Windows For Reduction Of ACF Sidelobes Of Pseudo-NLFM Signal

Adithya Valli Nettem, Elizabath Rani Daniel, Kavitha Chandu

Abstract— Synthesis of suitable non linear frequency modulation (NLFM) signals still is a major research direction in radar pulse compression theory for sidelobe reduction. NLFM signals can be generated using simple two-stage and tri-stage piece wise linear frequency modulation (PWLFM) functions. The autocorrelation function of this NLFM signal exhibited low peak sidelobe level ratio (PSLR) value compared to its counterpart LFM signal. In this paper an attempt is made to reduce the PSLR values using special window function having spectral characteristics with low leakage factor and high relative sidelobe attenuation. The simulation results confirm a significant side lobe reduction by the NLFM signal designed using PWLFM functions when a more flexible Power of Cosine window function is applied compared to all other window functions.

Index Terms— LFM, mainlobe width, NLFM, Spectral characteristics, Windows

1 Introduction

Estimation of target characteristics is still an important research direction as moving targets and closely spaced targets are hard to detect. Pulse compression is a favorable technique which influences target parameter estimates by employing different signal models such as Frequency modulation (FM), Phase modulation (PM), short radio pulses and unsinusoidal signals. The typical approach in pulse compression is to correlate the received signal with a delayed copy of transmitted signal in a matched filter (MF) [1]. Most significant request imposed in the design of radar signals is to assure the lowest sidelobe levels to the response of MF. Since 1940's Linear Frequency Modulated (LFM) signal is the most used pulse compression waveform as it can be generated easily and bandwidth can be effectively used as the frequency is linearly swept over to cover the entire signal bandwidth. The compressed waveform at the receiver has a sidelobe at -13dB which can be a hindrance while detecting closely spaced targets [2]. So an important research direction refers to the design of improved methods to synthesize radio pulses with rectangular envelope but with suitable modified FM laws (non linear frequency modulation (NLFM) signals) so that the expected MF response is achieved [3]. NLFM signals have a vast applicability in radar systems with a good range resolution, good interference mitigation, better signal to noise ratio (SNR), low-cost, and has a spectral weighting function inherently in their modulation function which effectively which gives a pure matched filter output with low side-lobe levels [3]. This paper focuses on the design of a NLFM signal and applying window functions to this signal to reduce the autocorrelation (ACF) sidelobes. The first part of the paper describes the design of NLFM signal using two-stage and tri-stage PWLFM functions followed by the simulations using different window functions.

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2 Pseudo-NLFM signal

NLFM signals are part of an important family of continuous modulation functions which plays a significant role in pulse compression radar systems. Non linear frequency modulation favorably shapes the power spectral density in such a way that the MF response has reduced sidelobes from its LFM counterpart to a large extent. An NLFM signal also provides a better detection characteristic and is more precise in determining the range compared to other methods available in literature [4] as (dual apodization (DA), spatially variant apodization (SVA) and leakage energy minimization (LEM)). Conversely accurate NLFM signal design and processing is still a difficult task as generally radar designer aims at having an easily produced and processed signal to meet the bandwidth constraints, target performance characteristics and sidelobe reduction goals [5]. All time desire for research would be looking forward for improved methods to design radar pulses with rectangular envelope but with appropriate FM laws such that the matched filter output shows favorable results. In radar systems theory numerous research work has been done to design optimum (level of sidelobe suppression) NLFM signals, all the work done generally can be categorized into two directions. One is based on design of NLFM signal using LFM signals introducing predistortion on short intervals into temporal domain or spectral domain and the other is the design by using predefined power spectral density function using different methods as stationary phase principle, iterative methods and explicit functions cluster method [5]. In this paper, a NLFM signal is generated using simple two-stage and tri-stage PWLFM functions which are described below.

$$f(\tau) = \begin{cases} \alpha_0\tau & 0 \le \tau \le T_1 \\ B_1 + \alpha_1(\tau - T_1) & T_1 \le \tau \le (T_1 + T_2) \end{cases} \quad (1)$$

Equation (1) represents the instantaneous frequency variation of NLFM signal formed by concatenating two piece wise LFM functions with a sweep rate of α_o in the first stage and α_1 in the second stage. The total pulse width of the chirp signal τ is divided into two time slots with respective pulse widths T_1 and T_2. If B_1 and B_2 are the corresponding bandwidths of the first and second stage LFM functions, then the corresponding sweep rates can be defined as

$$\alpha_0 = \frac{B_1}{T_1} \qquad \alpha_1 = \frac{B_2}{T_2}$$

The corresponding phase variation of this concatenated NLFM function can be obtained by integrating (1)

$$\varphi(\tau) = \int f(\tau) = \begin{cases} \alpha_0 \frac{\tau^2}{2} & 0 \le \tau \le T_1 \\ B_1\tau + \alpha_1(\frac{\tau^2}{2} - T_1\tau) & T_1 \le \tau \le T_1 + T_2 \end{cases} \quad (2)$$

Similarly tri-stage NLFM function can be obtained by concatenating the instantaneous frequency functions of three piece wise LFM function with each different sweep rates. The instantaneous frequency of this NLFM can be written as following (3)

$$f(\tau) = \begin{cases} \alpha_0\tau & 0 \le \tau \le T_1 \\ B_1 + \alpha_1(\tau - T_1) & T_1 \le \tau \le (T_1 + T_2) \\ B_1 + B_2 + \alpha_2(\tau - (T_1 + T_2)) & (T_1 + T_2) \le \tau \le (T_3 + (T_1 + T_2)) \end{cases} \quad (3)$$

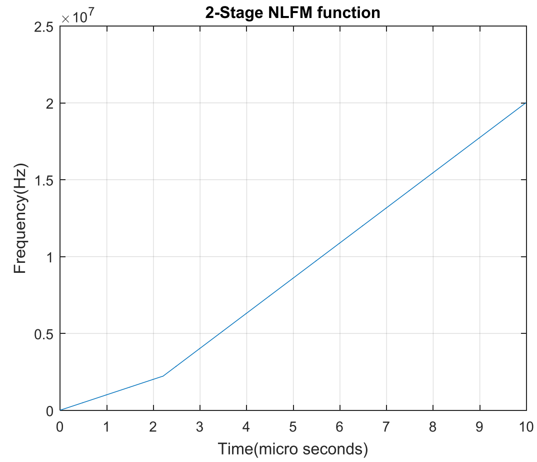
and corresponding sweep rates can be defined as follows

$$\alpha_0 = \frac{B_1}{T_1} \qquad \alpha_1 = \frac{B_2}{T_2} \qquad \alpha_3 = \frac{B_3}{T_3}$$

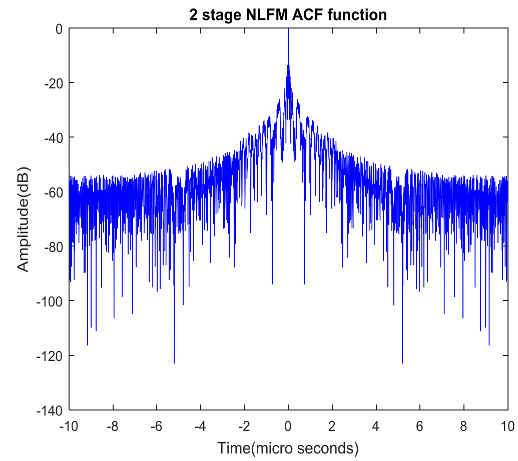
Thus, the phase of this two stages NLFM signal can be derived by integrating (3)

$$\varphi(\tau) = \int f(\tau) = \begin{cases} \alpha_0 \frac{\tau^2}{2} & 0 \le \tau \le T_1 \\ B_1\tau + \alpha_1\left(\frac{\tau^2}{2} - T_1\tau\right) & T_1 \le \tau \le T_1 + T_2 \\ B_1\tau + B_2\tau + \alpha_2\left(\frac{\tau^2}{2} - (T_1 + T_2)\tau\right) & (T_1 + T_2) \le \tau \le (T_3 + (T_1 + T_2)) \end{cases} \quad (4)$$

Simulations have been carried out for T = 10 µs and B = 20 MHz with different combinations of T_1, T_2, B_1 and B_2. All the possible combinations are examined by choosing different sweep rates for both two-stage and tri-stage PWLFM functions. Amongst all the combinations, Fig. 1(a) shows the frequency variation of two-stage NLFM function which achieved highest PSLR of -25.88dB at the output of matched filter (MF) as shown in Fig. 1(b). This is achieved at specific values of T_1=2.7µs, T2=7.3µs, B_1=2.2MHz and B_2=17.8MHz. Similarly Fig. 2(a) shows the frequency variation of tri-stage NLFM function which achieved highest PSLR of - 26.36dB as shown in Fig. 2(b). This is achieved at specific values of T_1=1µs, T_2=8µs, T_3=1µs, B_1=2MHz, B_2=8MHz and B_3=10 MHz

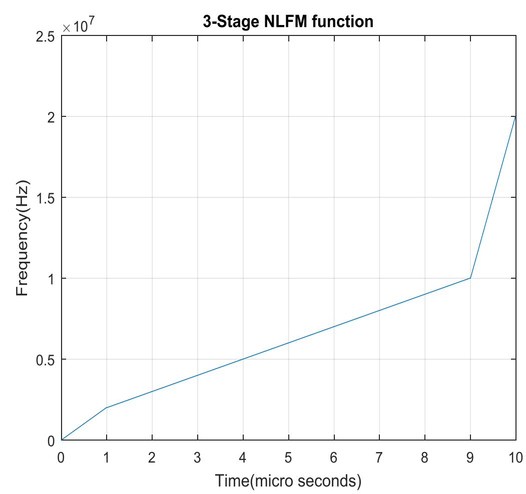


(a)

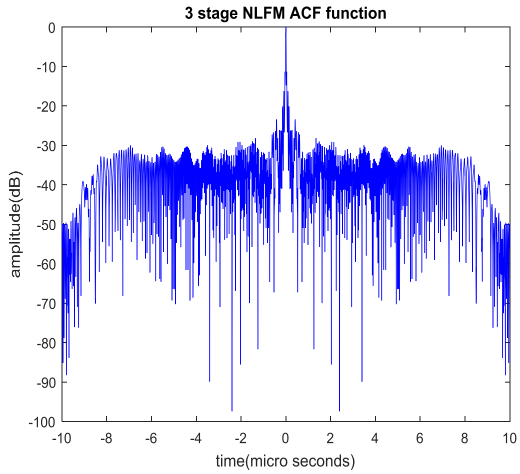


(b)

Fig. 1(a) Two-stage NLFM function (b) Autocorrelation function



(a)



(b)

Fig. 2(a) Tri-stage NLFM function (b) Autocorrelation function

3 WINDOW WEIGHTING

Basically window is a mathematically limited function which exists within given interval and is zero valued anywhere else and is used to reduce the well known Gibbs oscillations caused by the abrupt truncation of a Fourier series [8, 9, 10]. A window function is a basic signal processing tool that is needed in many signal processing fields such as radar/sonar. In most of these applications window function is assumed to have all the spectral power into extremely narrow band with zero sidelobes which is impossible both theoretically and practically. Window function always has a mainlobe with sidelobes. No window is ideal and it should be selected based on the requirement of the application [11, 12]. Thus, a method for designing window functions with flexible spectral characteristics is greatly needed. Commonly used spectral characteristics of a window function include the mainlobe width (MW), the peak sidelobe level ratio (PSLR) and relative sidelobe attenuation which are closely related to the resolution and spectrum leakage [13]. The essential building block of pulse compression matched filtering is FFT. The FFT computation takes on the signal as periodic or repeats itself for each block of data. If the signal is non-periodic, it results in the leakage in frequency spectrum of the signal causing the spectrum to spread out over a wide frequency range which arise difficulty in identifying the exact frequency content of the measured signal. So window function is employed to reduce the leakage factor. When a window function is multiplied to a non periodic signal it makes the signal to be periodic and suppress the side lobes to a certain extent [14, 15]. Window weighting can be applied both in time/frequency domain, the former method is preferred to later as it produces low PSLR values [16]. Generally windows can be categorized into fixed and variable having all parameters fixed or variable. Designers must make trade-offs among the mainlobe width (MW), the peak sidelobe level ratio (PSLR) of windows by carefully adjusting these parameters [15]. In this paper both the fixed and flexible window functions in time domain are used which are described in below Table I. It shows the used window functions with formulae and variable parameters.

__Table I__ Window functions used with formulae and variable parameters

Windows	Formula	Parameters
Hamming	$w(n) = 0.54 - 0.46 * \cos\left(\frac{2\pi n}{N}\right)$ $0 \le \lvert n \rvert \le N/2$	--
Hanning	$w(n) = 0.5 - 0.5 * \cos\left(\frac{2\pi n}{N}\right)$ $0 \le \lvert n \rvert \le N/2$	--
Bartlett	$w(n) = 1 - \frac{\lvert 2n \rvert}{N}$ $0 \le \lvert n \rvert \le N/2$	--
Connes	$w(n) = (1 - ((x * (\frac{x}{\alpha}))/\alpha))\wedge 2$ $x = n/N$ $n = -\frac{(N-1)}{2} : \frac{(N-1)}{2}$	α=0.4296
Papoulis	$w(n) = \left(1 - \frac{\lvert n \rvert}{N/2}\right)\cos\left(\frac{\pi n}{N/2}\right) + \frac{1}{\pi}\sin\left(\frac{\pi \lvert n \rvert}{N/2}\right)$ $0 \le \lvert n \rvert \le N/2$	--
Parzen	$w(n) = \{$ [illegible]	--
Cauchy	$w(n) = \frac{1}{[illegible]}$ $0 \le \lvert n \rvert \le N/2$	$\alpha = 2,3,4$
Power of Cosine	$w = cos^{p}\left(\frac{\pi n}{N}\right)$ $n = -\frac{(N-1)}{2} : \frac{(N-1)}{2}$	$p = 2,5,7$

Table II shows the spectral characteristics of windows namely, leakage factor which determines the ratio of power in the sidelobes to the total window power and it is always desirable to have leakage factor as low as possible, relative sidelobe attenuation which is the difference in height from the mainlobe peak to first highest sidelobe peak. Sidelobe roll off ratio depends on this value, as better the sidelobe attenuation better the PSLR values but from below table it is observed that there is always a tradeoff between main lobe width and PSLR values.

Table II Spectral characteristics of window functions

Windows		**Leakage factor (%)**	**Relative Side lobe Attenuation (dB)**	**Main lobe width(-3dB)**
Hamming		0.04	-42.7	0.0024
Hanning		0.05	-31.5	0.0026
Bartlett		0.28	-26.5	0.0024
Connes		0.27	-27.4	0.0029
Papoulis		0	-46	0.0031
Parzen		0	-53	0.0034
Cauchy	α=3	0.14	-31	0.0024
	α=4	0.28	-26	0.0029
	α=5	0.09	-31.3	0.0031
Power of Cosine	p=2	0.05	-31.5	0.0026
	p=5	0	-54	0.003
	p=7	0	-67.9	0.0046

4 SIMULATIONS AND RESULTS

The efficiency of the waveform can be proved by lower peak side lobe level ratio (PSLR). The Peak side lobe level ratio (PSLR) is given by

PSLR= $20*\log_{10}$ (Peak sidelobe amplitude/mainlobe amplitude)

Simulations are carried out for both the two-stage and tri-stage NLFM functions using aforementioned window functions. The following figures represent the matched filter output with and without window functions. It's evident from the figures that the windowed signal output yields lower PSLR values.

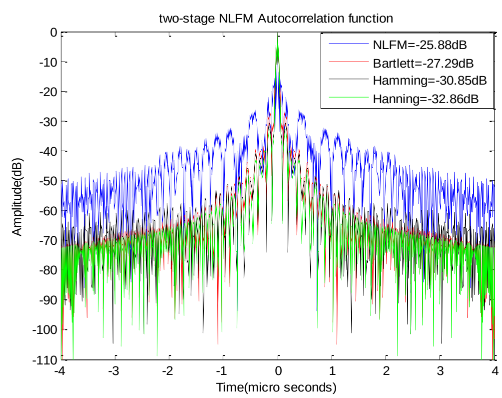


Fig. 3 (a) Two-stage NLFM matched filter output with (Hamming, Hanning and Bartlett) and without windows

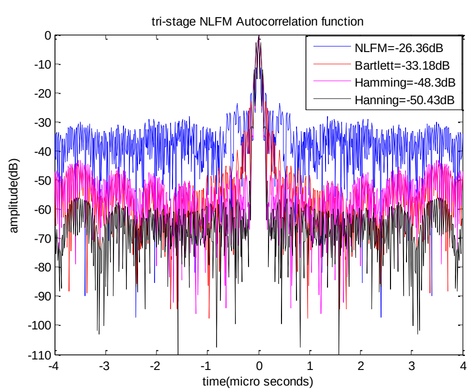


Fig. 3 (b) tri-stage NLFM matched filter output with (Hamming, Hanning and Bartlett) and without windows

Figure 3 shows the matched filter output with regularly used Hamming, Hanning and Bartlett windows. The PSLR values exhibited by these windows ranged from -27.29dB to -32.86dB for two-stage and from -33.18dB to -50.43dB for tri-stage NLFM signal

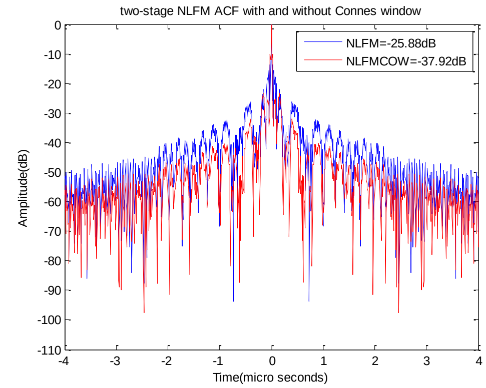


Fig. 4 (a) Two-stage NLFM matched filter output with Connes window

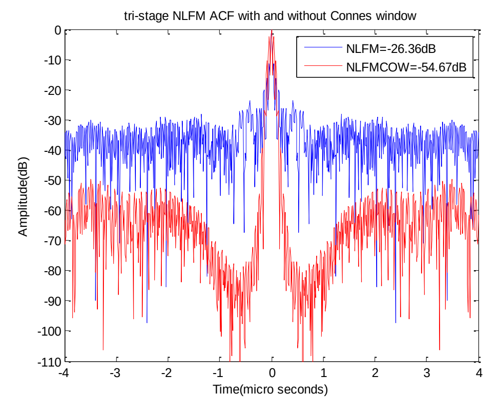


Fig. 4 (b) tri-stage NLFM matched filter output with Connes window

Figure 4 shows the matched filter output with Connes window which has almost similar spectral characteristics as of Bartlett window as shown in Table II, but yielded a better PSLR value (10-20dB higher) because of it high relative sidelobe attenuation. These results in a slight increase in the mainlobe width compared to that of Bartlett window.

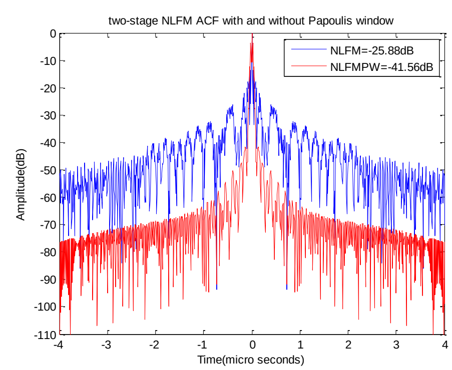


Fig. 5 (a) Two-stage NLFM matched filter output with Papoulis window

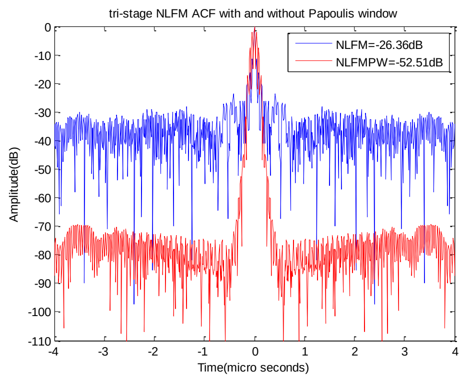


Fig. 5 (b) tri-stage NLFM matched filter output with Papoulis window

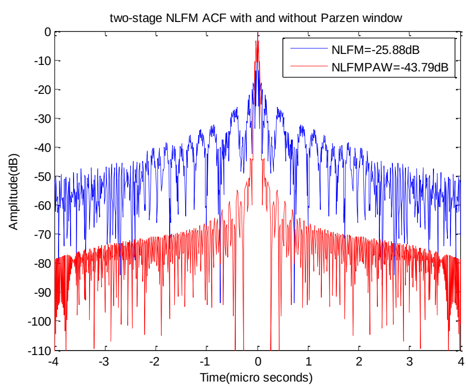


Fig. 6(a) Two-stage NLFM matched filter output with Parzen window

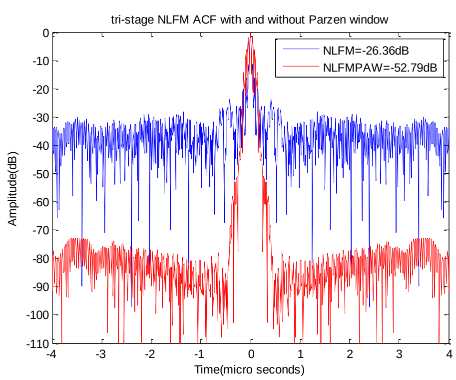


Fig. 6(b) tri-stage NLFM matched filter output with Parzen window

Figures 5&6 show the matched filter output with Papoulis and Parzen window which resulted in better PSLR values compared to regularly used Hamming and Hanning windows because of their spectral characteristics zero leakage factor and high relative sidelobe attenuation as shown in Table II.

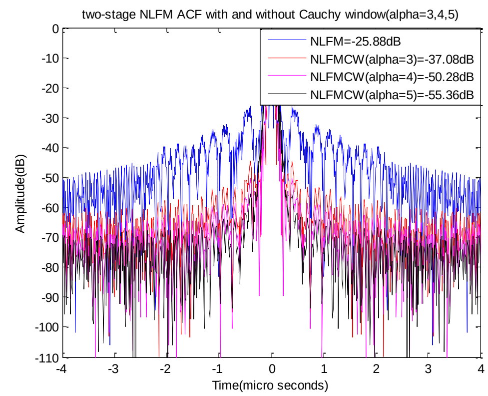


Fig. 7 (a) Two-stage NLFM matched filter output with Cauchy window

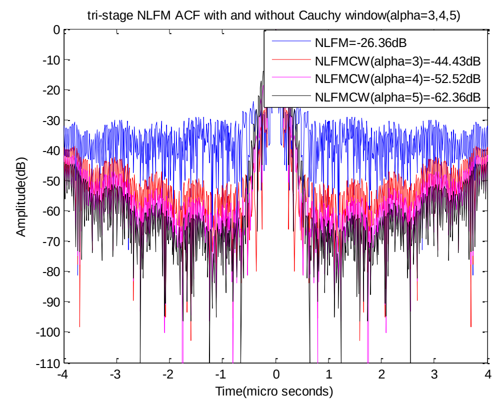


Fig. 7 (b) tri-stage NLFM matched filter output with Cauchy window

All the above functions used are fixed window functions; Figure 7 shows the window function with flexible parameter named as Cauchy. Its MF output characteristics depend upon the variable parameter alpha. There is a significant improvement in PSLR values with different alpha (α=3, 4, 5) values as such from -37.08dB to -55.36dB for two-stage NLFM signal and -44.43dB to -62.36dB for tri-stage NLFM signal.

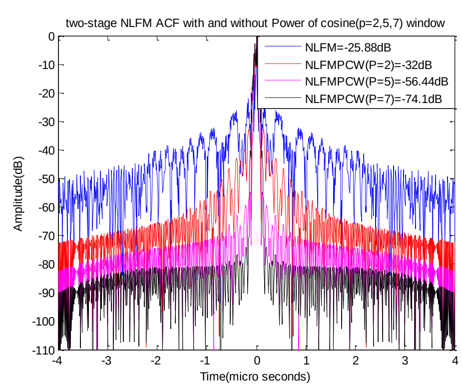


Fig. 8 (a) Two-stage NLFM matched filter output with Power of Cosine window

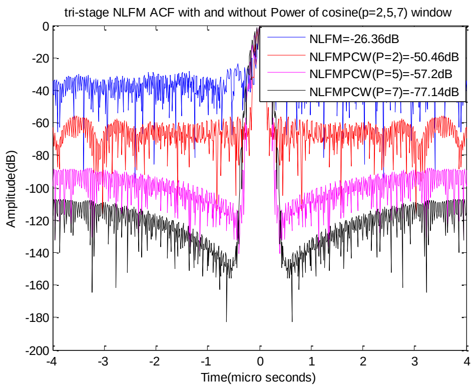


Fig. 8 (b) tri-stage NLFM matched filter output with Power of Cosine window

Finally the flexible window function named Power of cosine (P=2, 5, 7) is discussed and its output of the MF is shown in Figure 8. It is evident form Table II, that with P=5&7 window exhibits MF characteristics better than all other window functions used. Table III shows the list of PSLR values for two-stage and tri-stage NLFM signals with all the fixed and flexible window functions discussed. Among all the fixed and variable window functions used in this paper, a drastic reduction in PSLR values can be seen with a variable window (power of cosine, P=7) having spectral characteristics of zero leakage factor and a high relative sidelobe attenuation of -67.9dB, which yielded lowest PSLR values of -74.1dB and -77.14dB for two-stage and tri-stage NLFM signals respectively.

Table III PSLR values of two-stage and tri-stage NLFM signal with different windows

Windows		Two-Stage NLFM PSLR(dB)	Three-Stage NLFM PSLR(dB)
Hamming		-30.85	-48.3
Hanning		-32.86	-50.43
Bartlett		-27.29	-33.18
Connes		-37.92	-54.67
Papoulis		-41.56	-52.51
Parzen		-43.79	-52.79
Cauchy	α=3	-37.08	-44.43
	α=4	-50.28	-52.52
	α=5	-55.36	-62.36
Power of Cosine	p=2	-32	-50.46
	p=5	-56.44	-57.2
	p=7	-74.1	-77.14

4 Conclusion

The pseudo NLFM function designed using simple two-stage and tri-stage PWLFM functions is attractive since it is capable of reducing sidelobe level better than their counterparts. A highest sidelobe suppression of around -26 dB is achieved. The higher sidelobe level introduced by two and tri-stage NLFM (-26dB) is suppressed using different windows. A drastic suppression from -26dB to -74dB is achieved in both the cases by using a variable window function Power of cosine (P=7). Based on the inference drawn from Table III, this signal can be used in the various applications where low sidelobe levels are desired.

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