Establishment Of The Shortest Route – A Prototype For Facilitation In Road Network

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Abstract: Calculating the shortest path between two locations in a road network is a significant problem in network analysis. Roads play a pivotal role in day to day activities of masses live in places and areas. They travel for various purposes that are, to study, to work, to shop and to supply their goods and the like, from one place to another place. Even in this modern era, roads remain one of the mediums used most frequently for travel and transportation. Being ignorant of the shortest routes people sometimes have to travel long distances, consume extra precious time, money and bare undesirable mental stress. Karachi is the second most populated and the seventh biggest city of the world. It is the central place of Pakistan, which is famous for industry, banking, trade and economic activity, and there are places which frequently visit by the inhabitants for their miscellaneous requirements. Federal Board of Revenue (FBR) is one of the departments that deal with taxation and revenue generation in the country. A common man residing near or distant areas often visit FBR to settle their business / property and tax related issues. In order to facilitate the masses, an effort is being made to develop a prototype based on Dijkstra's Algorithm (DA) to establish a shortest route, that will help individuals in navigation and subsequently alleviate difficulties faced by them in travelling/ road networks.

Index Terms: Dijkstra's Algorithm (DA), Shortest Route, Optimal path, Network Design

1. Introduction

Calculating the shortest path between two locations in a road network is a vital issue that finds a wide range of applications in the fields of electric navigation, telecommunication, pipe network designs, electricity, communication, urban planning, transportation planning, route guidance and so on [1,2]. The shortest route issue is a substantial problem in network optimization, which sees the track between two nodes (vertices) in a graph, in a manner that the amount of the weights of its constituent edges is minimized. It not only refers to the shortest distance in the general geographical significance, but also extends to other measures like time, cost, safety and the capacity and so on.

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Correspondingly, the shortest path problem is transformed into the fastest path, quickest way and the lowest cost problem and so on [3]. There are several other algorithms that can be utilized to find the shortest route and shortest distance between two locations in a network. The DA is the theoretical foundation to solve shortest path problem and is considered as the grandmother of all label setting algorithms [4, 5]. This graphical search algorithm was published in 1959 and invented by Edsger Dijkstra, a Dutch computer scientist in 1956. It gives solution of the single source shortest path problem with non-negative edges, path costs, creating a shortest path tree for a graph. This algorithm is rather useful in routing and other web related protocols. For a given source vertex in the graph, the DA determines the path with minimum cost (i.e. The shortest path) between that vertex and every other vertex. It can likewise be applied for finding costs of the shortest paths from a single source vertex to a single destination vertex in a network by stopping the algorithm when the shortest course to the destination vertex has been obtained [7]. In literature, DA is considered as a classical and antiquated algorithm for the shortest route problem (SRP). On the basis of literature, it is generally accepted that different variants of DA have been developed and a number of many other efficient algorithm is implemented to solve one-to-one shortest path problem. It is simple, easy to understand and implement, and highly efficient algorithm [13,14]. DA has been widely used in industrial transportation, emergency navigation and navigation planning, waste management, logistic distribution and so on [9,10]. However, the most common applications of this algorithm are solving the mathematical (graph) problems and finding the shortest route in personal navigation to perform the analysis. Balstrom (2000) listed some of the applications of the same routine e.g. looking for the nearest hospital, restaurant, jungle trekking on the mountains/hills, a petrol station, shopping place, identifying the emergency exit doors for any disaster in a building [11]. In this paper, we applied DA in real road network to find an optimal route between two nodes to achieve desired objectives. For a consistency of the study, we focused on two different areas of the city i.e. Nagan Chowrnagi (NC) as a source or starting node and FBR (located at tower) as a destination node. The first chosen area is one of the densely populated areas of Karachi, Pakistan. A large number of middle and lower-middle class resides there and its adjoining areas. On the other hand, the second chosen area is a business hub, which includes many national and international offices like FBR. It is the supreme federal agency of Pakistan, which audits, enforces and collects revenue for the Government of Pakistan and also conducts audits of taxpayers [6].Therefore, a large number of people visit FBR regularly for different purposes such as employee to work, people to resolve their tax related issues, students and researchers to gather information and to collect data used in their research and so on. In this regard an effort has been made to find out the optimal path from NC to FBR by using DA. The remaining of the manuscript is arranged as; following the introduction section two discusses the methodology. In section 3 we describe the problem. Then, in section 4 we consider the DA for obtaining the shortest route of a given network. Next, in section 5 and 6, the analysis of the results and conclusion are discussed.

2. METHODOLOGY

DA is considered an appropriate in finding the optimal path between two nodes in a network. As such, it is used in this study to achieve the desired objectives.

The description of the DA is as follows.
Let u_i be the shortest distance from source node 1 to node i, and the length of arc (i, j) is defined as d_{ij}(≥0). Then the algorithm labels an immediate successor of node j as [u_j, i] =

[u_i+ d_{ij}, i], d_{ij}≥0.

The starting node is labeled as [0, -], which indicates the node has no predecessor. There are two types of labeling nodes in DA: temporary (TEM) and permanent (PER). If the shortest route to a node can be determined, then the status of the temporary label (TEML) is modified, and converted to PER.
Step I: Initialize by assigning a permanent label (PERL) [0, -] to source node (node 1). Set i = 1,
Step II: (a) Calculate the TEMLs [u_i+ d_{ij}, i], for every node j that can be approached from node i, provided j is not PERL. If node j is already assigned [u_j, k] through another node k and if u_i+ d_{ij}, < u_j, then node j is labeled as [u_i+ d_{ij}, i] instead of [u_j, k].
(b) If PERL's are assigned to all the nodes, stop. Or else, choose the label [u_r , s] having the shortest distance (=u_r) among all the TEMLs. Set i = r and go to step I [8].

3. Problem Formulation

The given network provides routes with their distances in Kilometers [12] between area 1 (node 1) and ten other areas (nodes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11). The selected nodes are those areas of Karachi, Pakistan which is traversed more frequently. So, we consider this road network to implement the DA to obtain the shortest path from node 1 to node 11.

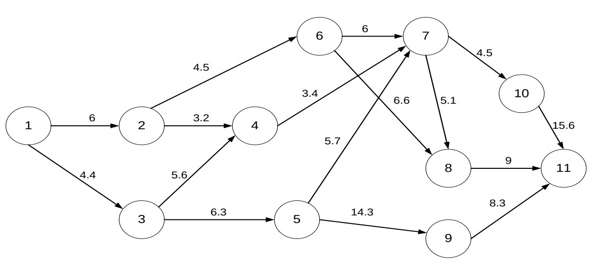

Fig 3.1: Network Diagram

Detail of the nodes is listed in table 3.1

List of Nodes	Location of Nodes
Node 1	Nagan Chowrangi (NC) - Source
Node 2	KDA Chowrangi (KDAC)
Node 3	Sohrab Gothi (SG)
Node 4	Karimabad (KB)
Node 5	Nipa Chowrngi (NPC)
Node 6	Nazimabad No. 4 (NZ)
Node 7	Hassan Square (HS)
Node 8	Mazar-e- Quaid (MZ)
Node 9	Financial Trade Center (FTC)
Node 10	Karsaz (KZ)
Node 11	Federal Board of Revenue (FBR) - Destination

Table 3.1 : Detail of Nodes

4. Solution of the Problem

According to the DA we start with iteration I.
Iteration 1: Assign node 1 as PERL [0,-] i.e. the starting node.
Iteration 2: Nodes 2 and 3 are successors of node 1. Thus, the status of labeled nodes is

Node	Label	Status
1	[0 , -]	PER
2	[6 , 1]	TEM
3	[4.4 , 1]	TEM

Between the two TEML's [6, 1] and [4.4, 1], node 3 has the minimum distance ($u_3 = 4.4$) . Thus, node 3 is assigned as PER.

Iteration 3: Nodes 4 and 5 are successors of node 3, and the detail of labeled nodes is

Node	Label	Status
1	[0 , -]	PER
2	[6 , 1]	TEM
3	[4.4 , 1]	PER
4	[10, 3]	TEM
5	[10.7 , 3]	TEM

Since min {6, 10, 10.7} = 6. Thus, the status of TEML [6, 1] at node 2 is changed to PER ($u_2 = 6$).
Iteration 4: Nodes 4 and 6 are successors of node 2, so the status of labeled nodes is

Node	Label	Status
1	[0 , -]	PER
2	[6 , 1]	PER
3	[4.4 , 1]	PER
4	[9.2, 2]	TEM
5	[10.7 , 3]	TEM
6	[10.5 , 2]	TEM

Node 4's TEML [10, 3] in iteration 3 is modified to [9.2, 2] in iteration 4 indicating that a smaller distance has been determined through node 2. Since min {9.2, 10.7, 10.5} = 9.2

the status of TEML [9.2, 2] at node 4 is converted to PER ($u_4 = 9.2$).

Iteration 5: Node 7 is a successor of node 4 and the status of labeled nodes is

Node	Label	Status
1	[0 , -]	PER
2	[6 , 1]	PER
3	[4.4 , 1]	PER
4	[9.2, 2]	PER
5	[10.7 , 3]	TEM
6	[10.5 , 2]	TEM
7	[12.6 , 4]	TEM

Since min {10.7, 10.5, 12.6} = 10.5 the status of node 6 is converted to PER ($u_6 = 10.5$).

Iteration 6: Nodes 7 and 8 are the successors of node 6, so the status of labeled nodes is changed as

Node	Label	Status
1	[0 , -]	PER
2	[6 , 1]	PER
3	[4.4 , 1]	PER
4	[9.2, 2]	PER
5	[10.7 , 3]	TEM
6	[10.5 , 2]	PER
7	[12.6 , 4]	TEM
8	[17.1 , 6]	TEM

Since min {10.7, 12.6, 17.1} = 10.7 the status of TEML [10.7, 3] at node 5 is converted to PER ($u_5 = 10.7$).

Iteration 7: Nodes 7 and 9 are the successors of node 5, and then the detail of labeled nodes is

Node	Label	Status
1	[0 , -]	PER
2	[6 , 1]	PER
3	[4.4 , 1]	PER
4	[9.2, 2]	PER
5	[10.7 , 3]	PER
6	[10.5 , 2]	PER
7	[12.6 , 4]	TEM
8	[17.1 , 6]	TEM
9	[25 , 5]	TEM

Since min {12.6, 17.1, 25} = 12.6 the status of node 7 is converted to PER ($u_7 = 12.6$).

Iteration 8: Nodes 8 and 10 are the successors of node 7. Thus the detail of labeled nodes is modified

Node	Label	Status
1	[0 , -]	PER
2	[6 , 1]	PER
3	[4.4 , 1]	PER
4	[9.2, 2]	PER
5	[10.7 , 3]	PER
6	[10.5 , 2]	PER
7	[12.6 , 4]	PER
8	[17.1 , 6]	TEM
9	[25 , 10]	TEM
10	[17.1 , 7]	TEM

Since min {17.1, 25, 17.1} = 17.1 so the status of TEML [17.1, 7] at node 10 is converted to PER ($u_{10} = 17.1$).

Iteration 9: Node 11 is the successor of node 10. So the status of labeled nodes updated

Node	Label	Status
1	[0 , -]	PER
2	[6 , 1]	P ER
3	[4.4 , 1]	P ER
4	[9.2, 2]	PER
5	[10.7 , 3]	PER
6	[10.5 , 2]	PER
7	[12.6 , 4]	PER
8	[17.1 , 6]	TEM
9	[25 , 10]	TEM
10	[17.1 , 7]	PER
11	[32.7 , 10]	TEM

Since min {17.1, 25, 32.7} = 17.1 the status of node 8 is converted to PER ($u_8 = 17.1$).

Iteration 10: Node 11 is the successor of node 8. Thus, the detail of labeled nodes is

Node	Label	Status
1	[0 , -]	PER
2	[6 , 1]	PER
3	[4.4 , 1]	PER
4	[9.2, 2]	PER
5	[10.7 , 3]	PER
6	[10.5 , 2]	PER
7	[12.6 , 4]	PER
8	[17.1 , 6]	PER
9	[25 , 10]	TEM
10	[17.1 , 7]	PER
11	[26.1 , 8]	TEM

Node 11's TEML [32.7, 10] in iteration 9 is modified to [26.1, 8] in iteration 10 indicating that a smaller distance has been obtained through node 8. Between [25, 10] and [26.1, 8], node 9 has the smallest distance.Thus, the status of node 9 is changed to PER ($u_9 = 25$).

Iteration 11: Now node11 is the only TEML left.

Node	Label	Status
1	[0 , -]	PER
2	[6 , 1]	PER
3	[4.4 , 1]	PER
4	[9.2, 2]	PER
5	[10.7 , 3]	PER
6	[10.5 , 2]	PER

7	[12.6 , 4]	PER
8	[17.1 , 6]	PER
9	[25 , 10]	PER
10	[17.1 , 7]	PER
11	[26.1 , 10]	TEM

So its status is converted to PER ($u_{11} = 26.1$). Because it does not lead to any other node .

Iteration 12:

Node	Label	Status
1	[0 , -]	PER
2	[6 , 1]	PER
3	[4.4 , 1]	PER
4	[9.2, 2]	PER
5	[10.7 , 3]	PER
6	[10.5 , 2]	PER
7	[12.6 , 4]	PER
8	[17.1 , 6]	PER
9	[25 , 10]	PER
10	[17.1 , 7]	PER
11	[26.1 , 10]	PER

Therefore, the shortest route of the discussed road network is

(11) → [26 .1 , 8] → (8) → [17.1 , 6] → (6) → [10.5 , 2] → (2) → [6 ,1] → (1)

5. Empirical Results

With the help of DA, the shortest route/optimal path has been calculated from node 1 (NC) to node 11 (FBR) and the required route is 1 → 2 → 6 → 8 → 11 with a total length of 26.1 Km. Thus, the masses residing in NC and its surrounding areas may follow this route NC → KDAC → NZ → MQ → FBR. The route so established will be extremely valuable and address all the difficulties being faced by the people while travelling to their desired destinations and spend substantial money which is a burden on the pockets of financially hard-pressed people.

6. Conclusion

In this study, we considered a real application of the classical Dijkstra's Algorithm for searching for an optimal path, i.e. the shortest route between two important places of the mega city Karachi, Pakistan, has been established. This practice of identifying the shortest path for transportation purpose in the fast developing environment is also considered essential, as it minimizes transportation cost, reduce time and distance. It further gives reliability and flexibility in movement, particularly in natural calamities, disasters, accidents and enable one's to make the right decision for the best route in certain conditions.

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