Determinism Of Hydrological Recession Processes On Oueme Basin Catchment And Application Of Least Actions Principle

Avahounlin Ringo F., Alamou Eric, Obada Ezékiel, Afouda Abel

ABSTRACT: This work aim to analyze the hydrodynamic process of oueme basin catchment (basin located in Benin between 7°58N and 10°12N latitude and 1°35 and 3°05E longitude). From rainfall and discharge data chronic rates over the period 2000-2009, empirical hydrological modeling based on linearization of Boussinesq's equation and least actions principle methods were used to predict the mechanism of the water drain, to determine the streamflow recession curves to the watershed scale and compare the modeling findings with hygrogram obtained by applied of principle of the least actions. An analysis of drying up showed a varied trend in four sub-basins. At Beterou and Bonou sub-basin, the non-linear character observed reflects a succession of phases of drying up. A conceptual linearization formulation of the basic equations of Boussinesq considering the non-linear character of the drying up of the two sub-basins helped simulate low flow rates with high efficiency and to determine the types low flow curves. Successfully comparing analyzed of modeling findings with recession curves obtained by least actions principle confirm the heterogeneity of recession nature at oueme basin scale.

Key-words: basic equations of Boussinesq, hydrodynamic process, least actions principle, oueme basin catchment recession analysis

1 INTRODUCTION

THe exceptional drought observed in recent decades that have been seen a sharp decrease in flow was impacted land water and groundwater and productions that depend caused a highly increased are drying rivers. During this period, Sudano-Sahelian's river low flows are systematically lower of chronic data observed [1] According to the same author the flow has even stopped of the Niger's river in Niamey in 1985 and Senegal's river in Bakel in 1984. Senegal's river would disappear without the artificial support of some water projects. In Benin, Oueme's watershed has undergone as well as all the watershed of the West African. A water stress which came from climatic and anthropogenic causes [2]. The flow deficit of about 40% corresponds to a decrease in rainfall of 15 to 20% [3]. This has had significant and sometimes dramatic consequences for other populations resulting from a decrease in water resources. These consequences can be sustainable in the hydrological cycle especially when we have a long period of drought or excess water. To address the concern when managing water resources during critical periods associated with low flow, it is necessary to have a sound knowledge of dieting of low water across river basins. This study can provide a good understanding of the processes of the groundwater discharge and other delayed runoff; an estimate of the watershed vulnerability to drought and a search for statistical evidence of a possible modification of low flows. This study aim to analyze the recession processes of Oueme's river, to predict the mechanism of the water drain, to determine the streamflow recession curves to the watershed scale and compare the modeling findings with hygrogram obtained by applied of principle of the least actions.

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2 DATA AND METHODS

2.1 Framework and study data

The Oueme catchment covers an area of 46.920 km^2 at the hydrometric station of Bonou. With a distance of 523 km representing 47.2% of the area covered [**4**]. It extends from latitude 7°58 to 10°12 and from longitude 1°35 to 3°05 [**5**]. The rainfall mainly controlled by the atmospheric circulation of two air masses and their seasonal movement (the Harmattan and the monsoon). It characterized by three types of climate: from the bimodal climate in the south to unimodal climate in the north with a transition phase in the center (Le Barbé et al. 1993). The averages of annual rainfall (1960 - 2010) are 1204.77 mm at the Bétérou rainfall station and 1098.40 mm at Savè. The dynamic of the flow is characterized by a high discharge during the rainy season. The maximum flow statement of 1960 to 2010 between May and September are the order of 267.88 m^3/s at Bétérou and 478.87 m^3/s at Savè's bridge. From November to May almost all the rivers dry up and the averages of low flows go from 49 to 5m^3/s at Savè station and 17 to 2m^3/s at Bétérou [**5**]. This study was conducted at different spatial scales (Fig.1). As such four watersheds (Affon, Beterou, Savè and Bonou) were considered. The data used in this study are the daily series of discharges and precipitations. This series is taken from the database of the Water office in Benin and that of the National of Meteorology office of Benin for what concerns the rainfall. Using data from the different rainfall stations distributed across each watershed the average rainfall was made by applying the Thiessen's statistical method.

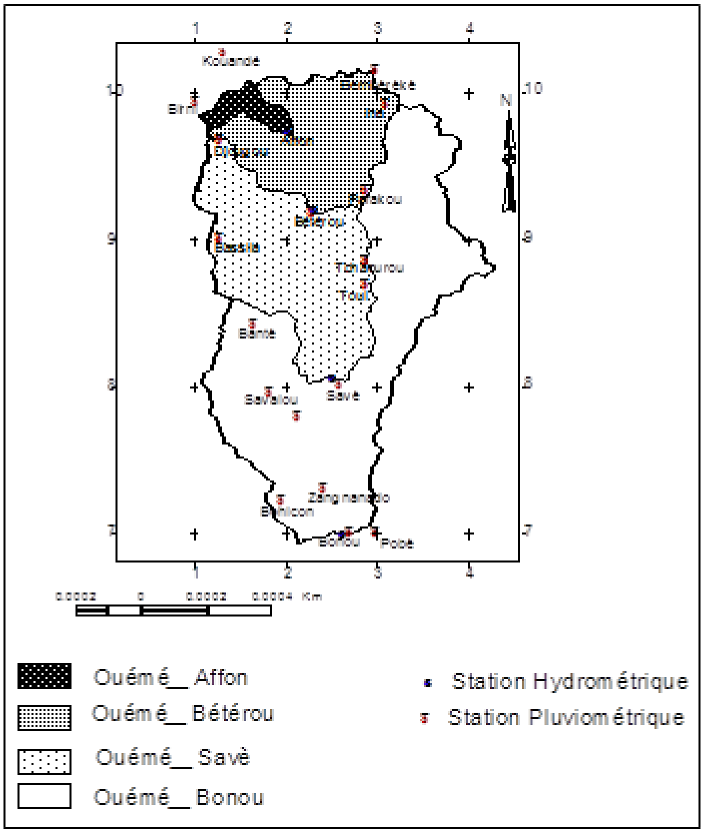

Fig. 1*: Oueme hydrological map and different sub- catchment*

2.2 Analysis Methods

- **Extraction of recession events**

The identification of data sets was based on the definition of recession asking two conditions. The first states that the discharges are decreasing consecutively. It is to identify sequences of consecutive decreasing discharges. For a sequence can have meaning. It must not be too short. For this a minimum of five days is set for recession events [**6**]. As for the second condition the discharges are not influenced by precipitation. To eliminate the influence of rainfall, we are ensured that the identified sequences correspond to periods without precipitation. However we consider that below a certain threshold precipitation especially parasite rains are not able to influence the flow. This threshold was set at 1mm/day. To study the depletion in each catchment. The coefficient of depletion of each event of recession extract is determined by the law of Maillet.

- **Analysis of recession processes**

To study the hydrological operation of Oueme catchment during low water. Recession processes streams are analyzed at different spatial scales for modeling the discharge of groundwater. For this, a conceptual and parsimonious model is developed for the modeling of depletion rates in the watershed. Recession's analysis is based on [7] development model. This is a linearization of the Boussinesq's equation. By the fact that this method eliminates the time as a reference making it unnecessary to determine the precise beginning of the recession. More its application is easy. So it appears to be a better method study of low water. It is defined by:

$$-\frac{dQ}{dt} = aQ^b \qquad \text{(Eq.1)}$$

Where *b* and *a* are constant functions of the physical and hydraulic properties of the aquifer. Practically *a* is the numerical value of *-dQ/dt* when *Q* = 1 and b defines the slope of the relationship between - *dQ / dt* and *Q* in log-log space [**7**] and *dt* is a constant. We considered *dt* = 1 day. The *-dQ/dt* versus *Q* plots can be interpreted in terms of storage–discharge relationships. The values of b are set to 1, 2 or 3 (Vogel and Kroll. 1992). For *b* = 1 the relationship between -*dQ / dt* and *Q* so between the groundwater discharge and runoff is linear and non-linear when $b \neq 1$. *dQ/dt* and *Q* values are calculated from equations 2 and 3:

$$\frac{dQ}{dt} = \frac{Q_{i+1} - Q_i}{t_{i+1} - t_i} = \frac{Q_{i+1} - Q_i}{\Delta t} \qquad \text{(Eq. 2)}$$

$$Q = \frac{Q_{i+1} + Q_i}{2} \qquad \text{(Eq. 3)}$$

Q_i is the daily flow at the outlet. t_i is the corresponding day and $t_{i+1} - t_j = \Delta t$ = 1 day. After studying the linearity of the recession processes and identifies the contribution of the reservoirs in depletion rates, the corresponding depletion rates are estimated. The prediction of low flows is based on a formulation that considers the response of the catchment following precipitation as the sum of the responses of all the reservoirs of the catchment. The reservoirs are systems of drainage and aquifers. This formulation proposed by [**8**] is translated by:

$$q = \sum_{i=1}^{n} F_i v_i S_i \qquad \text{(Eq. 4a)}$$

(Eq. 4)

$$Q = qA \qquad \text{(Eq. 4b)}$$

q ($m.T^{-1}$) is the discharge per unit area of the watershed. F_i is the proportion of each reservoir in the watershed. The coefficient v_i is the reservoir depletion coefficient (T^{-1}) and S_i is the index of water storage on the each reservoir (m) and n is the number of reservoir. *A* is the area of the watershed contributing to the low flows and *Q* ($m^3.T^{-1}$) is the discharge of the watershed. The depletion rates are determined after recession processes analysis across each catchment. The water storage index *S* is etermined by assuming that at the beginning of recession. The initial index S_0 is equal to:

$$S_0 = S_T + \frac{r}{v} \qquad \text{(Eq. 5)}$$

Where S_T is the height of water that could fill the voids in the ground before the start of the flow, *r* is the daily recharge rate and *v* the coefficient of depletion. From S_0 the reservoir is depleted using the equation of G. Tison (Eq. 6). The volume of water stored per unit area at time t is defined by: $$V_t = \frac{V_0}{1+v} \quad or \quad S = \frac{V}{A} \Rightarrow S_t = \frac{S_{t-1}}{1+v}$$ (Eq. 6)

Where S_t is the water storage index at time *t*, S_{t-1} is the water storage index date t_{-1} and v is the coefficient of depletion. For each S_t corresponds one Q_t discharge calculated using Eq .4 b. Assuming that the water which fills the empty of the soil is retained in the soil, so do not contribute to recession flow. The S at each step of depletion is [**9**]:

$$S = (S_t - S_T)$$ (Eq. 7)

This assumption allows neglecting S_T. Some parameters are impossible to measure or evaluate. This incomplete identification requires the estimate by a calibration step. This is the case with the recharge rate *r* considered uniform throughout the watershed.

- **Least actions principle applying**

An application of the principle of Least Action allowed the team of Afouda and al (2006) to write:

$$\frac{d}{dt}(\lambda Q) + \vartheta Q^{2\vartheta-1} = 0$$ (Eq. 8)

Where Q is the out-flow, the parameter describes the humidity of soil and the physiographic and geological parameters of the basin while is a parameter of non linearity. This equation takes in account two shapes of non linearity. One distinguishes a shape of non linearity kinematics associated to the laminar character or turbulent of the out-flow and that is described by the parameter and a dynamic linearity shape associated to the evolution of the conditions of humidity of the basin pouring and that can be described λ =λ (t). [**10**] debated this equation then for the different values of. The gait based on the use of the principle of least action permits to recover the different empiric expressions in use in the survey of these curves and to give account of the behavior of the basin pouring during the low flow period. The analytic solution can be considered in two different cases. There is then λ ≠ 1 and λ= cst. This case has been clarified by [**11**]. According to these authors, what returns this particularly attractive approach is that three of the analytic solutions of the equation of Boussinesq in dimension -1 for the no - enclosed horizontal aquiferous (two exact solutions ([**12**], [13]) and a solution approached by linearisation ([12]) can be expressed under the shape of the Eq.1. The identification of the recession parameters driving to $\vartheta = \frac{\beta+1}{2}$ and $\lambda = \frac{\beta+1}{2\alpha}$ proves the explicit interpretation of ϑ as parameter of non linearity, according to the physical measurements and the hydraulic properties of the aquiferous, in agreement with the results knew [**12**], [**13**], [**7**], [**11**] and of other authors mentioned by these last. The analytic solution is given by the following relation (according to λ and ϑ):

$$Q(t) = Q(t_0)[\varsigma.(t-t_0)+]^{\frac{1}{2(1-\vartheta)}}$$ (Eq. 9)

with $\varsigma = \frac{2(1-\vartheta)\vartheta}{\lambda} Q_0^{2(\vartheta-1)} = cst$ and $\vartheta \neq 1$.

This relation is similar to the one gotten by [**14**] from the different considerations, to describe the dynamics of recession. And while considering the case of the non homogeneous model, the equation of Euler-Lagrange may be:

$$\frac{d(\lambda Q)}{dt} + \vartheta Q^{2\vartheta-1} = X(t)q(t)$$ (Eq. 10)

Where Q_t et Q_{t-1} is respectively the discharge pouring at the dates t and (t-1); X(t) describes the state of the basin on the day j and q_{t-1} the rainy state or no of the day (t-1). Therefore we deduce:

$$Q_t = Q_{t-1} - \frac{\vartheta}{\lambda} Q_{t-1}^{2\vartheta-1} + \frac{X(t)}{\lambda} q_{t-1}$$ (Eq. 11)

Since the parameters λ and ϑ are considered like constants one will use the period of recession deducted of the Eq. 11 to determine their values. The equation of the recession deducted of the Eq. 11:

$$Q_t - Q_{t-1} = -\frac{\vartheta}{\lambda} Q_{t-1}^{2\vartheta-1}$$ (Eq. 12)

what drives to:

$$\ln(Q_{t-1} - Q_t) = \ln\left(\frac{\vartheta}{\lambda}\right) + (2\vartheta-1)\ln(Q_{t-1})$$ (Eq. 13)

Either while putting $A = (2\vartheta - 1)$ et $B = \ln\left(\frac{\vartheta}{\lambda}\right)$ we have:

$$\ln(Q_{t-1} - Q_t) = A\ln(Q_{t-1}) + B$$ (Eq. 14)

The daily mean discharge using permits thanks to a good linear adjustment between $\ln(Q_{t-1} - Q_t)$ and $\ln(Q_{t-1})$ to get the values of A and B. The values of the parameters of the model are determined by $\vartheta = \frac{A+1}{2}$ and $\lambda = \frac{A+1}{2\exp^{B}}$.

These two values permit to simulate the recession period. In this survey the methods of based modelling on the principle of least action (ModHypMA) have been applied to compare and to validate the results of the streamlined modelling of the recession process in oume sub-catchment.

3 DISCUSSION OF FINDINGS

3.1 Statistical analysis of recession events

The recession of a river course mainly due to two processes that are the destocking of water stored in the river system and the contribution of groundwater to rivers [**4**]. This hydrological phenomenon can be evaluated as soon as the interval between two rains becomes long enough to observe the groundwater empty. But because of the multiplicity of events in the flood hydrograph. It is very complex to define recession flows in full rainy season [**15**]. And to avoid the phenomena of interferences rains during rainfall recession, the depletion events were determined by the combined of hydrographs and corresponding daily rainfall. These correspond to the periods of decay discharges without the influence of precipitation i.e. the periods of no precipitation or extremely low (less than 1mm) (Fig. 2). Most of information derived from the statistical analysis of the extraction at each watershed scale is reported in Table 1.

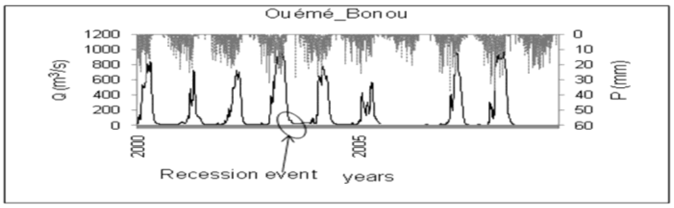

Fig 2: *Superposition of rainfall and hydrograph for recession events extraction*

TABLE 1
Describe Statistics of low flow at each catchment

Watershed	Affon	Beterou	Savè	Bonou
Events number	11	18	16	17
Qmax (m3/s)	9.85	58.14	56.03	133.5
Qmin (m3/s)	0.001	0	0.17	3.6
Qmoy(m3/s)	2.51	6.02	8.94	24.78
Var	5.64	86.06	124.93	456.44
δ	2.37	9.27	11.17	21.36
CV	0.94	1.54	1.25	0.86

The average recession discharges vary from 2.51m^3/s (at the outlet of Affon) to 24.78m^3/s (at the outlet of Bonou). Hydrographs for each recession event (Fig. 3) show a variation of the recession duration according to the watershed area. More recession speed thus draining groundwater is fast at the beginning of recession process to become very slow towards the end of recession. By applying the Maillet's model, the coefficient of recession for each event were calculated reports a space and temporal fluctuation (Fig. 4) of this with a very low coefficient of variation on Affon and Savè watershed and relatively high on of Bétérou and Bonou.

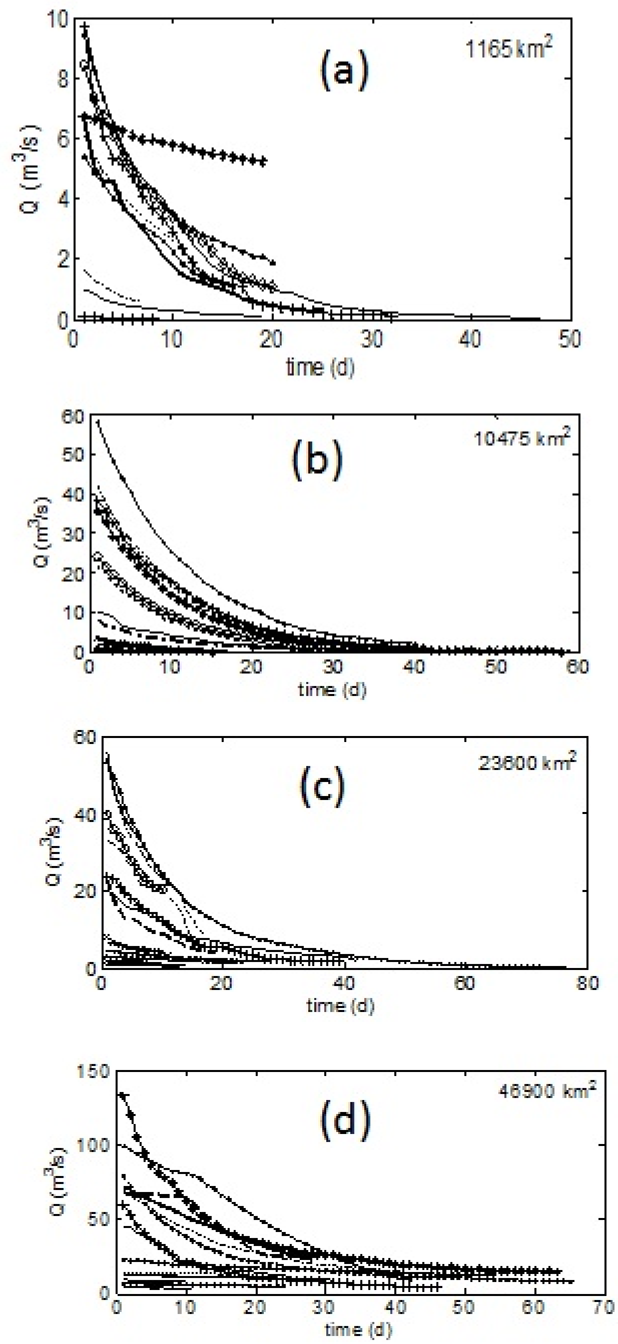

Fig. 3: *Hydrographs of recession events*

(a) Oueme sub-catchment at Affon
(b) Oueme sub-catchment at Beterou
(c) Oueme sub-catchment at Savè
(d) Oueme sub-catchment at Bonou

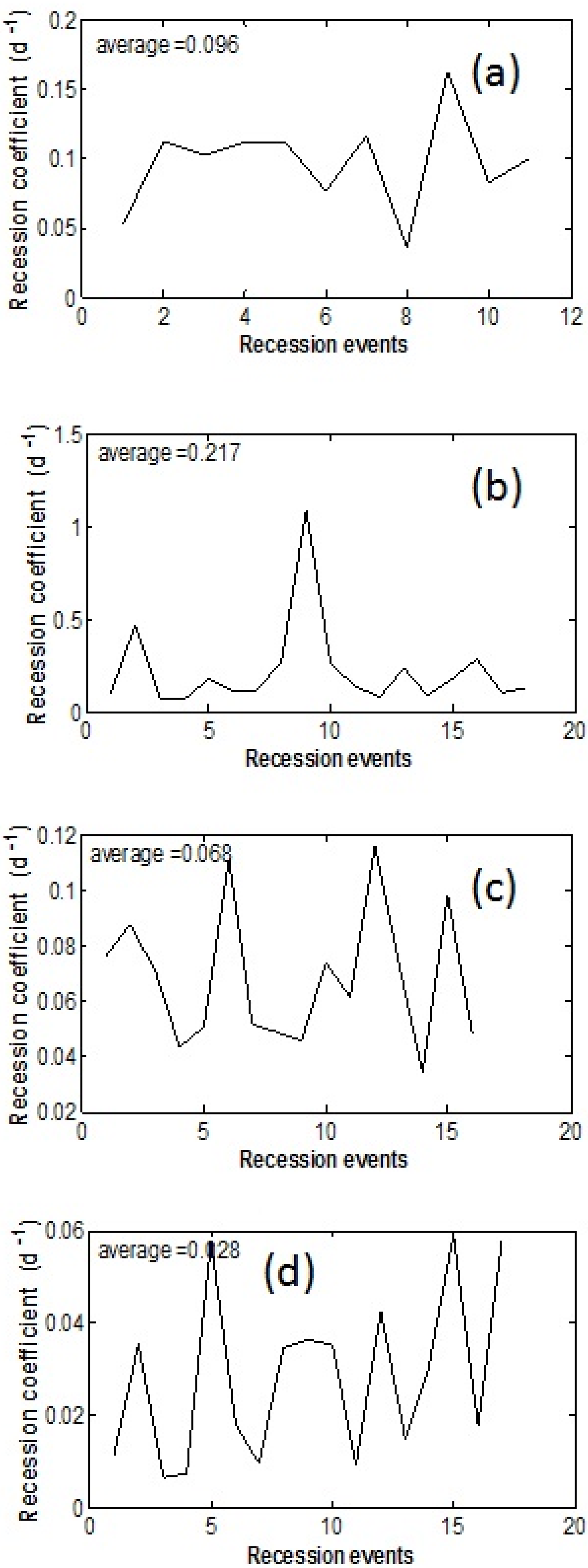

Fig. 4: *Variation of recession coefficients*

(a) Oueme sub-catchment at Affon
(b) Oueme sub-catchment at Beterou
(c) Oueme sub-catchment at Savè
(d) Oueme sub-catchment at Bonou

At Affon the depletion has duration of about 10.41 days (α value are between 0.035 d^{-1} and 0.162 d^{-1}). At Bétérou, the depletion coefficient α varies from 0.064d^{-1} to 1.084d^{-1} (recession time of approximately 4 days). Here there has been a greater range of variation coefficient of depletion with a peak of 1.084d^{-1}. These results confirm those of Le Barbé et al (1993) that highlight the existence of two types of recession coefficients on this watershed. These distinguish a phase of recession more or less slow with a coefficient of around 0.06d^{-1} when the flow exceeds a turning discharge of about 0.1m^3/s and a second phase after the hinge flow is exceeded where the recession is very fast and the coefficient of recession is about 1.42d^{-1}. At Savè (α is between 0.034 d^{-1} and 0.116d^{-1}) and Bonou (α is between 0.006d^{-1} and 0.058d^{-1}) the average duration of the recession is respectively 14.7 and 37.7 days. The high values of α obtained at Affon, Bétérou and Savè are associated with large slopes reservoirs made of areas of cracks and weathering characteristics of the base. The low values of depletion coefficient recorded at Bonou reveal areas of low hills with low flows are mainly due to the contribution of groundwater corresponding to the sedimentary area.

3.2 Analysis of recession processes

The study of depletion curves by the method of Brusaet and Nieber allow checking if the low water provide from one or more reservoirs. In the case of feeding a single reservoir. The curve representing dQ/dt and Q in loglog space is linear and in the case of a supplying by several reservoirs. This curve is not linear. The recession curves across different watersheds (Fig.5) show that the process of draining groundwater is linear at Affon and Savè (dispersion of the cloud of points (Q, -dQ/dt) around the right (eq.1) for a = 0.088 and b = 1). So low flows mainly would come from one reservoir. At Bétérou, the recession process is linear for high discharges (dispersion of the cloud of points (Q, -dQ/dt) around the right of (eq.1) for a = 0.088 and b = 1). But there is a slight difference between the curve and points towards the end of recession reflecting a non-linear process for low discharges mean an acceleration of the depletion in this watershed when low flows are below 0.1m^3/s. But at Bonou, the nonlinearity observed is due to the contribution of several reservoirs to flow. And to understand the cause of this non-linearity, the process of draining groundwater is modeled across Bétérou and Bonou by combining several linear reservoirs.

3.3 Simulation of recession processes

Assuming that the base flow is provided by a linear reservoir (n = 1), the recession rates were simulated. Thus a simulation with a linear reservoir at Affon and Savè gives results consistent with the observed discharges. But at Bétérou and Bonu, one linear reservoir could not give consistent results with the observed discharge (Fig. 6). The results obtained at Affon and Savè could be justified by the membership of these to the same geological unit that is the base. The recession discharges therefore come from the side of the hillslopes flows under the influence of slopes. These results should be obtained at Bétérou. Located on base but there is a lag between the simulated and observed discharge at the end of recession. A non-linear recession processes obtained at Bonou for the simulation with a reservoir would be justified by the existence of at least two sets geological (the base and the sedimentary basin) contributing to low water flow.

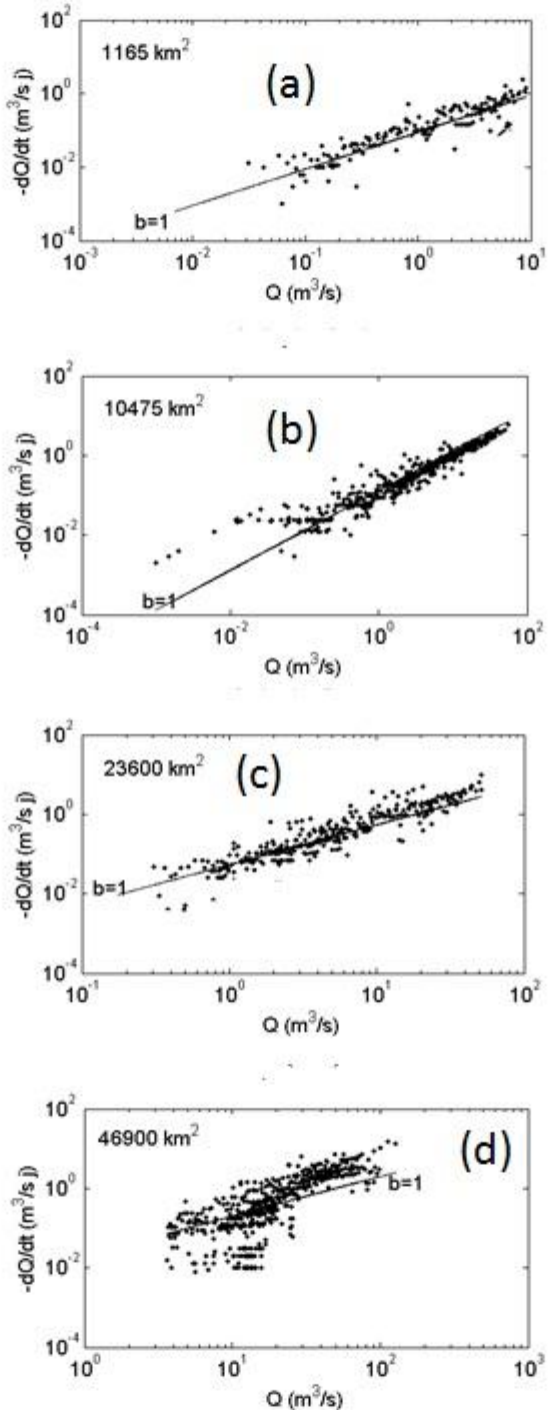

Fig. 5: *Relation between dQ/dt and Q in loglog space*

(a) Oueme sub-catchment at Affon
(b) Oueme sub-catchment at Beterou
(c) Oueme sub-catchment at Savè
(d) Oueme sub-catchment at Bonou

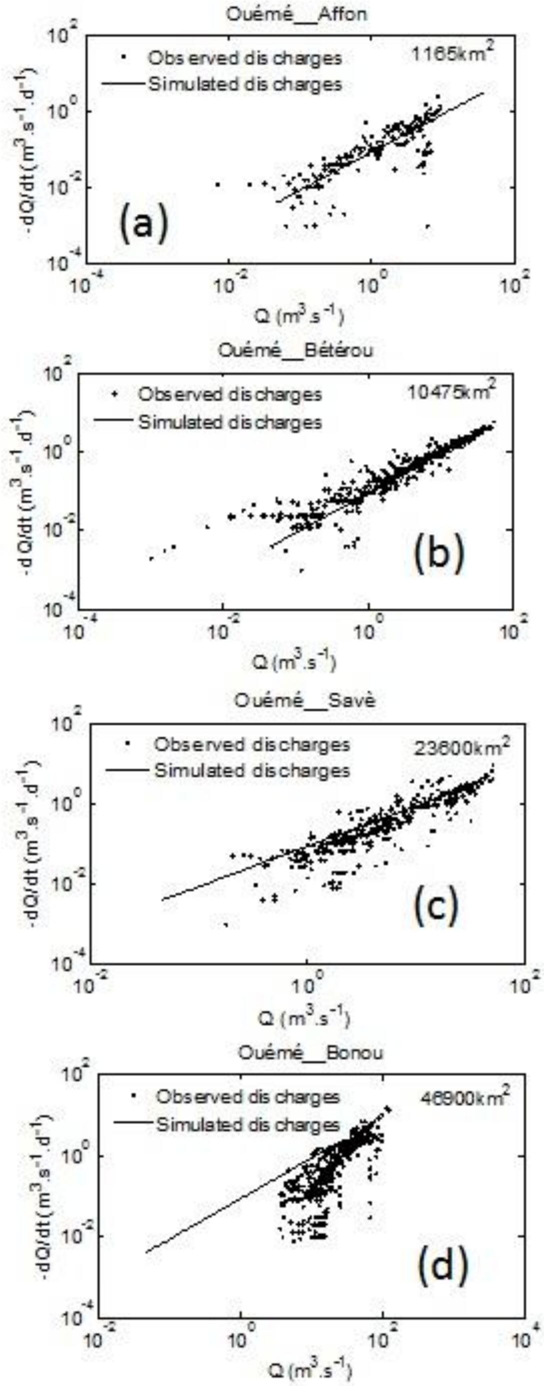

Fig. 6: *Relation between observed discharges and one event discharges simulated*

(a) Oueme sub-catchment at Affon
(b) Oueme sub-catchment at Beterou
(c) Oueme sub-catchment at Savè
(d) Oueme sub-catchment at Bonou

In order to verify if the non-linearity of the recession processes observed at Bétérou and Bonou is the consequence of the combination of several linear reservoirs simulations with different combinations of reservoirs were made. The graph of fig.7 shows synthetic recessions obtained by adding streamflow from two linear reservoirs with different depletion rates ratios (v_1 and v_2) = (0.05 and 0.088); (0.02 and 0.088) and (0.01 and 0.088). We assume that the two reservoirs have the same proportion in the watershed ($F_1 = F_2 = 0.5$) and the same recharge rate *r*. The only dissimilarity is the ratio of the two depletion coefficients. It is concluded that the non-linearity of recession process increases with the ratio of depletion rate. The model is not sensitive to very small differences between the recession coefficients [**16**].

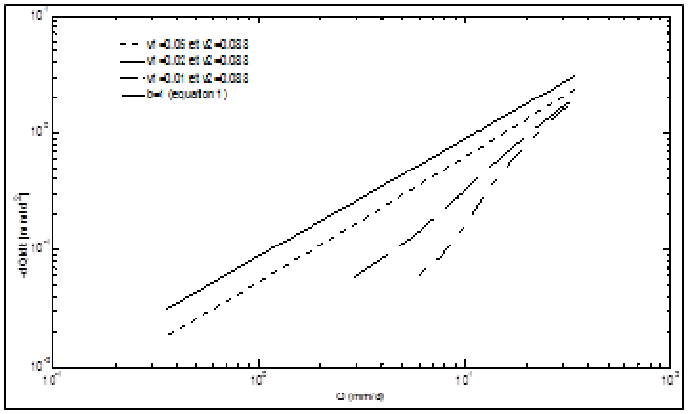

Fig. 7: *Synthetic simulations with the combination of two linear reservoirs*

The linear Behavior of recession at Affon and non-linear at Bétérou show that the spatial scale influences the recession process because the two watersheds have the same geomorphological and pedology characters [**17**]. But if that was the case the recession process should be non-linear at Savè. This is not the case. The spatial scale is not alone responsible for the linearity of recession processes. Basing on the geologic composition of the watershed, the assumption that the non-linear process in the watershed is the sum of linear processes at the watershed provides a likely explanation [**8**] of recessionary processes. The simulation results confirm those obtained by [16] using the same model. We try to predict the non-linear recession processes obtained at Bétérou and Bonou using one hand information from the analysis of the recession and other information related to the proportion of different geological units in the catchment and their contribution to low flows diet. At Bétérou, the data suggest a depletion coefficient $v \simeq 0.088 d^{-1}$ (graph 2 in Fig.5 where *a* = 0.088 and *b* = 1) at the beginning of the recession and a coefficient $v \simeq 1\ d^{-1}$ at the end (analysis of recession events for very low flows). Then we assume the existence of a second linear reservoir which is successively assigned the proportions 50%, 30%, 20% and 10% of the watershed area. Fig.8 illustrates the similarities between the observed and calculated discharges. The analysis of the graph (a) of fig.8 shows that none of the combinations of linear reservoirs product the linear character of recession processes at the beginning of the recession and non-linear character of recession processes at the end. The hypothesis of a second reservoir at Bétérou is not verified. This is why we assume the existence of two phases of depletion with a single reservoir (graph (b) of fig.8). The first phase is the high flow rates with a recession coefficient $v = 0.088\ d^{-1}$ and the second phase for low flows less than 0.1 m^3/s [**8**] with a coefficient of the order 1 d^{-1}. The result shows that a linear reservoir with two stages of recession is the most plausible hypothesis for the interpretation of the behavior of depletion rates at Bétérou's watershed scale.

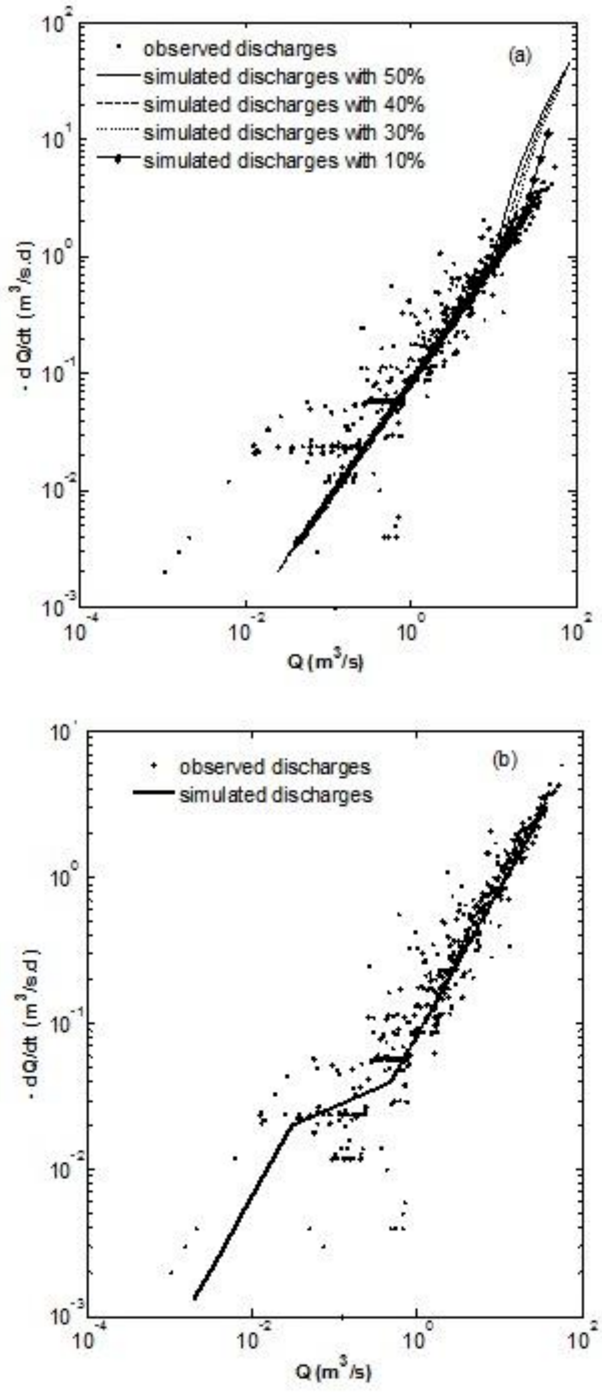

Fig. 8: *Simulation of low flow at Bétérou*

(a) Simulation with two linear reservoirs
(b) Simulation with one reservoir having two depletion phases

At Bonou, recession data suggest the recession coefficients of the order of $v \simeq 0.088\ d^{-1}$ (*a* = 0.088 and *b* = 1) and $v \simeq 0.02\ d^{-1}$ (*a* = 0 02 and *b* = 1). The coefficient $v \simeq 0.088\ d^{-1}$ is assigned to the base region characteristic of rapid depletion while $v \simeq 0.02\ d^{-1}$ is assigned to the sedimentary area where the depletion is often slow. According to the geological formation, two geological units may contribute to low water flows are retained: the base in proportions of 0.85 and the sediment in proportions of 0.15). The graph (a) of fig.9 shows the similarities between the observed and calculated discharges at Bonou.

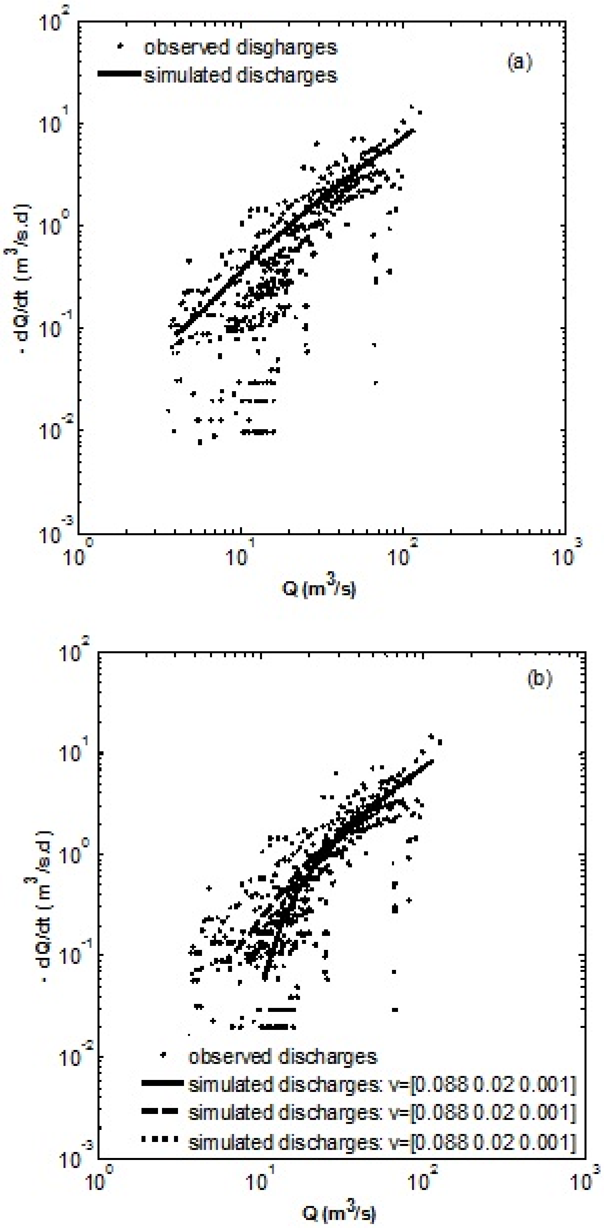

Fig. 9: *Simulation of low flow at Bétérou*

(a) Simulation with two linear reservoirs
(b) Simulation with three linear reservoirs

It appears that the model with two linear reservoirs (graph (a) of fig. 9) gives predictions that approximate reality. But this prediction is poor especially at very low flow. The hypothesis of two linear reservoirs is not sufficient to fully model the low water across Bonou. To overcome this deficiency a third reservoir is added to the system. Indeed. The sedimentary basin is made up of sedimentary deposits and alluvial deposits [**17**] which would certainly contribute in different ways in the supply of low water. The recession coefficient *v* = 0.02 is attributed to sedimentary deposits and default values of the recession coefficient to alluvial deposits. Both deposits have the same area and therefore an amount of 0.075 to each deposit relative to the watershed. The results from the simulations (graph (b) of fig.9) are consistent with the discharges measured at the outlet of this watershed. The combination of three linear reservoirs with coefficients 0.088; 0.02 and 0.001 remains the most plausible linear reservoirs combination for estimating depletion rates in Bonou's watershed scale.

3.4 Analysis of the modeling performance

The graphs of figures 10 present the correlations between simulated discharges and observed discharges at each watershed scale for different periods of calibration and validation.

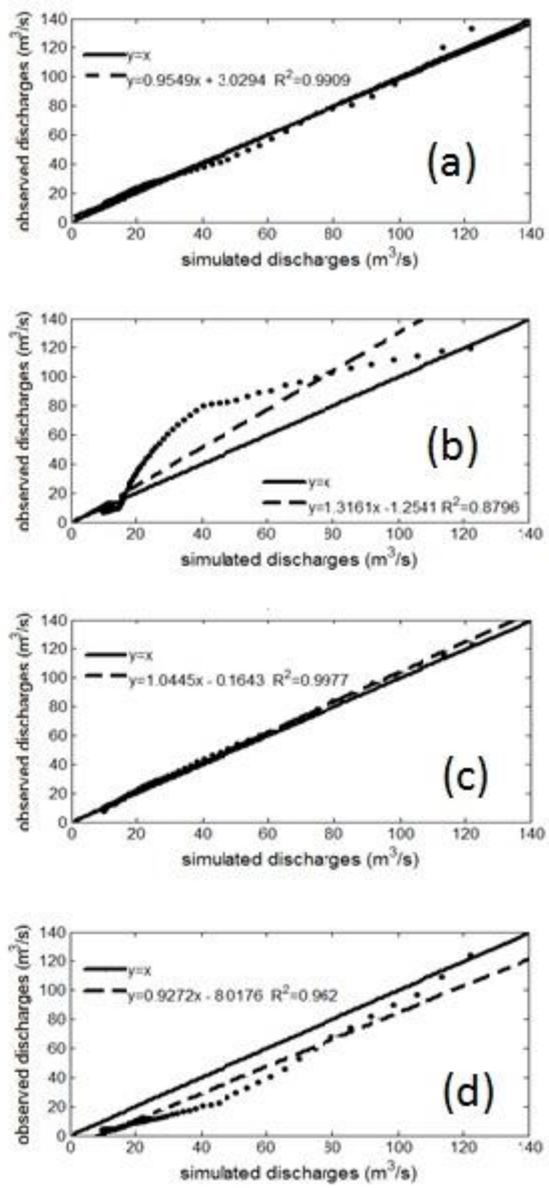

Fig. 10: *Correlation betwen observed discharges and simulated at Bétérou (a and b) and Bonou (c and d)*

The values of the coefficient of determination R^2 (99% at Bétérou. between 87 and 99% at Bonou) obtained show a very strong correlation between observed discharges and those simulated at each watershed scale. In addition to the coefficient of determination, the Nash-Sutcliffe usually used in hydrology has been selected to measure the fit between simulated and observed discharges across the two watersheds of the different sub-periods and timing validation. The Nash's values are very satisfactory and confirm the performance of the model to simulate low flows across the Oueme catchment.

3.5 Determination of standard depletion curves

To check if the assumptions indicated above namely: existence of a reservoir with two phases of depletion at Bétérou and three linear reservoirs at Bonou one hand and the recession coefficients are of the order of v1 = 0. 088d-1, v2 = 1d-1 at Bétérou and v1= 0. 088d-1, v2 = 0. 02d-1, v3= 0.001d-1 at Bonou other hand. Standard recession curves were plotted from the simulated data set (fig. 10).

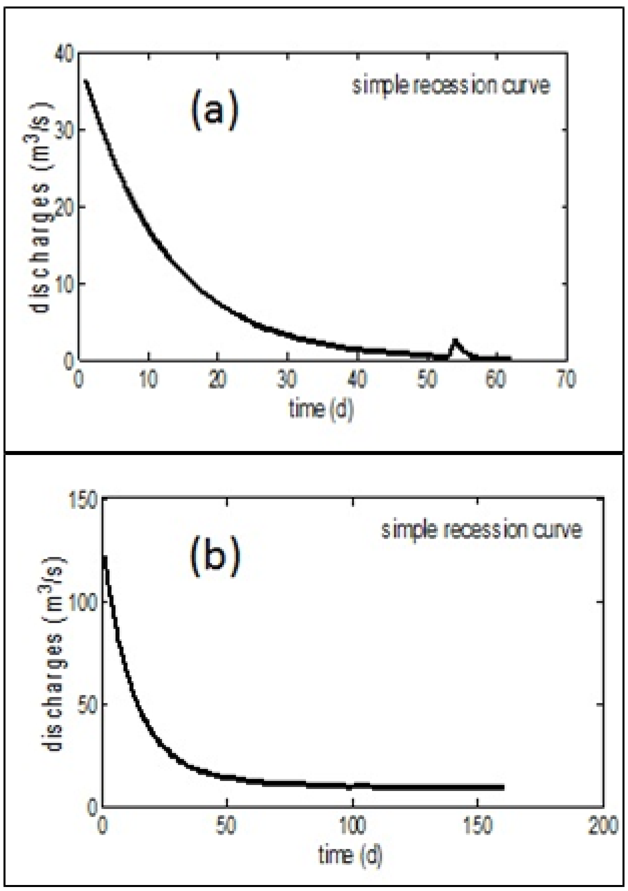

Fig. 10: *Standard recession curves at Bétérou (a and a') and Bonou with simulated discharges (b and b')*

It is clear from this figure that there are two phases of depletion at Bétérou and three at Bonou. The estimation of coefficients depletion from data sets of each phase of recession gives at Bétérou the values 0.07 d-1 at the beginning of recession and 0.98 d-1 at the end. At Bonou, the first recession stage has a coefficient of the order of 0.073 d-1 characteristic of the base. While the second phase corresponds to a coefficient of about 0.03 d-1 and the last ph²ase (the end of the recession) a coefficient substantially equal to 0.004 d-1. These results confirm the hypothesis of three linear reservoirs at Bonou and a one linear reservoir with two distinct phases of depletion at Bétérou.

3.6 Comparing analyzed of modeling performance

The comparison of model results remains a delicate exercise and conclusion of such experiences usually depends on the methodology used for comparison and the characteristics of the watershed in. If several previous trials comparing models have in the past been [**10**] states that in most cases, methodologies and results lead to the conclusion that it is not possible to draw firm conclusions about the differences in the models. According to the same author, some comparison studies were based on watersheds from several hydroclimatic regions with a variety of data. Other studies have relied on a limited number of data, there by restricting their conclusions to hydroclimatic conditions. In this case, the objective is to compare the results of the parameterization modeling low flows with the results of the parameters obtained after modeling based on the least action principle called ModHyPMA. The ModHyPMA model was subject to an application on one of our study sub-basin (the Beterou sub-basin) and its performance relative to the comparison made with the model GR4J made him a reference model for regarding the modeling of recession events. Various studies in hydrology have been proposed to quantify the recession process. These use the exponential model to constant coefficient corresponding to a linear behavior of recession [**18**]. Furthermore nonlinear models such as that developed by the Bétérou and Bonou sub-basin made subject of research and the most likely reason for the non-linearity in the rainfall-runoff transformation process is moisture off the ground. ModHyPMA, the model taking into account these humidity conditions represents it by λ parameters which determine physiographic and geological properties of the basin. The values of this parameter which reflects the flow rate of decay without the effective rain gave results of the same order as the values obtained by [**4**].

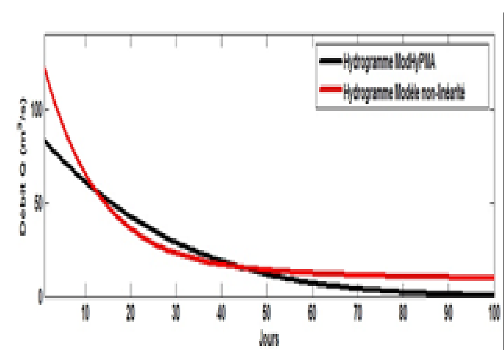

Fig. 11: *Compared recession hydrogram obtained by ModHypMA model and non-linear modeling*

These results confirmed that the onset of the low water period varies from one year to another and a single recession curve is considered as representing the playing field in terms of transmissivity basin and storage property, which is often not verified. Our results are obtained comply with that finding. Having considered the non- linear nature of the recession scale Bétérou like sucession of a series of linear character, the parameter values are α whose inverse is equal to λ. [**11**] consider that the linearization of the equation of the recession curve is appropriate and that λ can be explicitly evaluated by size and hydraulic properties of the saturated layer, such as conductivity hydraulic, porosity, surface drainage, the initial thickness of the aquifer and the length of the drainage system. This consideration was taken into account in modeling low flows with reference to partitioning Bonou basin according to the different hydrogeological formations in place. If the presence of several aquifer systems contributes to variability that is found in the form of recession curves, characterizing the nonlinear behavior of processes drainage tablecloths across Bétérou sub-basin and Bonou is reflected in the types of drying curves and coefficient values of drying off are the same as those previously obtained. The Fig.11 presents hydrographs modeled drying in Beterou sub-basin. Indeed, the daily middle debits observed to the outlet of the basin pouring Bétérou have been used to draw the recession hydrogram according to the methods extolled by the model based on the principle of the least action. The parameters of the model are determined by $y = 0.96$ and $\lambda = 11.2$. To the total, beyond of the variation in the time of the recession curve, extolled by the ModHyPMA model, the modelling of the low flow put here stroving, successfully determines the different episodes of recession in the period of recession and the coefficient corresponding.

4 Conclusion

In the nature, the basins present a big variety of shapes of recession curves. This big variety translates a non linearity of the recession process of in the basin. Oueme catchment hydrogical functional in low flow is put in evidence to different spatial scales. So at Affon and Savè drains it of the tablecloths takes place in a linear manner. In Beterou and Bonou this phenomenon translates a non-linear character. This behavior of non linearity remains the succession of several recession phases (two to Beterou sub-basin and three in Bonou sub-basin) sustained by different hydrogeological systems (one to Beterou sub-basin and two in Bonou sub-basin). A conceptual formulation of the model considering the non-linear character of the recession permitted to simulate the low flow with a big efficiency. The curves types of recession gotten in Beterou and Bonou sub-basins confirm the two-phase and three-phase natures of the recession phenomenon to the scale of the two under - basins. The low flow curves and the successfully comparing analyzed of modeling findings with recession curves obtained by least actions principle confirm the heterogeneity of recession nature at oueme basin scale.

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