Performance Of WF Precoding Combined With LDPC Coding

T.R. Rajaonarison, A.A. Randriamitantsoa, P.A. Randriamitantsoa

Abstract -The MIMO system (multiple input multiple output)constitutesa solution to increase the capacity and/or to guarantee the quality of a wireless communication. There are many scheme MIMO making it possible to bring diversity of emission with an aim of making the transmissions robust in a difficult environment of propagation. There are two different approaches to bring this diversity: open-loop system and closed-loop system. A WF precoder is a closed-loop system which increases the capacityof the MIMO channel.In addition, the application of the error correcting codes improves the performances of the transmission. We propose in this article to combine the WFprecoder WF with a code LDPC. This new scheme makes it possible to have better a BER/capacity compromise.

Keywords - BER, code, LDPC,MIMO, precoder, receiver, WF.

1. Introduction

The benefits of using multiple antennas at both thetransmitter and the receiver in a wireless system arewell established. Multiple-input multiple-output(MIMO) systems enable a growth in transmissionrate linear in the minimum number of antennas ateither end. MIMO techniques also enhance link reliability and improve coverage. MIMO is now entering next generationcellular and wireless LAN products with the promise ofwidespread adoption in the near future. Iterative decoding of low-density parity-check (LDPC)codes is a powerful method for approaching capacity onnoisy channels. In this paper, we will be interested in the combination of WF precoder which increases the capacity MIMO, and the LDPC code which ensure reliability.

2. MIMO Antenna System

The MIMO system uses multiple antennas in transceiver and multiple antennas in receiver. The figure 1 represents a MIMO system with nTtransmitter antennas and nR receiver antennas, which gives $nT \times nR$ ways created. Each way is characterized by a complex coefficienthij. The MIMO channel created is modelled by a matrix H whose the coefficients correspond to the distortions of amplitude and phase on the way going of the transmitterantenna j towards the receiver antenna$i$. The expression of the matrix H is given by:

$$y = Hs + n \qquad (1)$$

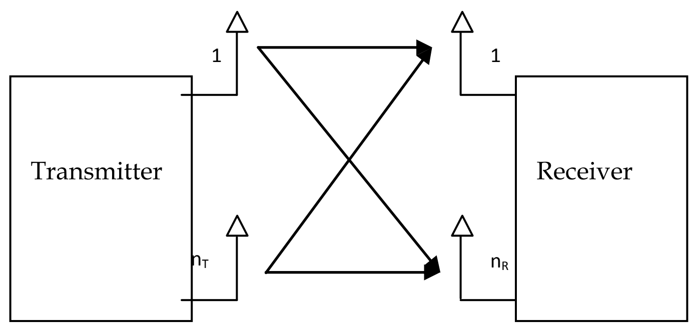

Fig.1 : Representation of the MIMO system.

3. WF Precoder

The precoding is a MIMOtechnique with CSI-Tapplied to the emission to optimize the performance of the channel using the SVD. The SVD makes it possible to obtain the diagonalisation of the channel i.e. obtaining the virtual channel by modeling a MIMO channel in parallel SISO system. It also makes it possible to determine the capacity of MIMO systems and to apply a linear pretreatment to the data to be transmitted and a post processing to the received signal. The SVD is given by:

$$H = U \sum V^H \qquad (1)$$

U and V are the unitary matrix, $\sum$ is a diagonal matrix such as the number of eigenvalues r is the rank of the channel matrix H and it is equal to $\min(n_T, n_R)$.
By replacing the matrix H in yby $U \sum V$, it obtains :

$$y = (U\sum V^H)\, s + n \qquad (3)$$

By applying a preprocessing to the symbols transmitted (s) on the side of the transmitter and a postprocessing to the reception, i.e. by multiplying member to member by U^H, we obtain the relation:

$$U^H y = \tilde{y} = U^H\, ((U\sum V^H)\, s + n)$$
$$= \sum V^H\, s + U^H\, n$$

Let us pose $U^H n = \tilde{n}$ (noise vector) and

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$s = V\tilde{s}$ (symbol vector)
We obtain :

$$\tilde{y} = \Sigma V^H V \tilde{s} + \tilde{n}$$

$$\tilde{y} = \Sigma \tilde{s} + \tilde{n} \qquad (4)$$

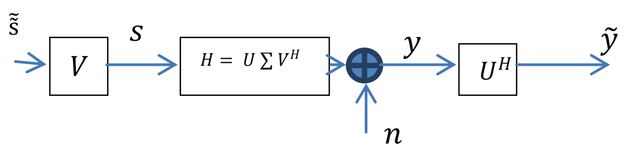

Fig.2 : Representation of the received signal estimated $\tilde{y}$ after decomposition SVD.

A precoder WF is a precoder which maximizes the capacity of a MIMO system. After the diagonalisation of the channel, we obtain b independent ways.
According to the constraint of following power to the emission :

$$trace\{F_d F_d^*\} = P$$

Then :

$$P = \sum_{i=1}^{b} f_i^2$$

And

$$\lambda_i = \sum_{i=1}^{b} \sigma_i^2$$

Pis the total power available.
The solution optimizing the capacity is given by:

$$f_i^2 = \left\{ \begin{array}{c} \Psi - \left(\frac{1}{\sigma_i}\right)^2 \; si \; \Psi > \left(\frac{1}{\sigma_i}\right)^2 \;\; pour \; i = 1, \dots, b \\ 0 \; ailleurs \end{array} \right\} \qquad (5)$$

With :

$$\psi = \frac{P + \gamma_\psi}{b_\psi} \; and \; \gamma_\psi = \sum_{i=1}^{b_\psi} \frac{1}{\sigma_i^2}$$

$b_\psi$is the number of ways used by the precoder.

4 LDPC CODES

The low density parity check codes (LDPC) are linear block codes with sparse parity check matrices H. The originalGallager's LDPC codes are called regular LDPC codes in which the number of 1's is the same in every row andevery column. The figure 3 shows an example of the matrix H proposed by Gallager witha weight t=3 by column and a weight of t=4 per line.

1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0
0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1

Fig.3 :Example of matrix Hproposed by Gallager.

4.1 LDPC Encoding

The LDPC encoding is an operation which consists in generating a word of this code $x = [u, p]$ starting from the information word u, generating matrix G or matrix of parity check $H = [HsHp]$.

$$[H_s H_p].[u\ p]^T = 0^T \qquad (6)$$

$$p^T = H_p^{-1}.H_s.u^T = H_p^{-1} v^T \qquad (7)$$

$v$is a projection vector. This relation shows that the first constraint on the code is the existence of the Hp^{-1}. The matrix Hp can be of form triangular allowing by simple substitution the calculation of the redundancy bits. Particularly, the matricesHp of the type bi-diagonal are interesting to obtain a simple encoding. The determination of theredundancy bitscan be done by initially calculating the vector of projection then by an accumulation of this vector:

$$\mathrm{p_k} = \mathrm{p_{k-1}} + v_k \qquad (8)$$

The first redundancy bit is obtained by :

$$p_0 = \sum_i v_i \qquad (8)$$

The other bits of redundancy is then calculated by simple substitution.

4.2 LDPC Iterative Decoding

The LDPC iterative decoding uses the belief propagation (BP) between the nodes of the graph. The exchanged messages of belief, are calculated by an algorithm "sum multiplication" or one of its derived algorithms.The algorithm of BP can be decomposed into two steps, the update of the group of the nodes of parity and the update of the group of the nodes of variable. These steps constitute an iteration. These updates can have different orders.

5. MIMO LINEAR RECEIVER

5.1 Receiver Zero Forcing (ZF)

The receiver ZF is a simple receiver which is based on the inversion of the matrixH. His square and invertible.
The objective of this receiver is to minimize the noise.
The noise is given by :

$$n = y - Hs$$

If H is complex, the estimated symbols are given by :

$$\tilde{s} = (H^H H)^{-1} H^H y = H^{\dagger} y \qquad (9)$$

5.2 Receiver Mean Minimum Squared Error (MMSE)

With the difference of the ZF which reverses the matrix and which thus increases the level of noise, this receiver minimizes the total error caused by the contribution of the noise and the mutual interference of the signals as a result it resists the noise better by not separating perfectly the subchannels. Like the ZF, the EQMM is a linear receiver, therefore the estimated symbol is formed by :

$$\tilde{s} = X^H y$$

The expression of the estimated symbols is

$$\tilde{s} = \left(H^H H + \frac{n_T}{\rho} I_{n_R}\right)^{-1} H^H y \qquad (10)$$

$\rho (= \frac{P}{\sigma^2})$is the mean SNR per antenna receiver.

6. Performance of WFPrecoder Combined With LDPC Code

6.1 WF precoder combined with LDPC code

The figure 4 represents the system of transmission proposed.

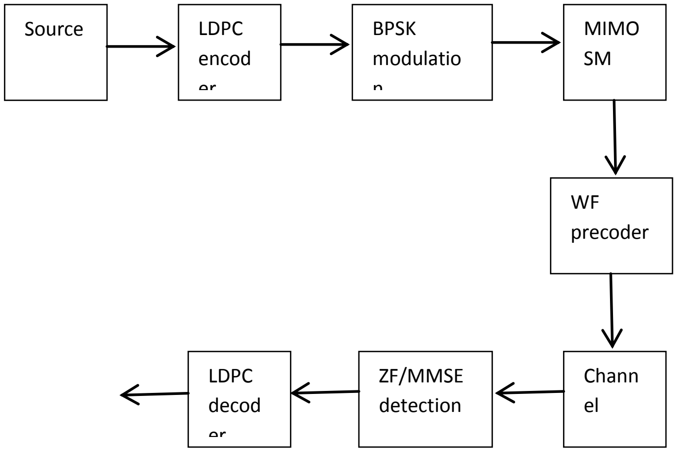

Fig. 4 :The system of transmission proposed.

6.2Simulations, results and discussion

The figure 5 represents the BER for MIMO system 2X2 without LDPC :

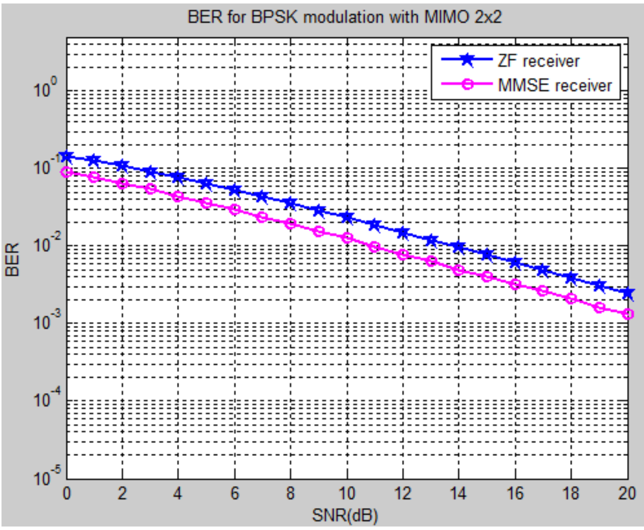

Fig. 5 :for MIMO system 2X2 without LDPC.

The figure 6 represents the BER for MIMO system 2X2 with LDPC :

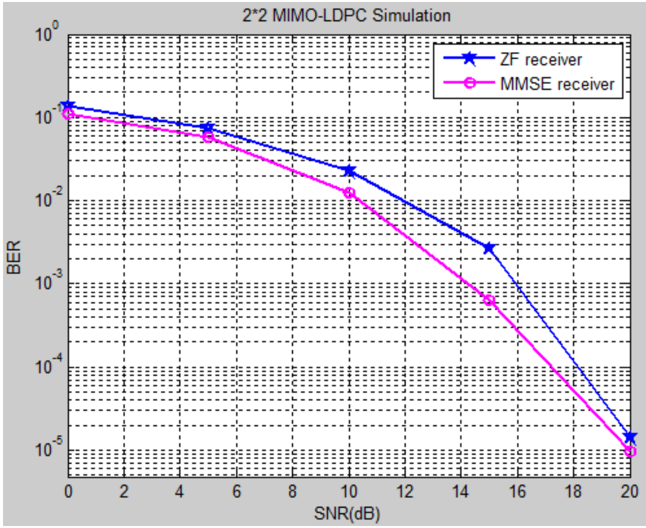

Fig. 6 :BER for MIMO system 2X2 with LDPC.

The figure 7 represents the BER for MIMO-WF system 2X2 with LDPC :

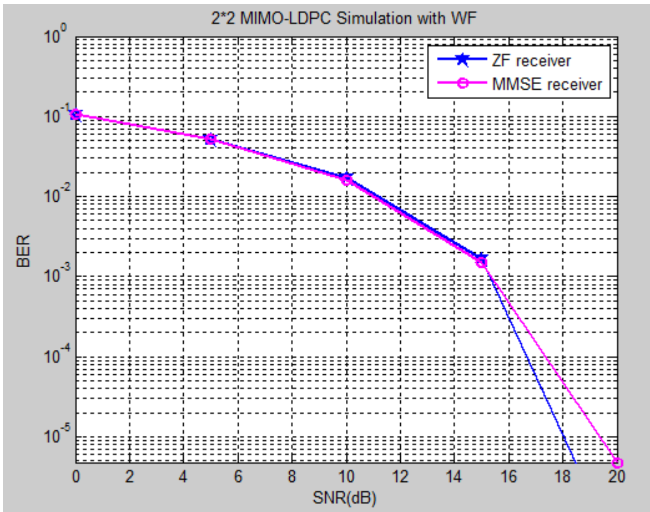

Fig. 7 :BER for MIMO-WF system 2X2 with LDPC.

The table 1 represents the comparison of results for ZF receiver:

TABLE 1 :Comparison of results for ZF receiver.

SNR (dB)	MIMO	MIMO-LDPC	MIMO-WF-LDPC
0	1,5.10^{-1}	1,5.10^{-1}	10^{-1}
4	8.10^{-2}	8.10^{-2}	6.10^{-2}
8	3,5.10^{-2}	3,5.10^{-2}	2,5.10^{-2}
12	1,5.10^{-2}	9.10^{-3}	7.10^{-3}
16	3.10^{-3}	9.10^{-4}	3.10^{-4}

The table 2 represents the comparison of results for MMSE receiver:

TABLE 2 : Comparison of results for MMSE receiver.

SNR (dB)	MIMO	MIMO-LDPC	MIMO-WF-LDPC
0	10^{-1}	10^{-1}	10^{-1}
4	4.10^{-2}	6.10^{-2}	6.10^{-2}
8	2.10^{-2}	2.10^{-2}	$2{,}5.10^{-2}$
12	9.10^{-3}	4.10^{-3}	6.10^{-3}
16	2.10^{-3}	$2{,}5.10^{-4}$	4.10^{-4}
20	10^{-3}	10^{-5}	10^{-6}

These results show us that:

- The MIMO system with LDPC presents a low bit error rate
- For weak SNR, the MIMO-LDPC is more performant than MIMO- WF - LDPC
- For raised SNR, the MIMO-WF-LDPC is more performant than MIMO-LDPC

7. Conclusion

To send the signals with MIMO system, a coder should be used. In the case of the MIMO, the most adequate coder is the coder time spaces. In order to still increase the performances of MIMO systems in terms of robustness, flows, quality of service, there are methods to pre-equalize the data before the emission. To carry out a precodage, it is necessary to know the state of the channel on the level of the receiver. The precoder WF maximizes the capacity [1]. We saw that the combination of the precoder WF with coding LDPC makes it possible to have a weak BER especially when the SNR is high. Consequently, this new scheme that we proposed is very interesting.

REFERENCES

[1] T.R. Rajaonarison, A.A. Randriamitantsoa, P.A. Randriamitantsoa, « Performance of MIMO precoders and MIMO receivers in term of capacity», : International Journal of Engineering & Scientific Research, ISSN 2347-6532, Vol.2 Issue 7, pp.111-121, July 2014.

[2] V. Tarokh, H. Jafarkhani «A differential detection scheme for transmit diversity », IEEE journal on selected areas in communications, Vol. 18, No. 7, pp. 1169 – 1174, July 2000.

[3] R. GGallager., « Low Density Parity Check Code », Ph.D Dissertation, M.I.T, 1963.

[4] A.Sanderovich, M.Peleg, S.Shamai, « LDPC coded MIMO multiple access with iterative join decoding», IEEE Transactions in Information Theory, Vol. 51 n°4, pp. 1437-1450 : 2004.

[5] E.Sharon, A.Ashikmin, S.Litsyn, « EXIT functions for binary input memoryless symmetric channel », IEEE Transactions on Communications, Vol. 54 n°7, pp. 1207 – 1214, Jul. 2006.

[6] D.Reynolds, X.Wang, « Low complexity turbo equalization for diversity channels», IEEE Transactions on signal processing, Vol. 81 n°5, pp. 989-995 : 2001.

[7] M. Vu, A. Paulraj, « MIMO wireless linear precoding», IEEE Transactions on signal processing magazine, pp. 86-105 : Sept 2007.