“Mathematical Modeling Of Drug Transport From Contact Lens To Anterior Segment Of The Eye”

Aadesh Kumar, Dr. Ram Avtar , Dr. Deepti Seth

Abstract: The motive of this paper, to show the overview of drug delivery from contact lens to anterior segment via pre corneal area, cornea. We use the diffusion based mathematical model in which parameters can be adjusted, based on experimental result obtained under controlled conditions. The contact lenses drug delivery system have been developed a model to increase the time of drug availability at the surface of eye.

Keywords: Contact Lens, Drug delivery, Biological Tissue, Anterior Chember, Hydrogel, Transport Phenomena.

---

I. INTRODUCTION

The use of contact lenses increasingly laboratories for in vivo animals retinal concept and preclinical studies. A flexible method to fabricate customized hydrogel contact lens. We observe that the fabricated gel has maximum transparency with refractive index range 1.42 – 1.45 nano meter and in spectra range 400 – 600 nm[1]. The soft contact lenses are made of hydrogel, capable of absorbing requisite volume of medium of aqueous. The cornea remains in contact with high concentration of drug for long period and drug penetration is more efficient. The drug reservoir ability depends on the water contents, thickness of lenses and molecular weight of drug. Controlled and release drug delivery have moved phenomenally in recent years and open new ways in the field of drug delivery system. In this paper, Mathematical model that elucidates the joint process of drug release from polymeric matrix [5] and consequent transport of drug particles in biological tissue. In present investigation, the mathematical modeling and biological physiology for the sake of bridging the gap between biological perspective and transport phenomena. Therapeutic contact lenses for the sake of increase of ocular bioavailability of ophthalmic medicines together dermal and transdermal delivery [3,4].

In mathematical point of view, the FAQ that how can we predict of drug concentration in anterior chamber of the eye. Here mathematical models describing the behavior of drug concentration across the Precorneal and cornea. When a drug dropped on contact lens then the absorption of drug depend on the concentration of lens. The mathematical modeling of the different mechanism responsible for controlled released from hydrogel such as diffusion is well described in literature. The aim of this work to describe and characterize the diffusion based mathematical model to design of multilayered drug load lenses. Contact lenses are emerging as an alternative ophthalmic drug delivery system to resolve the weakness of conventional topical method[2].

Model:
In this model we will find the concentration of contact lens, precorneal Area, Cornea and Anterior chamber by using mathematical calculations.

* Correspondence Author
Aadesh Kumar*, Assistant Professor Department of Applied Science, Shri Bhawani Niketan Institute of Technology & Management Jaipur, India
Email: asharma.sngi@gmail.com
Dr. Ram avtar, Department of Mathematics HBTU Kanpur U.P. India. Email: rautar62@gmail.com
Dr. Deepti Seth, Assistant Professor Department of Applied Science, KIET Ghaziabad, U.P., India. Email: deepti_hbti@yahoo.com

---

FIG(1)
Schematic diagram of Eye
Drug transport in lens

Governing equations:

\[ \frac{\partial c_L}{\partial t} = D_L \frac{\partial^2 c_L}{\partial x^2} - \delta_0 c_L \]  \hspace{1cm} (1) \hspace{1cm} [4]

\[ c_L(x,t) = c_1 \cos \left( \frac{p^2 - \delta_0}{4D_t} x \right) + c_2 \sin \left( \frac{p^2 - \delta_0}{4D_t} x \right) e^{-\delta_0 t} \]  \hspace{1cm} (1a)

Where \( p \) is constant and \( \delta_0 \) is also a constant depends the behavior of contact lens.

Solution of pre corneal area equations:

\[ \frac{d c_2}{dt} = -\frac{F_p f_p A_p}{V_d} c_2 - \frac{C_0 A_2}{V_d} \]  \hspace{1cm} (2)

Now equation 1 can be written as

\[ \frac{d c_2}{dt} \frac{V_d}{V_d} + \frac{V_d}{V_d} c_2 - \frac{F_p f_p A_p}{V_d} c_2 = -\frac{C_0 A_2}{V_d} \]  \hspace{1cm} (3)

We observe that 2 is linear differential equation so

\[ \text{I.F.} = \left( \frac{V_d}{V_d} e^{-\frac{F_p f_p A_p}{V_d} t} \right) \]

Then solution of equation (3) is

\[ c_2(t) = \left( \frac{F_p f_p A_p}{V_d} \right) e^{-\frac{F_p f_p A_p}{V_d} t} + \frac{C_0 A_2}{V_d} \]  \hspace{1cm} (4)

Solution of corneal area equation

\[ \frac{\partial C_{c2}}{\partial t} = D_C \frac{\partial^2 C_{c2}}{\partial x^2} - k_r C_c \]

Here \( C_c < k_r \) now the reaction term which approximate by \( k_r C_c \) (Kaku ji, 1988)

So equation (4) reduce as

\[ \frac{\partial C_{c2}}{\partial t} = D_C \frac{\partial^2 C_{c2}}{\partial x^2} - k_r C_c \]  \hspace{1cm} (5)

Solution of equation (5) is

\[ C_{c2}(x,t) = \left[ c_5 \cos \left( \frac{p^2 - k_1}{D_{c2}} x \right) + c_6 \sin \left( \frac{p^2 - k_1}{D_{c2}} x \right) \right] e^{-k_r t} \]

Where \( i = 1, 2, 3 \)

\[ C_{c2}(x,t) = \sum_{n=1}^{\infty} a_n \cos \theta_1 x + \sum_{n=1}^{\infty} b_n \sin \theta_1 x \]

Where \( C_{c2} \) is the concentration of corneal layers

Where \( a_n = c_5 c_7 \) and \( b_n = c_6 c_7 \) and \( \theta_1 = \left( \frac{p^2 - k_1}{D_{c2}} \right) \)

Then

\[ C_{c2}(x,t) = \sum_{n=1}^{\infty} \left[ a_n \cos \theta_1 x + b_n \sin \theta_1 x \right] e^{-k_r t} \]  \hspace{1cm} (9)

Where \( \theta_1 = \left( \frac{p^2 - k_1}{D_{c2}} \right) \)

\[ C_{c2}(x,t) = \sum_{n=1}^{\infty} \left[ \phi_n \cos \theta_1 x + \psi_n \sin \theta_1 x \right] e^{-\theta_1 t} \]  \hspace{1cm} (10)

Where \( \theta_3 = \left( \frac{p^2 - k_3}{D_{c3}} \right) \)

Equation of anterior chamber

\[ \frac{d}{dt} (V_d C_d) = -F_p f_p A_p - C_0 C_d \]

Equation 8 can be written as

\[ \frac{dc_2}{dt} + \frac{C_0 c_2}{V_d} = \frac{F_p f_p A_p}{V_d} \]

Solution of equation (13) is given by

\[ c_2(t) = \frac{F_p f_p A_p}{C_0} + C_0 e^{-\frac{F_p f_p A_p}{V_d} t} \]

Boundary Conditions:

\[ \frac{\partial C_1}{\partial x} = 0 \text{ at } x = -l_1 \]  \hspace{1cm} (a)

\[ C_1(t) = C_3 \text{ at } t = 0 \]  \hspace{1cm} (b)

\[ C_3(x,t) \big|_{x = 0} = 0 \]  \hspace{1cm} (c)

\[ \frac{\partial C_3}{\partial x} \big|_{x = l_1} = 0 \]  \hspace{1cm} (d)

\[ \frac{C_{c3}}{\phi_1} \big|_{x = l_1} = \frac{C_{c3}}{\phi_2} \big|_{x = l_1} \]  \hspace{1cm} (e)
Now applying the boundary condition (a) on equation (1) we get
\[ D_{a_0} \frac{\partial^2 c_3}{\partial x^2} = D_{a_0} \frac{\partial^2 c_3}{\partial x^2} \left|_{x=l_2} \right. \]
\[ = D_{a_0} \frac{\partial^2 c_3}{\partial x^2} \left|_{x=l_2} \right. \]
\[ = \phi_1 \left( c_3 \right)_{x=l_2} - \left( c_3 \right)_{x=l_2} \]
\[ = 0 \]  
\[ \left( c_3 \right)_{x=l_2} = 0 \]  

Now applying the boundary condition (b) on equation (4) we get
\[ c_i(x,t) = \left( b_1 \cos \left( \frac{\pi}{b_1} \right) \right) \sin \left( \frac{\pi}{b_1} \right) \]

Where \( b_1 = -c_2 c_3 \)

Again apply the boundary condition (b) on equation (4) we get
\[ c_i(x,t) = \left( \frac{F_{f_{p}} f_{p} e^{\frac{t}{2}}}{S} \left( 1 + \frac{V_1}{V_2} \right) \left( \frac{V_1}{V_2} \right)^{\frac{t}{2}} + c_0 \right) \]
\[ c_i \approx 1 \]  

Now apply the boundary conditions (d-h) on equation (7) we get
\[ a_2 = \rho_1 \]

Apply the boundary condition (j) on the equation (10) we get
\[ c_i(t) = \left( \frac{F_{f_{p}} f_{p} e^{\frac{t}{2}}}{c_0} \left( \frac{\rho_1}{\rho_2} - 1 \right) \right) \]

Table (1)[6,3]

<table>
<thead>
<tr>
<th>Coefficient of Corneal layers</th>
<th>( D_{a_0} )</th>
<th>Fickian diffusion flux of drug across corneal layer</th>
<th>Fickian diffusion flux of drug across conjunctiva</th>
<th>Solution of drainage rate constant</th>
<th>Normal tear volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storma, Epithelium, endothelium</td>
<td>( 2.81 \times 10^{-5} )</td>
<td>( 7.8-7.6 ) (For open Eyes) ( 6.0-6.2 ) (For closed Eyes)</td>
<td>( 4-6 )</td>
<td>( 1.45 ) (Normal)</td>
<td>( 7.0 )</td>
</tr>
</tbody>
</table>

Graph Between Concentration and Tear Volume
We observe from above graph \( C_z \) change according to \( V_L \). The concentration of lens is very high and as soon as drug reached at last layer of lens that's decrease and also parameter effect on concentration of lens \( C_z \) & \( C_{24} \) also. It means that as drug inject on the contact lens surface than drug soaked by lens in maximum quantity of drug and reached at Precorneal surface. Also the effect of \( V_L \) over concentration is very high, as value of \( V_L \) increase then total concentration decrease. It is shown in Fig:3.

According to fig. 4 we observe that the parameter effect on concentration of anterior segment, the concentration decrease as the value of \( V_A \) increase.

According to above graph the effect of \( Cl_a \) shows that concentration is also little bit decrease as the value of \( Cl_a \) increase.

RESULT:
From fig 3, 4, 5 it is clear that the concentration increases as time increases and also if the value of \( V_L \) increases then concentration of Precorneal surface decreases. To increase the concentration of Precorneal surface we use contact lens. The concentration of cornea depends on the diffusion coefficient, permeability contact lens \( k_t \). It is clear from all expression of concentration that concentration can't be negative. The concentration of anterior chamber is also decrease as the value of \( V_A \) increase. Also the concentration of anterior segments depends on the clearance. If clearance increase then less change in concentration of anterior segment from fig (8).

CONCLUSION:
By using of contact lens for therapeutically treatment may use many deficiencies seen with the typical administration of eye drop into eye. The resident time of drug at Precorneal surface area will be longer. All most toxicity will be soaked by can lens. By using soft contact lens, we can control the eye diseases like glaucoma, dry eyes etc. The contact lens can also use for change eye color modification or treatment of diabetic eye diseases, we can provide best treatment of artificial cornea and corneal wound healing.

APPENDIX

ACKNOWLEDGMENT
The authors gratefully acknowledge the constructive and fruitful comments of the reviewers of original manuscript of the paper.

REFERENCES
[3] Ferrira J.A. etal; “Drug Delivery: From a Contact Lens to the Anterior Chamber” Journal of Computer...

