that prospective teachers will now teach later. Having good covariational reasoning competence provides a good understanding for students to understand calculus.

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