# Minimax Optimization Of Dynamic Pendulum Absorbers For A Damped Primary System

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**Abstract**— In this paper, a minimax optimization procedure for dynamic vibration pendulum absorbers used with damped primary system is developed. An optimization problem is formulated providing the parameters of a pendulum absorber which can minimize the primary system vibration amplitude and decrease the sensitivity of the primary system response to uncertainties of excitation frequency. Three types of pendulum absorber are investigated: classical pendulum, pendulum-torsional spring and dual pendulum. The benefits of using the different types of pendulum vibration absorber are presented and the main system frequency response is compared leading to a recommendation about the most suitable type to a specific application.

**Index Terms**—Minimax optimization procedure, damped primary system, classical pendulum absorber, pendulum torsional spring absorber, dual pendulum absorber.

## 1 Introduction

THE vibration absorber was invented by Watts in 1883 [1] and Frahm in 1909 [2]. The dynamic vibration absorber (DVA) or tune-mass damper (TMD) is a passive device which is used to suppress the primary system vibration by attaching a DVA. Basically, linear vibration absorber consists of a mass and a spring, and usually viscous damper added to them. They have been widely used in tall building, structure and accurate machinery devices [3,4]. Since Den Hartog investigation of DVA carried out in 1928, a lot of work has been developed on the optimization design and tuning of DVA. Den Hartog derived a closed form of optimum DVA parameters for single degree of freedom (SDOF) DVA attached to undamped primary system. According to Den Hartog, the most favorable frequency response of the primary system should have two equal resonance magnitude peaks [5]. This makes the primary system response less sensitive to variations of excitation frequency. Randall proposed more realistic configuration of damped DVA attached to damped primary system [6]. His study submitted a numerical optimization scheme to determine the optimal DVA parameters which differ significantly from those achieved in case of classic system. Pennestri presented an application of the Chebyshev's criterion to the optimal design of the damped DVA and developed ready-to-use computational graphs. He handled seven unknown variables and six constraint equations in the minimax objective function [7]. Pade and Steffen (2000) studied the reduction of vibration levels of machinery systems using dynamic vibration absorbers. They proposed a general methodology for the optimal selection of the absorber parameters to guarantee their effectiveness over a frequency band [8]. Miller (2003) investigated the nonlinear mechanical properties of elastomers in reference to their potential application in adaptive-passive tuned vibration absorbers. He focused on changing the absorber natural frequency in the range 45 to 211 Hz [9]. Bonsel, Fey and Nijmeijer (2004) studied the application of a linear dynamic vibration absorber to a piecewise linear beam system to suppress its first resonance. They considered both the undamped and damped absorbers [10]. Varpasuo (2006) studied the response characteristics of stochastic vibration absorber. He assessed the effectiveness of the vibration absorber for various stiffness and damping configurations [11]. Jang et.al. (2007) studied the design of a cantilever type multi-DOF vibration absorber. They used three cantilever beams and a rigid body suspended from the beam as a spring-mass system to reduce resonant vibrations of the main system [12]. Ozkan (2010) studied the application of passive tuned mass damper (single or multiple) to Euler-Bernoulli beams and examined their effectiveness base on free and forced vibration characteristics of the beams [13]. Liao et.al. (2011) presented an active-adaptive tuned vibration absorber based on magneto rheological elastomer. They studied two feedback types of activation force and incorporated phase-lead compensator to eliminate the time delay effect during signal processing [14]. Mirsanei et.al. (2012) presented a design for an adaptive tuned dynamic vibration absorber based on a smart slider-crank mechanism [15]. Shen and Ahmadian (2013) studied analytically four semi-active dynamic vibration absorbers including the time delay induced by measurement and execution. They discussed the effect of time delay on the control performance [16]. Huang and Lin (2014) designed a vibration absorber called periodic vibration absorber for mechanical systems subjected to periodic excitation. They claimed that their design can absorb significant amount of higher harmonics in addition to the fundamental harmonic [17]. Hassaan (2014) presented a novel idea for using the mass-spring vibration absorber as an energy harvester. He defined a new frequency called harvesting frequency depending on the mass ratio of the absorber and independent of the main system damping [18]. The DVA has another configuration which is pendulum-like absorber. Pendulum dynamic vibration absorber (PDVA) consists of lumped mass suspended to primary system by massless rod as simplified configuration. PDVA may consist of single or dual pendulum. Moreover, PVDA can take many different configurations [19, 20].

## 2 FORMULATION OF MOTION EQUATIONS

### 2.1 Single Pendulum DVA

The mathematical model A of PDVA May be developed for the two degree of freedom system shown in Fig.1. For analysis simplification, it is assumed that the stiffness is linear, massless rod with lumped mass $m_a$. Two equations of motion can be expressed in matrix form as follows:

$$\begin{bmatrix} m_a + m_p & m_a l \\ m_a l & m_a l^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} c_p & 0 \\ 0 & c_a \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} k_p & 0 \\ 0 & m_a g l \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} F e^{jwt} \\ 0 \end{bmatrix} \quad (1)$$

Where x is the displacement of primary system and θ is the angular displacement of PDVA. $m_p, k_p$ and $c_p$ are the mass, stiffness, and damping coefficient of the primary system, respectively. $m_a$, $c_a$ and $l$ are the mass, damping coefficient and distance between centroid of mass and pivot axis respectively. F and ω are the amplitude and frequency of the excitation force acting on the primary system.

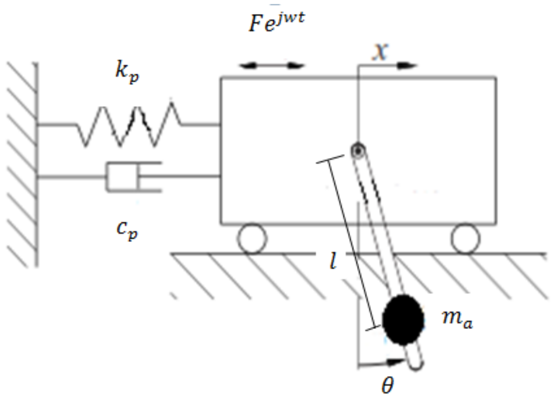


**Figure 1. Model A of a single pendulum.**

The steady state response of the main mass and the pendulum are given by:

$$x = Xe^{j\omega t} \tag{3}$$

$$\theta = \Theta e^{j\omega t} \tag{4}$$

Defining the following non-dimensional variables:

$$\begin{aligned} &\alpha = \frac{X}{F/k_p}, && \zeta_p = \frac{c_p}{2m_p\omega_p}, && \omega_p = \sqrt{\frac{k_p}{m_p}}, \\ &\gamma = \frac{\theta l}{F/K_p}, && \zeta_a = \frac{c_a}{2m_a l^2 \omega_a}, && \omega_a = \sqrt{\frac{g}{l}}, \\ &\Omega = \frac{\omega}{\omega_p}, && \mu = \frac{m_a}{m_p}, && T = \frac{\omega_p}{\omega_a}, \end{aligned} \tag{5}$$

Combining Eqs. 1, 3, 4 and 5 yields:

$$\begin{bmatrix}\alpha\\ \gamma\end{bmatrix} = \begin{bmatrix} -(1+\mu)\Omega^2 + 2\zeta_p j\Omega + 1 & -\mu\Omega^2 \\ -T^2\Omega^2 & -T^2\Omega^2 + 2\zeta_a Tj\Omega + 1 \end{bmatrix}^{-1} \begin{bmatrix}1\\0\end{bmatrix} \tag{6}$$

where α is the normalized vibration amplitude of the primary mass and γ is the normalized vibration amplitude of the pendulum.

### 2.2 Single Pendulum DVA with Torsional Spring

Model B has the same configuration of model A except an additional torsional spring is attached at the PDVA pivot. The amplitudes equation remains as Eq.6 except the value of $\omega_a$ in Eq. 5 which will be:

$$\omega_a = \sqrt{\left(\frac{m_a gl + k_a}{m_a l^2}\right)}. \tag{7}$$

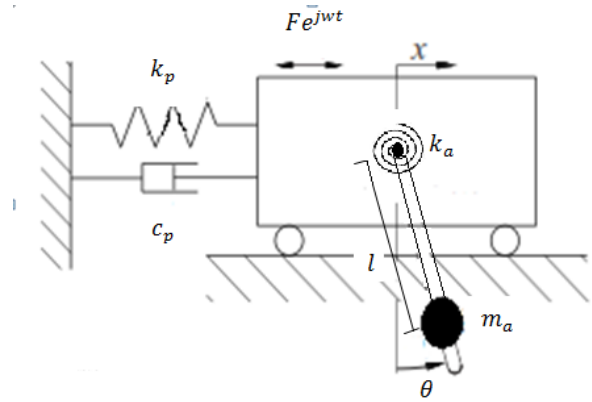


**Figure 2. Model B of single pendulum with torsional spring.**

### 2.3 Dual Pendulum DVA

Model C consists of a primary system and dual PDVA mounted on it as shown in Fig. 3. Dual pendulum has six parameters: $m_{ai}$, $l_i$ and $c_{ai}$, where $i = 1, 2$. It becomes a three degree of freedom system with three differential equations of motion taking the matrix form:

$$\begin{bmatrix} m_{a1}+m_{a2}+m_p & m_{a1}l_1 & m_{a2}l_2 \\ m_{a1}l_1 & m_{a1}l_1^2 & 0 \\ m_{a2}l_2 & 0 & m_{a2}l_2^2 \end{bmatrix} \begin{bmatrix}\ddot{x}\\ \ddot{\Theta}_1\\ \ddot{\Theta}_2\end{bmatrix} + \begin{bmatrix} c_p & 0 & 0 \\ 0 & c_{a1} & 0 \\ 0 & 0 & c_{a2}\end{bmatrix} \begin{bmatrix}\dot{x}\\ \dot{\Theta}_1\\ \dot{\Theta}_2\end{bmatrix} + \begin{bmatrix} k_p & 0 & 0 \\ 0 & m_{a1}gl_1 & 0 \\ 0 & 0 & m_{a2}gl_2\end{bmatrix} \begin{bmatrix}x\\ \Theta_1\\ \Theta_2\end{bmatrix} = \begin{bmatrix}Fe^{j\omega t}\\ 0\end{bmatrix} \tag{8}$$

The steady state response of the three masses after the transients have vanished is given by the equations:

$$x = Xe^{j\omega t} \tag{9}$$

$$\theta_i = \Theta_i e^{j\omega t}\text{, where } i = 1,2 \tag{10}$$

taking the following non-dimensional variables:

$$\begin{aligned} &\gamma_i = \frac{\theta l_i}{F/K_p} && \zeta_{ai} = \frac{c_{ai}}{2m_{ai}l_i^2\omega_{ai}}, && \omega_{ai} = \sqrt{\frac{g}{l_i}}, \\ &\Omega = \frac{\omega}{\omega_p}, && \mu_i = \frac{m_{ai}}{m_p}, && T_i = \frac{\omega_p}{\omega_{ai}} \end{aligned} \tag{11}$$

where i = 1,2

Combining Eqs.8- 11 yields:

$$\begin{bmatrix}\alpha\\ \gamma_1\\ \gamma_2\end{bmatrix} = \begin{bmatrix} -(1+\mu_1+\mu_2)\Omega^2 + 2\zeta_p j\Omega + 1 & -\mu_1\Omega^2 & -\mu_2\Omega^2 \\ -T_1^2\Omega^2 & -T_1^2\Omega^2 + 2\zeta_{a1}T_1 j\Omega + 1 & 0 \\ -T_2^2\Omega^2 & 0 & -T_2^2\Omega^2 + 2\zeta_{a2}T_2 j\Omega + 1 \end{bmatrix}^{-1} \begin{bmatrix}1\\0\\0\end{bmatrix} \tag{12}$$

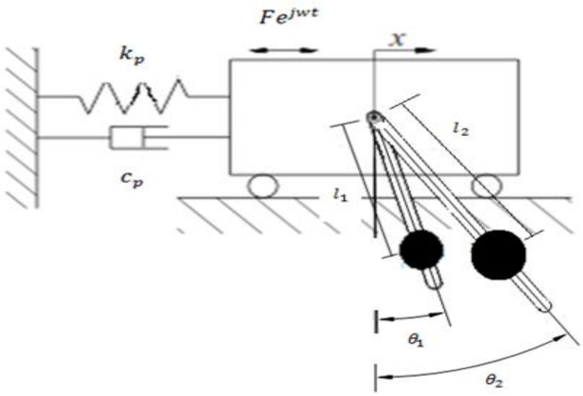


**Figure 3. Model C of dual pendulum DVA.**

## 3 MINIMAX OPTIMIZATION FORMULATION

### 3.1 Single Pendulum DVA

The vibration amplitude of the primary system which is the objective function is a function of frequency $(\Omega)$ and four parameters: $\zeta_a, \zeta_p$, T and μ. The normalized frequency $(\Omega)$ has a range $\Omega \in (0, 2)$. Randall has considered that $\zeta_p$ and μ are independent parameters characterized by primary system design, so the last parameters $\zeta_a$ and T are considered optimization parameters [6]. The frequency response curve (FRC) of a 2-DOF system is specified by two peaks, i.e. point p and a as shown in the Fig.4. It can be observed that the difference between the two peaks amplitude $(\alpha_p, \alpha_a)$ is high in addition to rapid variation of vibration amplitude between $\alpha_p$ and $\alpha_a$. In other words, the vibration amplitude is sensitive to the variation of the excitation frequency. In practical application, it must be avoided. Therefore, many authors discuss how to minimize the maximum vibration amplitude of the primary system which is the objective function

$$\min \max \alpha \tag{13}$$

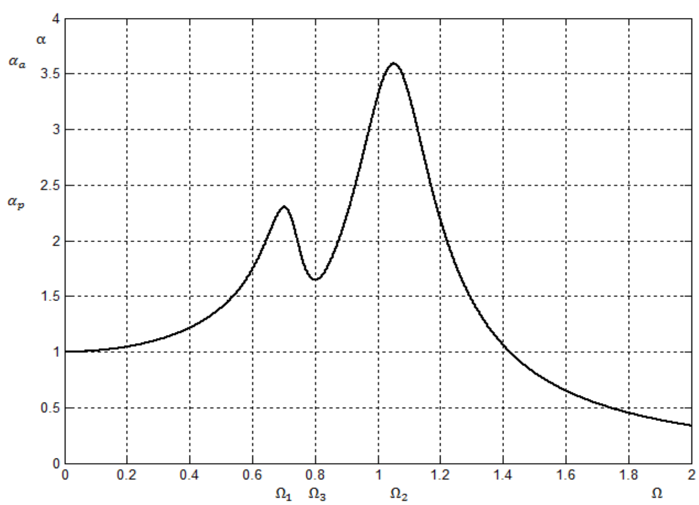


**Figure 4: FRC of the primary system at μ=0.1 and $\zeta_p$=0.1**

Den Hartog supposed that the most favorable response curve should have the same maximum amplitude [5]. That is:

$$\alpha_p = \alpha_a \tag{14}$$

Then Pennestri formulated additional constraints of optimization problem according to Chebyshev's min-max criterion as follow [7]:

$$\begin{gathered} \left.\frac{d\alpha}{d\Omega}\right|_{\Omega=\Omega_1} = \left.\frac{d\alpha}{d\Omega}\right|_{\Omega=\Omega_2} = \left.\frac{d\alpha}{d\Omega}\right|_{\Omega=\Omega_3} = 0 \\ -\alpha(\Omega_1) + L + \Delta = 0 \\ -\alpha(\Omega_2) + L + \Delta = 0 \\ -\alpha(\Omega_3) + L + \Delta = 0 \end{gathered} \tag{15}$$

Where Δ is the maximum deviation of the response curve from the value $\alpha = L$ and $\Omega_1, \Omega_2$ and $\Omega_3$ are the frequency ratios where such a curve attains a maximum or a minimum. Another formulation of the constraint condition was developed by Fang and Wang as follows [21]:

$$\begin{gathered} \left.\frac{d\alpha}{d\Omega}\right|_{\Omega=\Omega_1} = \left.\frac{d\alpha}{d\Omega}\right|_{\Omega=\Omega_2} = 0 \\ \alpha(\Omega_1) = \alpha(\Omega_2) \\ \Omega_1 < \Omega_s < \Omega_2 \end{gathered} \tag{16}$$

where $\Omega_s = \sqrt{1 - 2\zeta_p^2}$ is the normalized frequency according to peak amplitude of single degree of freedom damped system. In this work, the difference between two peaks of the FRC of the 2DOF system is observed. It substantially dependent on optimization parameter T and slightly dependent on $\zeta_a$ as shown in Fig.5. This difference can be expressed as:

$$\Delta\alpha = \alpha_p - \alpha_a \tag{17}$$

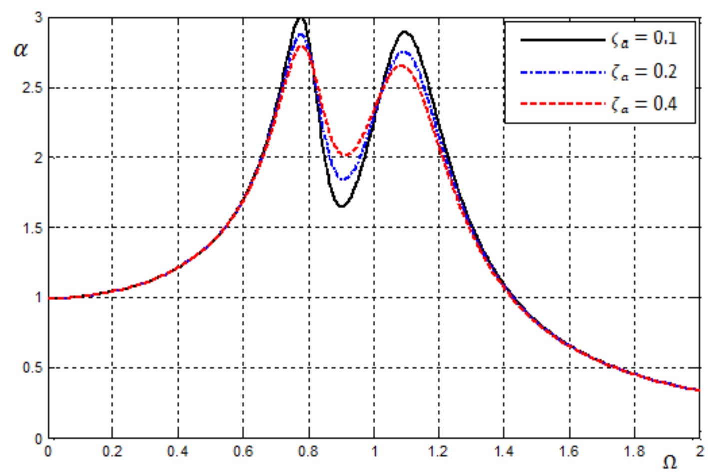


**Figure 5. FRC of the primary system at μ=0.1 and $\zeta_p$=0.1**

The difference between to resonant peaks has a positive value, i.e., $\Delta\alpha > 0$ at $T < T_{opt}$ and vice versa, i.e., $\Delta\alpha < 0$ at $T > T_{opt}$ as shown in Fig. 6. According to Den Hartog [5], the most favorable FRC should have $\Delta\alpha = 0$. Consequently, the optimum FRC should have one optimum parameters $T_{opt}$ and $\zeta_{opt}$.

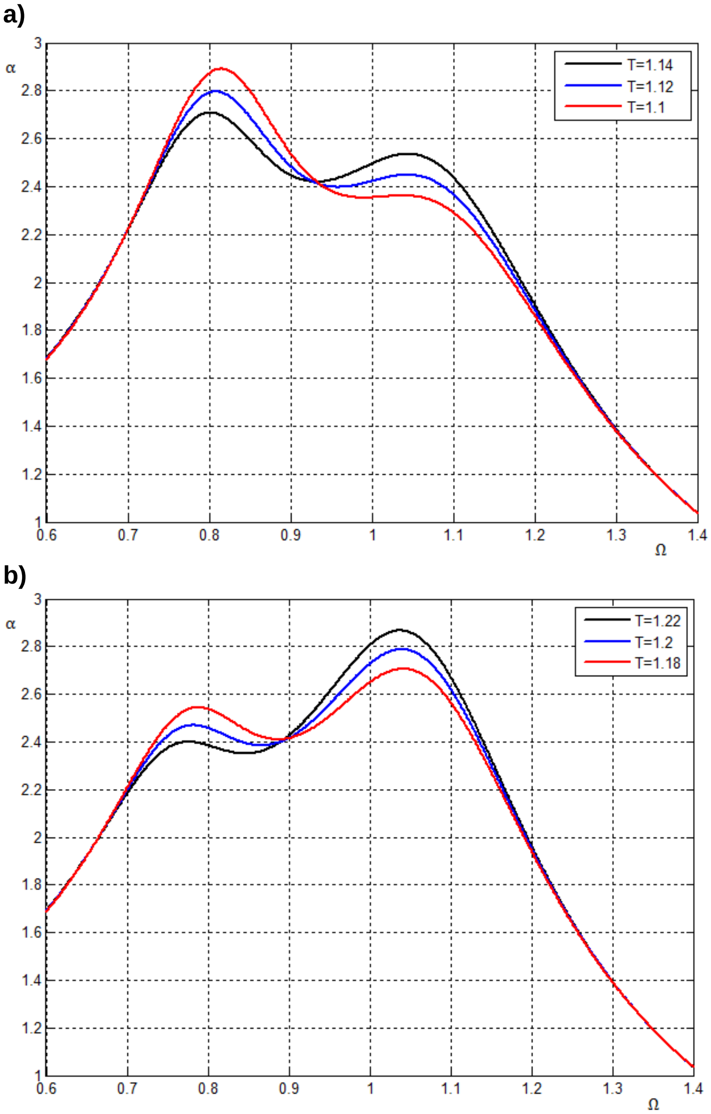


**Figure 6: The FRC of the primary system at $\mu$=0.1 and $\zeta_p = 0.1$ (a) $T < T_{opt}$. (b) $T > T_{opt}$.**

The optimization constraint equations don't need all of this complexity where the objective function of Eq.13 may have the constraint Eqs.14-16 inherently. In other words, same optimum parameters can be achieved by the following simple constraint equations:

$$\zeta_a > 0 \ , T > 0 \tag{18}$$

### 3.2 Single Pendulum DVA with Torsional Spring

The vibration amplitude of the primary system for Model B doesn't have a great modification except the value of $\omega_a$ in Eq.7, the optimum pendulum parameters $T_{opt}$ and $\zeta_{opt}$.

### 3.3 Dual Pendulum DVA

The vibration amplitude of the primary system for model C which is the objective function is function of the frequency $(\Omega)$ and seven parameters: $\zeta_{ai}$, $T_i$, $\mu_i$ and $\zeta_p$, where $i = 1, 2$. Also the normalized frequency $(\Omega)$ varies from 0 to 2. Randall has considered that $\zeta_p$ and $\mu_i$ are independent parameters characterized by primary system design [6]. Ikeda examined the dynamics of the two identical pendulums dynamic vibration absorber [19]. Brzeski et.al .suggested that differentiation of the lengths of the pendulum can be lead to improve DVA efficiency [20]. His optimization problem was to deal with only the second pendulum length $(l_2)$ and the other five parameters are fixed. In this work, the optimization problem is dealt with four normalized pendulum parameters: $\zeta_{ai}, T_i$ where $i = 1, 2$ which minimizes the maximum amplitude of the primary system meanwhile reduce the sensitivity. We can achieve the optimal choice of parameters: $\zeta_{a1opt}$, $\zeta_{a2opt}$, $T_{1opt}$, $T_{2opt}$ by applying objective function in Eqs.12 and 13 and constraint equation:

$$\zeta_{ai} > 0 \ , T_i > 0 \ , \text{where } i = 1, 2 \tag{19}$$

The FRC of primary system for model C which is three degree of freedom system should be having three peaks vibration amplitude. According to constraints Eq.14, outputs of our simple optimization algorithm must satisfy the following constraints:

$$\begin{gathered}
\left.\frac{d\alpha}{d\Omega}\right|_{\Omega=\Omega_1} = \left.\frac{d\alpha}{d\Omega}\right|_{\Omega=\Omega_2} = \left.\frac{d\alpha}{d\Omega}\right|_{\Omega=\Omega_3} = \left.\frac{d\alpha}{d\Omega}\right|_{\Omega=\Omega_4} = \left.\frac{d\alpha}{d\Omega}\right|_{\Omega=\Omega_5} = 0 \\
-\alpha(\Omega_1) + L + \Delta = 0 \\
-\alpha(\Omega_2) + L + \Delta = 0 \\
-\alpha(\Omega_3) + L + \Delta = 0 \\
-\alpha(\Omega_4) + L + \Delta = 0 \\
-\alpha(\Omega_5) + L + \Delta = 0
\end{gathered} \tag{20}$$

Where $\Omega_i$,i=1 to 5 are the frequency ratios of three peaks and two valleys vibration amplitude.

## 4 NUMERICAL RESULTS AND DISCUSSIONS

### 4.1 Single Pendulum DVA

Consider a linear damped primary system with the following characteristics: $\zeta_p = 0.1, \mu = 0.1$. A MATLAB optimization code is written for this task producing the following optimal parameters:

$$\zeta_{a_{opt}} = 0.1989, \qquad T_{opt} = 1.1602 \tag{21}$$

Fig 7 shows the peak primary system amplitude $\alpha_{max}$ for $\zeta_a \in (0.05, 0.5)$ and $T \in (0.5, 1.5)$. The minimum value for $\alpha_{max}$ is 2.623 which is the same minimum value that can be achieved by linear DVA [7]. The mass ratio $\mu$ is an important factor on the maximum primary system amplitude $\alpha_{max}$ which is continuously decreases with increasing $\mu$, Fig. 8 shows that the constraint Eqs.15 and 16 are attained accurately.

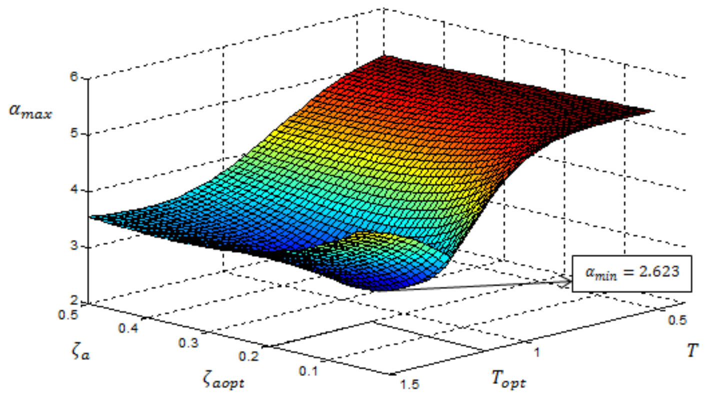


**Figure 7: $\alpha_{max} vs \zeta_a and T$ surface at $\mu$=0.1 and $\zeta_p = 0.1$**

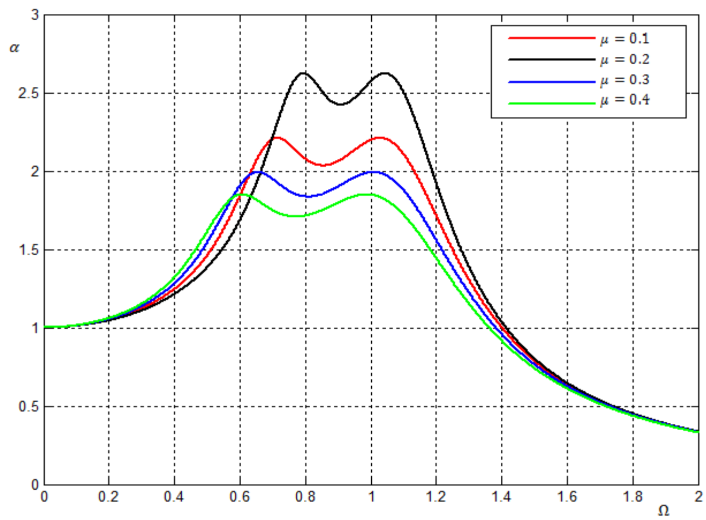


**Figure 8: The FRC of the primary system at $\zeta_{aopt}, T_{opt}$ for $\zeta_p = 0.1$**

As mentioned in section 3.1, $\zeta_a$ has a slight influence on $\alpha_{max}$. In practical applications, the vibration amplitude of PDVA ( $\gamma_{max}$ ) should be also restricted with certain magnitude. This value can be reduced by 10 % when the value of $\zeta_a$ raise up to 0.25 as shown in Fig. 10 while $\alpha_{max}$ increases by only 2 % (Figs.9 and 10).

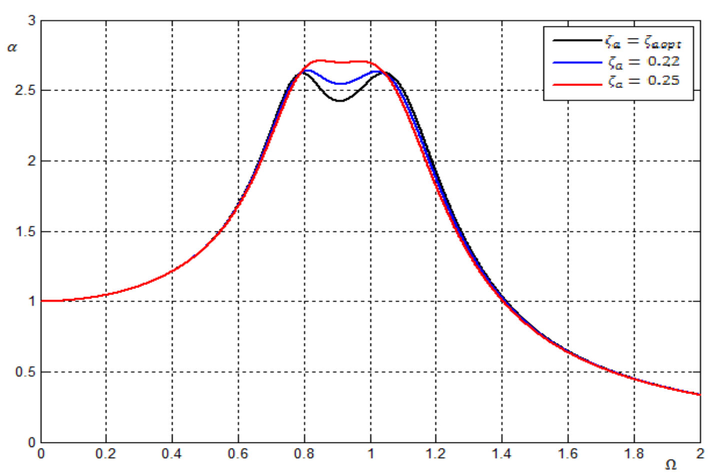


**Figure 9. The FRC of the primary system at $\mu$=0.1, $\zeta_p = 0.1$ and $T_{opt}$ for different $\zeta_a$**

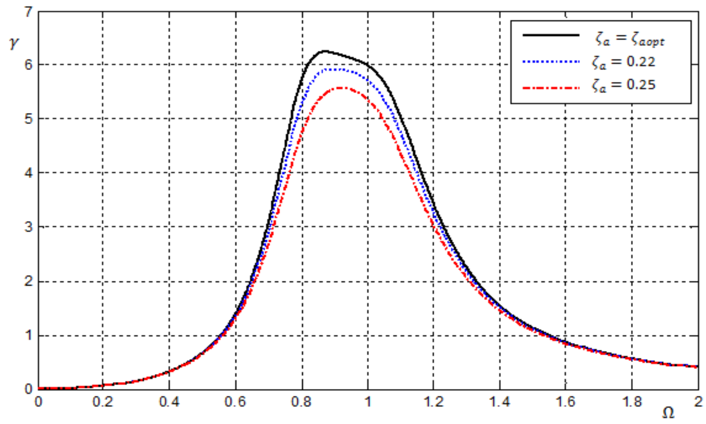


**Figure 10. The FRC of PDVA at $\mu$=0.1, $\zeta_p = 0.1$ and $T_{opt}$ for different $\zeta_a$**

## 4.2 Single Pendulum DVA with Torsional Spring

Optimum parameters for model B are similar to optimum one for model A where the optimization program of single pendulum can be re-used. The difference between each is the equation of $\omega_a$ in Eq.7. Therefore, the FRC of the main system and PDVA remain the same. The main purpose of torsional spring is to add extra stiffness for the system and meanwhile increase the design flexibility of PDVA.

**Design example:**

Consider a system with the following characteristics [6]:

$$m_p = 100 \text{ kg}, \zeta_p = 0.1, \mu = 0.1 \text{ and } \omega_p = 100 \text{ rad/s}$$

First, we apply the Model A to such system at the optimum parameters of Eq.20. Substitution into Eq.5 leads to an optimum pendulum length $l_{opt}$ of 1.32 mm. This value is not reasonable to establish. Therefore, we should look for another configuration with flexibility to choose a suitable pendulum length like model B. Fig. 11 shows the relationship of $k_a$ and l at optimum PDVA parameters and corresponding $\omega_{aopt}$ which is 86.2 rad/s.

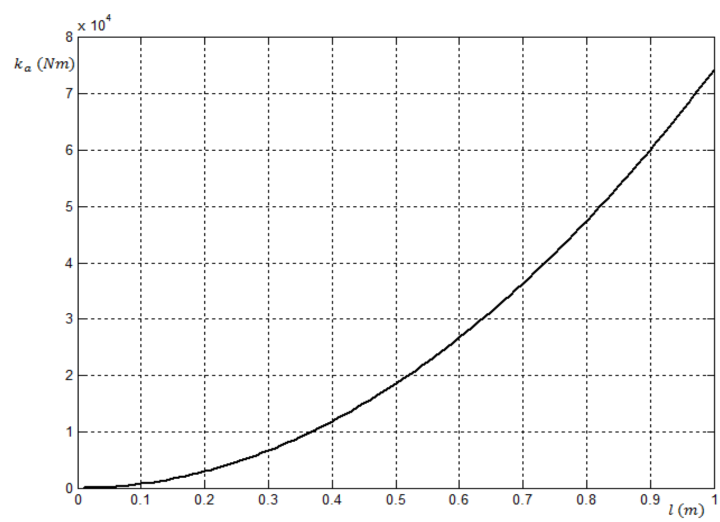


**Figure 11. $k_a$ & $l$ relationship for Model B at $\zeta_p = 0.1$, $\mu = 0.1$ and optimum parameters $\zeta_{aopt}, T_{opt}$**

### 4.3 Dual Pendulum DVA

We consider the same numerical example in the previous sections. The total normalized mass ratios of two PDVA are assumed equal to 0.1, i.e.$\mu_1 + \mu_2 = \mu = 0.1$. It is not necessary to take two identical pendulums as in reference [8]. Optimum parameters of each pendulum commensurate to some extent as reported in Table 1. Mass ratios $\mu_1$ & $\mu_2$ do not influence $\alpha_{max}$ as long as the summation of them is constant. Figs. 13 and 14 show the FRC of the two PDVA and emphasize that identical mass ratio is not a better choice for minimum $\gamma_{max1}$ & $\gamma_{max2}$ [8]. The FRC of model C has approximate flat region over three peaks as shown in Fig. 12. In comparison, The FRC of model B has great tortuosity shape (Fig. 8) which means that model C has less sensitivity to variation of the normalized frequency Ω.

**Table 1. The optimum choice of pendulum parameters for different $\mu_1$ & $\mu_2$**

| $\mu_1$&$\mu_2$ | $\zeta_{a1opt}$ | $\zeta_{a2opt}$ | $T_{1opt}$ | $T_{2opt}$ | $\alpha_{max}$ |
|---|---|---|---|---|---|
| 0.02 & 0.08 | 0.1028 | 0.1613 | 0.9869 | 1.2214 | 2.4921 |
| 0.03 & 0.07 | 0.1202 | 0.1481 | 1.0026 | 1.2377 | 2.4797 |
| 0.04 & 0.06 | 0.1342 | 0.1352 | 1.0188 | 1.2516 | 2.4756 |
| 0.05 & 0.05 | 0.123 | 0.1461 | 1.2637 | 1.0361 | 2.4773 |

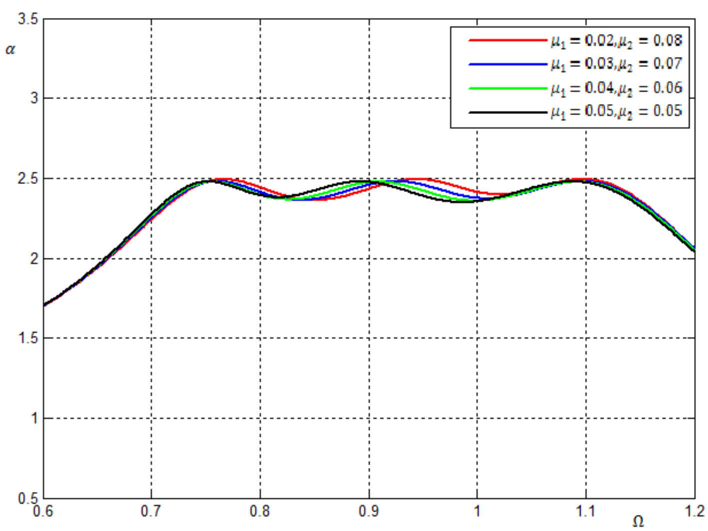


**Figure 12. FRC of the primary system for model C at $\zeta_p = 0.1$**

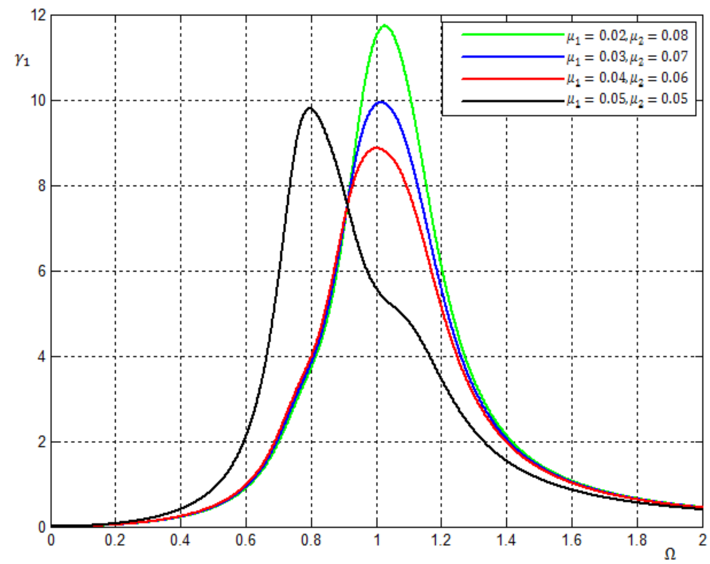


**Figure 13. FRC of first pendulum for model C at $\zeta_p = 0.1$**

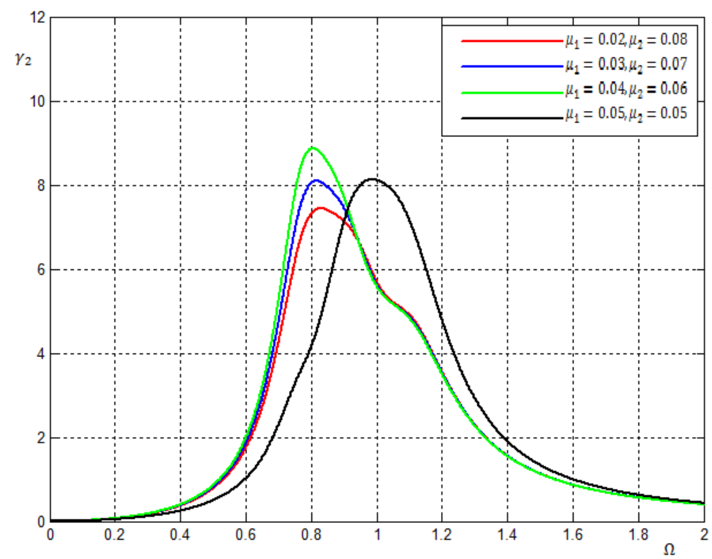


**Figure 14. FRC of second pendulum for model C at $\zeta_p = 0.1$**

## 5 CONCLUSIONS

In this investigation, the design of three different types of PDVA was presented . A simple optimization concept was developed leading to the optimum parameters efficiently for model A . Furthermore, the reliability of usage this optimization concept was emphasized where the FRC of the primary system for model C has achieved the constraint (Eq. 20) without adding it to the optimization program. This reduced the complexity of the optimization problem. Models A and model C could reduce the primary system amplitude by 47% and 50% respectively. The dual pendulum gave the best vibration attenuation with less sensitivity. Model B had a good advantage where the pendulum length can be adjusted to reduce the effect of nonlinearity.

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