Statistical Prediction Of Laser Generation For A High-Powered Copper Bromide Vapor Laser

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Abstract: Based on multivariate methods of factor analysis and principal component regression, an approach is proposed for predicting the laser generation of a copper bromide vapor laser with a wavelength of 510.6 and 578.2 nm. The influence of 6 independent variables on the increase of laser output power has been considered. New values have been given to the geometric dimensions of the laser tube, the supplied electric power, and hydrogen pressure in order to improve laser generation by up to 17%. Two-dimensional nomograms with statistically valid areas in order to facilitate predictions are presented.

Index Terms: Copper bromide laser, factor analysis, nomogram, parametric model, prediction, principal component analysis, regression model.

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1 INTRODUCTION

COPPER vapor lasers, including copper bromide vapor (CuBr) lasers, continue to be the focus of scientific studies due to their wide range of practical applications in various fields and scientific investigations. In medicine, it is mainly utilized in dermatology and photocoagulation. It is used to absorb oxyhemoglobin, treat spots, pigmentations, for photogoagulation of blood vessels, in precision surgery, for skin rejuvenation [1]. High-powered copper bromide vapor lasers are used to work different types of materials: for drilling, cutting, marking, etching, etc. Laser sources with output power over 130 W are suitable for wider range of industrial applications [2]. The laser is often used for isotope isolation of various chemical elements, to study different properties of materials, etc. [3]. Copper bromide vapor lasers have a number of other applications - in advertising and show business, laser microscopy, military, air and underwater navigation and location, in ecology for the study of air and ocean pollution, etc. [4]. In order to improve the efficiency of scientific studies, various numerical and analytical methods are used to develop new laser sources and to perfect existing ones. In recent years, there have been publications related to the use of statistical methods [5]. In this regard, numerous new statistical parametric and non-parametric [5-7] models have been developed. Statistical parametric methods are especially suitable for practical engineering [8]. This is due to their simplicity and accessibility. The goal of this paper is to use the accumulated experience in the field of parametric modeling to predict the laser output power of a CuBr laser. This would allow for new results, expanding experiment planning capabilities, and enabling the construction of new laser devices. All results have been obtained with the help of SPSS software package [9]. The object of investigation in this study is a copper bromide vapor laser, emitting in the visible spectrum. This type of laser is an original Bulgarian invention, protected by Bulgarian and international patents. It was developed by a team at the Laboratory of Metal Vapour Lasers at the Georgi Nadjakov Institute of Solid State Physics, Bulgarian Academy of Science, Sofia [10, 11].

2 TECHNICAL DESCRIPTION

Copper bromide vapor lasers are known as sources of laser generation in the visible spectrum emitting at two wavelengths: green 510.6 nm and yellow - 578.2 nm. Neon is usually used as an added gas. In order to improve output laser power, small quantities of hydrogen are added. This type of laser has numerous advantages in comparison with classic high-temperature pure copper lasers. Its operating temperature in the active zone is over 700°C. This allows for the laser tube to be made from quartz glass without high-temperature ceramics, thus reducing its cost significantly and making production much simpler. It enters its operating mode faster, operates more reliably, provides better durability, and longer service life. There is no need for forced water cooling, making it easier to operate. These capabilities exceed the ones of pure copper lasers. A general schematic of the laser tube is given in Fig. 1.

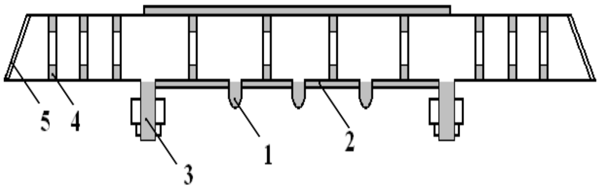

Fig. 1. Construction scheme of CuBr laser tube: 1- reservoirs with copper bromide, 2- heat insulation of the active volume, 3- copper electrodes, 4- inner diaphragms, 5- tube windows.

3 DESCRIPTION OF DATA AND STATISTICAL PROCEDURES

The statistical examination of the CuBr laser is performed on the basis of historical data (see [5] and the bibliography cited there, concerned CuBr laser).

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TABLE 1
Examined CuBr Laser Characteristics.

Variable	Independent input characteristics	Measure
D	Inside diameter of the laser tube	mm
Dr	Inside diameter of the rings	mm
L	Distance between the electrodes	cm
Pin	Supplied electric power	kW
*PH*2	Hydrogen pressure	Torr
PL	Input power per unit lenght	kW/cm
Prf	Pulse repetition frequency	kHz
PNe	Neon pressure	Torr
C	Equivalent capacity of the condenser battery	pF
Tr	Temperature of the CuBr reservoir	^{o}C
	Dependent output characteristic	
Pout	Output laser power (laser generation)	W

A total of 11 variables have been considered. Ten of these are independent and one is dependent. Their description is given in Table 1. In statistical analysis 93 experiment results for high-powered CuBr laser have been considered. After taking a 75% random sample, 74 experiment data were used. The statistical examination of experimental data was carried out by using the classical techniques of factor analysis and principal component regression (PCR) [5, 8]. Factor analysis is applied in order to resolve the problem of multicolineariry between the ten input variables (see Table 1). This procedure gives the opportunity to combine the highly collinear variables in a group, called factor. This way the data variance is distributed between the factors. It is recommended that the involved factors are accounted for at least 80% of all data [12]. Usually, the non-correlating factors are generated by using the method of principal component analysis (PCA) [12]. They can be further used as new artificial variables for performing multiple regression analysis, which is known as PCR. The regression method expresses in an explicit equation the dependence between the dependent variable *Pout* and the factors, used as predictors. The factor and regression analysis for different samples of data for CuBr laser, performed in [5], shows that of the 10 independent variables, only six have significant influence: *D* – inner diameter of the laser tube, *Dr* – inner diameter of the rings, *L* – distance between the electrodes (length of the active zone), *Pin* – input electric power, *PH*2 – hydrogen pressure, and *PL* – specific power per unit length. For the other variables it was found that their influence on laser output power *Pout* is insignificant within the experimentally established values and for this reason they can be ignored. Multivariate regression analysis is used to construct an equation demonstrating the dependence of laser generation *Pout* on the upper mentioned 6 independent variables. Studies indicate [5] that between these independent variables there is a high degree of correlation and therefore, they can not be used explicitly to obtain the parametric regression equation. For this reason, we will use the PCR. The general form of the regression equation in the case with three factors is:

$$\hat{P}out = b_0 + b_1 F_1 + b_2 F_2 + b_3 F_3 \,, \tag{1}$$

where b_0, b_1, b_2, b_3 are regression coefficient and are subject to regression analysis, and F_1, F_2, F_3 are the factors, obtained by means of factor analysis. It has to be noted that the application of statistical methods requires the adherence to numerous additional conditions and hypotheses. The random sample used meets the necessary conditions completely.

4 RESULTS FROM APPLICATION OF FACTOR ANALYSIS AND PCR METHOD

Factor analysis is performed at the start of our statistical analysis. Its objective is to group the 6 independent variables (predictors) into groups (facto rs). There must be strong correlation between the independent variables in each factor. Between the predictors participating in individual factors there must be weak correlation or none at all. As a result of the performed factor analysis, the 6 independent quantities are divided into three mutually independent groups (factors). These factors account for about 94% of all investigated data. Using Principal Component Analysis and Varimax rotation, the rotation matrix for the factors is obtained - Table 2. In this case, factor loadings show the relative degree of influence of each variable within a given factor. Loadings lower than 0.5 are ignored [5].

TABLE 2
Factor Analysis Results: Rotated Component Matrix^{a}.

Variable	Components (factors)		
	F_1	F_2	F_3
Pin	0.950		
Dr	0.906		
L	0.804		
D	0.792		
PL		-0.918	
*PH*2			0.941

^{A} *Extraction Method: Principal Component Analysis.*
Rotation Method: Varimax with Kaiser Normalization.
Rotation converged in 5 iterations.

Using the data from Table 2, the following equations can be written down for the factors F_1, F_2, F_3, in which the variables participate in linear combinations. We get:

$$\begin{aligned} F_1 &= 0.950Pin + 0.906Dr + 0.804L + 0.792D \\ F_2 &= -0.918PL \\ F_3 &= 0.941PH2 \end{aligned} \tag{2}$$

The obtained 3 factors are used for PCR analysis to build a linear regression model. The results are given in Table 3. Using unstandardized coefficients B from this table, the regression equation (1) takes the following specific form:

TABLE 3

RESULTS FROM PCR METHOD: COEFFICIENTS OF THE REGRESSION MODEL WITH CONFIDENCE INTERVALS^{A}.

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Intervals for B	
	B	Std. Error	Beta			Lower Bound	Upper Bound
(Constant)	33.594	0.931		36.083	0.000	31.737	35.450
F_1	29.207	0.937	0.912	31.159	0.000	27.338	31.077
F_2	3.752	0.937	0.117	4.003	0.000	1.883	5.622
F_3	9.821	0.937	0.307	10.477	0.000	9.951	11.690

^{A} Dependent Variable: Pout.

TABLE 4

Adequacy Test of the Regression Equation (3).

D, mm	*Dr*, mm	*L*, cm	*Pin*, kW	*PL*, kW/cm	*PH* 2, Torr	*Pout*, W exper.	F_1	F_2	F_3	b_0	b_1	b_2	b_3	$\hat{P}out$,W , from (3)
58	58	200	5	12.5	0.5	120	2.686	-1.463	0.251	33.594	29.207	3.752	9.821	109.02

$$\hat{P}out = 33.594 + 29.207F_1 + 3.752F_2 + 9.821F_3 \qquad (3)$$

Also, using standardized coefficients Beta from Table 3 we write down the standardized regression equation

$$\overline{\hat{P}out} = 0.912F_1 + 0.117F_2 + 0.307F_3 \qquad (4)$$

This equation shows the relative importance of different factors in the magnitude of the dependent variable. In our case the first factor dominates and has the highest effect in the model. Also regarding (2), the more significant influence in decreasing order exert laser characteristics *Pin*, *Dr*, *L*, and *D*. The last two columns of Table 3 show 95% confidence intervals of model coefficients which will be necessary to verify the admissibility of the prediction models. The resulting regression model (3) has a coefficient of determination R2=0.935, or describes 93-94% of all utilized experiment data. The model and all regression coefficients are statistically significant at level 0.05 (Sig.=0.000). In order to test the adequacy of the obtained regression equation, an actual physical experiment is calculated. With the highest value of laser output power *Pout*=120 W known, this is predicted using equation (3) to be 109.02 W, i.e. with an error of about 9%. The details are given in Table 4. This result is consistent with the experiment measurements, which can be considered within the limits of 5-10% relative error.

5 RESULTS OF THE STATISTICAL PREDICTIONS FOR NEW EXPERIMENTS

Table 5 shows the calculated predicted values of laser generation *Pout*. For each variation of the theoretical new experiment v, a new factor analysis procedure is carried out and a corresponding PCR equation of type (1), (3) is found, so that the limits of the confidence interval are observed with the coefficients b_0, b_1, b_2, b_3, given in Table 3. The magnitudes of the obtained predicted values for output power are up to 140W which is 17% more than the maximum achieved experiment power of 120W (see for comparison Table 4). Figs. 2, 3, 4 show diagrams of some of the results from Table 5. One coordinate system shows the relationship to *Pout* for each pair of independent variables: (*D* and *L*), (*D* and *Pin*) and (*L* and *Pin*), respectively. All variables are chosen with respect to their influence in equations (4) and (2). In each of the figures, a darker background is used to outline the total area (cross-section) of each pair of independent variables. These figures are nomograms, which can easily be used to develop new laser sources and simulate new experiments.

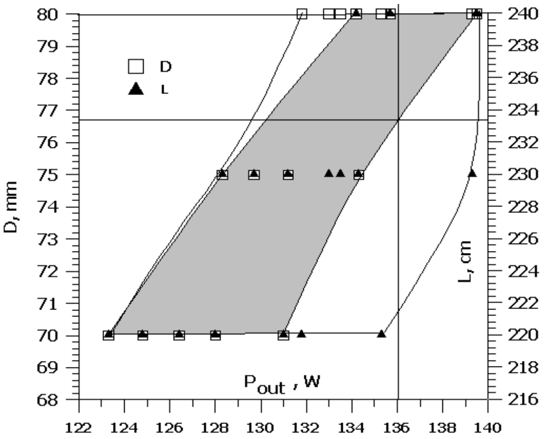

Fig. 2. Nomogram of predicted values of *Pout* depending on independent variable values in the case D and L.

TABLE 5
RESULTS OF THE NUMERICAL EXPERIMENTS

Varient	*D*, mm	*Dr*, mm	*L*, cm	*Pin*, kW	*PL*, kW/cm	*PH2*, Torr	F_1	F_2	F_3	b_0	b_1	b_2	b_3	$\hat{P}_{out}$,W, from (3)
v1	70	68	220	5	11.36	0.6	2.787	-0.908	0.605	34.789	30.575	3.320	10.389	123.3
v2	75	72	230	5	10.87	0.6	2.789	-0.640	0.468	34.854	30.756	3.405	10.387	128.3
v3	80	78	240	5	10.42	0.6	3.153	-0.483	0.309	34.934	30.992	3.501	10.394	134.2
v4	70	68	220	5.1	11.59	0.6	2.834	-1.010	0.623	34.761	30.494	3.258	10.410	124.8
v5	70	68	220	5.2	11.82	0.6	2.881	-1.110	0.642	34.831	30.692	3.194	10.430	126.4
v6	70	68	220	5.3	12.05	0.6	2.926	-1.209	0.660	34.852	30.752	3.128	10.451	128.0
v7	70	68	220	5.5	12.50	0.6	3.014	-1.401	0.694	34.893	30.874	2.994	10.489	131.0
v8	75	72	230	5.1	11.09	0.6	3.002	-0.786	0.476	34.875	30.817	3.346	10.406	129.7
v9	75	72	230	5.2	11.30	0.6	3.047	-0.882	0.495	34.896	30.877	3.287	10.425	131.2
v10	75	72	230	5.4	11.74	0.6	3.136	-1.077	0.532	34.937	31.002	3.161	10.463	134.3
v11	80	68	220	5.2	11.82	0.6	3.099	-0.999	0.417	34.903	30.891	3.181	10.439	131.8
v12	80	75	220	5.2	11.82	0.6	3.206	-1.000	0.390	34.950	31.032	3.192	10.461	135.3
v13	80	75	230	5.2	11.30	0.6	3.201	-0.829	0.372	34.952	31.043	3.291	10.440	135.5
v14	80	70	230	5.2	11.30	0.6	3.125	-0.827	0.392	34.918	30.941	3.282	10.424	133.0
v15	80	75	230	5.2	11.30	0.7	3.074	-0.959	1.111	35.003	31.040	3.343	10.881	139.3
v16	80	75	240	5.2	10.83	0.7	3.072	-0.803	1.095	35.006	31.051	3.430	10.870	139.5
v17	80	75	240	5.2	10.83	0.6	3.198	-0.674	0.359	34.955	31.058	3.383	10.419	135.7

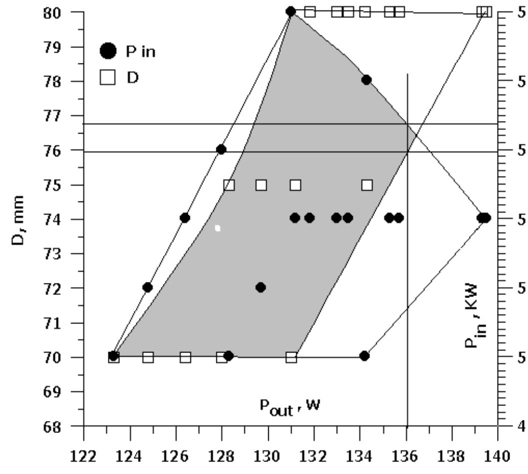

Fig. 3. Nomogram of predicted values of *Pout* depending on independent variable values in the case *D* and *Pin*.

As an example, for *Pout* = 136W, the possible values of the variables need to be determined. To this end, in each figure, a vertical line is drawn with *Pout* = 136W. For each of the figures, we get:

$$\begin{aligned} &D_1(77;80) \cap L_1(233;240) \\ &D_2(76;77) \cap Pin_1(5.30;5.33) \qquad (5) \\ &L_2(226;234) \cap Pin_2(5.14;5.34) \end{aligned}$$

After summarizing the interval values, the following cross-sections for each of the three variables are obtained:.

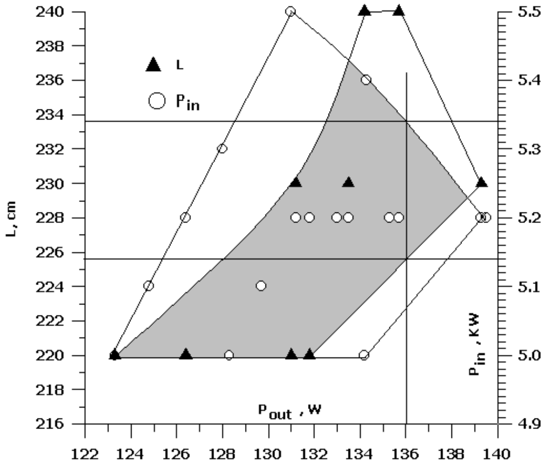

Fig. 4. Nomogram of predicted values of *Pout* depending on independent variable values in the case *L* and *Pin*.

$$\begin{aligned} D &= D_1 \cap D_2 = (77;77)\,\mathrm{mm} \\ L &= L_1 \cap L_2 = (233;234)\,\mathrm{cm} \qquad (6) \\ Pin &= Pin_1 \cap Pin_2 = (5.30;5.33)\,\mathrm{kW} \end{aligned}$$

This way, the presented nomograms and equations (5)-(6) can be used to predict statistically valid values of *Pout* for new experiments and laser sources. Finally, nomograms for other combinations of the input variables can be constructed and more detailed simulations can be carried out in order to improve the predictions and juxtapose all possible influences of laser characteristics.

6 CONCLUSION

Based on a new random statistical sample, containing 74 experiment datasets, factor and regression analysis is performed for the output power Pout of a CuBr laser. The influence of 6 independent quantities, which are known to significantly affect the output laser power *Pout*, is considered. A regression equation, which allows the calculation of laser output power, is obtained. The numerical calculations performed show that the developed model allows for an increase in laser output power by up to 17% of the maximum experiment value. Sample nomograms, which allow the application of the results from the developed model, are given. The applied approach enables the improvement of experimental studies for the perfection of existing laser sources as well as the development of new ones.

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REFERENCES

[1] S.E. McCoy, "Copper bromide laser treatment of facial telangiectasia: Results of patients treated over five years," *Lasers in surgery and medicine*, vol. 21, no 4, pp. 329-340, 1997.

[2] I.I. Balchev, N.I. Minkovski, N.V. Sabotinov, and I.K. Kostadinov, "Micromachining with copper bromide laser," in: Proc. of 12th International School on Quantum Electronics: Laser Physics and Applications, P.A. Atanasov, A.A. Serafetinides and I.N. Kolev, eds., *Proc. of SPIE*, vol. 5226, pp. 372-376, 2003.

[3] S.I. Yakovlenko, "Gas and Plasma Lasers," in: *Encyclopedia of Low-Temperature Plasma*, Series B, vol. XI(4), S.I. Yakovlenko, ed., Moscow: Fizmatlit, pp. 764-811, 2005.

[4] E. Metel, J. Podlinski, M. Dors, J. Mizeraczyk and N.V. Sabotinov, "CuBr Laser Visualization of the Bubbles Flow in a Pulsed Discharge in Water," in: Proceedings of 14th International School on Quantum Electronics: Laser Physics and Applications, P.A. Atanasov, T.N. Dreischuh, S.V. Gateva, and L.M. Kovachev, eds, *Proc. SPIE*, vol. 6604, pp. 660412, 2007.

[5] S.G. Gocheva-Ilieva and I.P. Iliev, *Statistical Models of Characteristics of Metal Vapor Lasers*, New York: Nova Science Publ. Inc., 2011.

[6] S.G. Gocheva-Ilieva, "Application of MARS for the construction of nonparametric models," in: *Mathematics and Education in Mathematics*, P. Russev, ed., Proc. of the 39th Spring Conference of the Union of Bulgarian Mathematicians, Sofia: Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, 2010, pp. 29-38, 2010. http://www.math.bas.bg/smb/2010_PK/tom/pdf/029-038.pdf

[7] I.P. Iliev, D.S. Voynikova, and S.G. Gocheva-Ilieva, "Application of the classification and regression trees for modeling the laser output power of a copper bromide vapor laser," *Math. Probl. Eng.*, vol. 2013, Article ID 654845, pp. 1-10, 2013. http://dx.doi.org/10.1155/2013/654845

[8] *NIST/SEMATECH e-Handbook of Statistical Methods*, http://www.itl.nist.gov/div898/handbook/, Accessed 30 July 2013.

[9] SPSS IBM Statistics 19, http://www-01.ibm.com/software/analytics/spss/. Accessed 30 July 2013.

[10] N.V. Sabotinov, et al., Bulg. patent No.:28674, 1975.

[11] N.V. Sabotinov, N.K. Vuchkov, and D.N. Astadjov, *Gas laser discharge tube with copper halide vapors*, United States Patent 4635271, 1987.

[12] I.T. Jolliffe, *Principal component analysis*, 2nd edition, New York: Springer, 2002.