Unsteady Flow Of A Dust Particles Through Human Airways: Effects Of Uniform Magnet

M.Chitra, Radhakrishnan.S

Abstract: The unsteady unidirectional laminar flow of Newtonian, viscous, compressible fluid with uniform distribution of dust particles under the influence of uniform magnetic field during inhalation in the trachea of the human respiratory tract have been investigated. The effects of dust particles under influence of magnetic micro particles are described in three parameters, the mass concentration of the dust β , relaxation time γ' and magnetic field H, which measures the rate at which the velocity of the dust particles with magnetic field adjusts in the velocity of the clean air and depends upon the size of the individual particles. Analytical expression for the velocity of clean air , air with fine and coarse dust particle under the influence of uniform magnetic field with arbitrary constant pressure gradient have been derived. The velocities, wall shear stress distribution flow rate for clean air and air with fine dust particles, and air with coarse dust particles with uniform magnetic field in the human trachea for different radial coordinate and for different time due to effect of dust parameters β and γ' with magnetic field H are computed graphically.

Keywords: Mass concentration, Aerosol, Relaxation time, Magnetic field, Air ions, Number density.

1. INTRODUCTION

The human respiratory tract is subjected to a variety of pollutants that are inhaled and transported to the human trachea. These include particulates, aerosol and gases with a wide range of toxicants. One of the pollutants is under the influence of uniform magnetic field of aerosol particles in the airways specifically the human trachea. The trachea is usually of the geometry of the pipe and we consider the basic equation in cylindrical coordinates with approximation stated later, these dusty particles with magnetic field transported with air through human trachea eventually lead to lungs. The natural atmosphere that we breathe contains but also large number of liquid and solid particles. These are known by the genetic name aerosols. Dust is simply small particles of substances in a solid state. The particle size in the range between 0.1µm and 2µm are known as fine particles. The particle size in the range above 2µm is known as coarse particles. Dust most often comes into contact with human organs because it circulates in the air, where it may be breathed in, or settle on the eyes or skin. Dust from combustible materials such as paper, grain or coal can present the risk of explosion. Dust explosions have occurred in grain elevators, coal mines and wood-processing plants. Biological effects of harmful exposure dust are:

(i) Allergic reactions such as skin rashes,
(ii) Lung diseases like cancer asbestosis and silicosis,
(iii) Systemic effect such as lead poisoning of the blood on the excretory system.
(iv) Particles ≤ 1µm in size can greatly exacerbate health problems. In excess of 90% of PM_{10} particles can be in this size range. Such particles can be composed of dust, lint, tobacco smoke, pollen grains, diesel soot, fresh combustion particles, ozone and terpene-formed aerosols, nitrates and sulphates, heavy metals, mineral fines, respiratory droplets, skin squamae and a variety of other substances.

- M.Chitra,Associate Professor ,Thiruvalluvar University, Vellore,Tamilnadu,India, E-mail: chitratvu@mail.com
- Radhakrishnan.S,Research Scholar, Thiruvalluvar University, Vellore,Tamilnadu,India, E-mail: srisrk1979@mail.com

The size of the dust particles ranges from $PM_{2.5}$ to PM_{10} lies in the upper respiratory tract. Saffman (1961) has discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Srivastava(1963) has obtained the expression for the velocity of a viscous incompressible fluid in a circular pipe under the influence of pressure gradient decreasing exponentially with magnetic field. Nirmala P.Ratchagar and Chitra.M,(2007) has discussed the Effects of fine and coarse dust particle on the transport of air in the trachea. Sambasiva Rao (1969) has studied the unsteady flow of a dusty fluid through uniform pipe under the influence of experimental pressure gradient with respect to the time. Gupta and Gupta (1976) and Michael and Rao have discussed flow of a dusty gas through a channel with arbitrary time varying pressure gradient. Considerable electrostatic and alternating current(AC) electric fields, poor specification of materials and relative humidity (RH)/dew-point temperature levels, "Faraday cage"-lie conditions plus failure to appropriately ground conductive objects (including humans), can create highly localized incidents of electromagnetic pollution capable of significantly reducing concentrations of biologically vital and microbiocidal small air ions (SAI), such as charged oxygen.studied by Ghaly and Teplitz(2004). In this paper, we have considered unsteady, laminar flow of clean air and air with dust particles during inhalation in the symmetric form of uniform pipe in the human trachea. The flow is unsteady under the assumption of constant pressure gradient with influence of uniform magnetic field,

(i)The dust particles are assume to be spherical in shape and are uniformly distributed.
(ii)The flow is fully developed (i.e. the velocity profile in the tube is independent of the axial coordinate of the tube).
(iii)The number density N of the dust particles are constant throughout the airways.
(iv)The buoyancy force has been neglected (Since pp_1 is very small).

The dust is represented by the number density N of small dust particles whose volume concentration is small, but has appreciable mass concentration. It is assumed that the individual particles of dust are so small that a stokes flow approximation to their motion relative to flow of dust particles is appropriate. The equation of motion gives rise to two additional independent parameters due to the presence of the dust, which may be the mass concentration of the dust β and relation time γ' . we are concern with the equation of motion concerned with some general results on the effects of air with fine and coarse dust particles. Making the dust fine decreases γ', and making it coarse increases γ', in a manner proportional to the surface area of the particles. For the two extremes of τ relatively large or small, it is possible to simplify the equation of motion and we shall examine these two limiting cases. we investigate the effects of velocity, flow rate and wall shear stress of clean air in the presence of fine dust and coarse dust particle for varying radial coordinates and for varying time considering the dust parameters β and γ' are computed graphically.

2. MATHEMATICAL FORMULATION

We assume the dust particles in the air and flow of clean air through the symmetric form of cylindrical tube of trachea. We denote the clean air and dusty air velocities u_a^*(r,z,t), v_a^*(r,z,t) respectively in the direction of the axis of the tube. The equation of motion for dusty air and clean air in the human respiratory tract are:

$$\frac{\partial u_a^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial Z^*} + \gamma\left[\frac{\partial^2 u_a^*}{\partial r^2} + \frac{1}{r}\frac{\partial u_a^*}{\partial r}\right] + \frac{KN_0}{\rho}(v_a^* - u_a^*) - \frac{\sigma B_0^2 u_a^*}{\rho} \quad (1)$$

$$m\frac{\partial v_a^*}{\partial t} = -\frac{K}{M}(u_a^* - v_a^*) \quad (2)$$

with Boundary Conditions:

$$\text{(i)}\ \frac{\partial u_a^*}{\partial r} = \frac{\partial v_a^*}{\partial r} = 0 \ \text{ at } r^* = 0 \quad (3)$$

$$\text{(ii)}\ u_a^* = v_a^* = 0 \text{ at } r^* = a \quad (4)$$

Initial conditions:

$$\text{(iii)} u_a^* = v_a^* = u_0 \text{ at } t^* = 0 \quad (5)$$

These equations are made dimensionless using,

$$r^* = \frac{r}{a}; Z^* = \frac{Z}{L}; P^* = \frac{pa^2}{\rho\gamma^2}, t^* = \frac{t\gamma}{a^2}, u_a^* = \frac{a}{\gamma}u_a, v_a^* = \frac{a}{\gamma}v_a;$$
$$H = \frac{\sigma B_0^2}{\rho}; \beta = \frac{K_0}{\gamma} = \frac{mN_0}{\rho}\frac{a^2}{\gamma}; \tau = \frac{M\gamma}{Ka^2}; \gamma' = \frac{1}{\gamma};\ K_0 = \frac{mN_0}{\rho}; Q = \frac{\nu}{a},,$$

$$\frac{\partial u_a}{\partial t} = \frac{-\partial p}{\partial z} + a\left(\frac{\partial^2 u_a}{\partial r^2} + \frac{1}{r}\frac{\partial u_a}{\partial t}\right) + \beta(v_a - u_a) - \frac{Ha^2 u_a}{\gamma} \quad (6)$$

$$\frac{\partial v}{\partial t} = \gamma'(u_a - v_a) \quad (7)$$

Boundary Conditions:

$$\text{(i)}\ \frac{\partial u_a}{\partial r} = \frac{\partial v_a}{\partial r} = 0 \ \text{ at } r = 0 \quad (8)$$

$$\text{(ii)}\ u_a = v_a = 0 \text{ at } r = 1 \quad (9)$$

Initial conditions:

$$\text{(iii)} u_a = v_a = \frac{c}{4}(1 - r^2) \ \text{ at } t = 0 \quad (10)$$

It is known that the solution obtain by separation of variables will not satisfy the initial conditions. Therefore to find transient solutions, we decompose the velocity into steady part and unsteady part as given below.

$$u_a(r,t) = u_s(r) + u_t(r,t) \quad (12)$$

$$v_a(r,t) = v_s(r) + v_t(r,t) \quad (13)$$

Where $u_s$and v_s are the steady state part of the clean air and dust air respectively and u_t and v_t are the unsteady (i.e. transient) state of the clean air and dust air respectively.

$$\frac{d^2 u_s}{dr^2} + \frac{1}{r}\frac{du_s}{dr} + C = 0 \quad (14)$$

For steady state

The equation (14) is solved using the boundary conditions.

$$\frac{du_s}{dr} = 0 \ \ at\ r = 0 \quad (15)$$

$$u_s = 0 \ \ at\ r = 1 \quad (16)$$

Then the solution of (14) satisfying the boundary condition(15)&(16) is

$$u_s(r) = \frac{C}{4}(1 - r^2) \quad (17)$$

Similarly, the unsteady state(transient solution)of clean air is obtained by solving the unsteady equations (6)&(7) using the boundary conditions becomes

$$\frac{\partial u_a}{\partial t} = \frac{-\partial p}{\partial z} + a\left(\frac{\partial^2 u_a}{\partial r^2} + \frac{1}{r}\frac{\partial u_a}{\partial t}\right) + \beta(v_a - u_a) - \frac{Ha^2 u_a}{\gamma} \quad (18)$$

2.1 Clean Air:

In this model, the pressure gradient $\frac{-\partial p}{\partial z}$ is a constant.

$$\frac{\partial u_a}{\partial t} = \frac{-\partial p}{\partial z} + a\left(\frac{\partial^2 u_a}{\partial r^2} + \frac{1}{r}\frac{\partial u_a}{\partial t}\right) + \beta(v_a - u_a) - \frac{Ha^2 u_a}{\gamma}$$

We solve these equations using Laplace transform of the form $U(r,s) = L[u(r,t)]\int_0^\infty e^{-st}u(r,t)dt$ where U is the Laplace transform of u and s is the Laplace parameter. Applying Laplace Transform in equation(6),we get

$$\therefore \left(s + \beta + \frac{Ha^2}{\gamma}\right)u_a{}^2 = \frac{c}{4}(1 - r^2) + a\left(\frac{\partial^2 u_a{}^2}{\partial r^2} + \frac{1}{r}\frac{\partial u_a{}^2}{\partial r}\right) + \beta\widetilde{v_a} \quad 19)$$

$$(7) \Rightarrow \frac{\partial v_a}{\partial t} = \gamma'(u_a - v_a)$$

Applying Laplace Transform in equation(7),we get

$$(s + v_a{}')\tilde{v} = \frac{c}{4}(1 - r^2) + \gamma'(\widetilde{u_a}) \quad (20)$$

Eliminating (19) & (20),we get

$$\therefore a\frac{\partial^2 \widetilde{u_a}}{\partial r^2} + \frac{1}{r}\frac{\partial \widetilde{u_a}}{\partial r} - P\widetilde{u_a} = -\frac{A}{4}(1 - r^2) \quad (21)$$

$$\begin{cases} where\ P = \left[s + \beta + \frac{Ha^2}{\gamma} - \frac{\beta\gamma'}{s + \gamma'}\right] \\ A = c\left[1 + \frac{\beta}{s + \gamma'}\right] \end{cases}$$

We solve these equations using Hankel transform of the form

$\overline{U} = \int_0^1 u(r) r J_0(\varepsilon_n r)\, dr$ Where the Kernel $J_0(\varepsilon_n r)$ is the Bessel function of order zero.

$$a\int_0^1 \left(\frac{\partial^2 \widetilde{u_a}}{\partial r^2} + \frac{1}{r}\frac{\partial \widetilde{u_a}}{\partial r}\right) r J_0(\varepsilon r) dr - P\int_0^1 \widetilde{u_a}\, r J_0(\varepsilon r) dr = -\frac{A}{4}\int_0^1 (1-r^2)\, r J_0(\varepsilon r) dr$$

$$(a^2\varepsilon^2 + P)\overline{\overline{u_a}} = \frac{A}{\varepsilon^3}\left[J_1(\varepsilon) - \frac{\varepsilon}{2}J_0(\varepsilon)\right]$$

$$\therefore \overline{\overline{u_a}} = \frac{A}{\varepsilon^3}\frac{\left[J_1(\varepsilon) - \frac{\varepsilon}{2}J_0(\varepsilon)\right]}{(a^2\varepsilon^2 + P)}$$

$$\overline{\overline{u_a}} = \frac{A}{\varepsilon^3}\left[\frac{J_1(\varepsilon_n)}{a^2\varepsilon^2 + P}\right] \qquad (22)$$

Applying Hankel Inverse Transform of (22),we get

$$\widetilde{u_a} = 2\sum_{n=1}^{\infty}\left(\frac{A}{\varepsilon^3}\right)\left(\frac{J_0(\varepsilon_n r)}{(a^2\varepsilon_n^2 + P)(J_1(\varepsilon_n))}\right)$$

$$\widetilde{u_a} = 2\sum_{n=1}^{\infty}\left(\frac{A}{\varepsilon^3}\right)\left(\frac{J_0(\varepsilon_n r)}{(a^2\varepsilon_n^2 + P)(J_1(\varepsilon_n))}\right)$$

For clean parameter $\beta = 0, \gamma' = 0$,

$$=> A = c, \qquad P = s + \frac{Ha^2}{\gamma}$$

$$\therefore \widetilde{u_a} = 2\sum_{n=1}^{\infty} c\frac{J_0(\varepsilon_n r)}{\varepsilon_n^3\left(a^2\varepsilon_n^2 + \frac{Ha^2}{\gamma} + s\right)(J_1(\varepsilon_n))} \qquad (23)$$

Applying Inverse Laplace Transform, we get

$$u_a = 2\sum_{n=1}^{\infty} c\frac{J_0(\varepsilon_n r)}{\varepsilon_n^3(J_1(\varepsilon_n))} e^{-\left(a^2\varepsilon_n^2 + \frac{Ha^2}{\gamma} + s\right)t} \qquad (24)$$

The required velocity of the clean air from(12), using(17)&(24)take the form

$$u_a = \frac{c}{4}(1-r^2) + 2\sum_{n=1}^{\infty} c\frac{J_0(\varepsilon_n r)}{\varepsilon_n^3(J_1(\varepsilon_n))} e^{-\left(a^2\varepsilon_n^2 + \frac{Ha^2}{\gamma} + s\right)t} \qquad (25)$$

2.2 Air with Fine and coarse Dust Particles

The effect of the dust enters through the two parameters β and γ'. Making the dust fine decrease γ', and making it coarse increase γ', in a manner proportional to the surface area of the particles. For the two extremes of γ' relatively large or small it is possible to simplify equations.

2.3 Fine Dust Particles:

$$\gamma' \ll {}^{L}/_{u^*}, \quad v_a = u_a$$

$$=> v_a - u_a = -\left(\frac{\partial u_a}{\partial t}\right)\gamma'$$

$$\frac{\partial u_a}{\partial t} = -\frac{\partial P}{\partial z} + \left(\frac{\partial^2 u_a}{\partial r^2} + \frac{1}{r}\frac{\partial u_a}{\partial t}\right) + \beta\left(\frac{\partial u_a}{\partial t}\right)\gamma' - \frac{Ha^2}{\gamma}u_a$$

$$(1+\beta\gamma')\frac{\partial u_a}{\partial t} = C + \left(\frac{\partial^2 u_a}{\partial r^2} + \frac{1}{r}\frac{\partial u_a}{\partial t}\right) - \frac{Ha^2}{\gamma}u_a$$

$$Let\ u_a = u_f,\ C = -\frac{\partial P}{\partial z}$$

$$(1+\beta\gamma')\frac{\partial u_f}{\partial t} = C + \left(\frac{\partial^2 u_f}{\partial r^2} + \frac{1}{r}\frac{\partial u_f}{\partial t}\right) - \frac{Ha^2}{\gamma}u_f$$

Boundary and initial condition

$$\frac{du_f}{dr} = 0 \ \ at\ r = 0; u_f = 0 \ \ at\ r = 1; u_f(r) = \frac{c}{4}(1-r^2)$$

Eliminating C, we get

$$(1+\beta\gamma')\frac{\partial u_f}{\partial t} = a\left(\frac{\partial^2 u_f}{\partial r^2} + \frac{1}{r}\frac{\partial u_f}{\partial t}\right) - \frac{Ha^2}{\gamma}u_f \qquad (26)$$

Applying Laplace Transform, we get

$$\left[(1+\beta\gamma')s + \frac{Ha^2}{\gamma}\right]\widetilde{u_f} = (1+\beta\gamma')\,{}^{c}/_{4}(1-r^2) + a\left[\frac{\partial^2\widetilde{u_f}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{u_f}}{\partial t}\right] \qquad (27)$$

Applying Hankel Transform,we get

$$\left[(1+\beta\gamma')s + \frac{Ha^2}{\gamma}\right]\widetilde{u_f} = \frac{c}{4}(1+\beta\gamma')\int_0^1 (1-r^2) r J_0(\varepsilon r) dr -$$

$$a^2\varepsilon^2 u(\varepsilon)\widetilde{u_f} = \frac{c(1+\beta\tau)}{\varepsilon^3\left[\left(a^2\varepsilon^2 + \frac{Ha^2}{\gamma}\right) + (1+\beta\tau)s\right]} \qquad (28)$$

Applying Hankel Inverse Transform,we get

$$\widetilde{u_f} = 2\sum_{n=1}^{\infty}\frac{c(1+\beta\gamma')J_0(\varepsilon_n r)}{\varepsilon_n{}^3\left[\left(a^2\varepsilon^2 + \frac{Ha^2}{\gamma}\right) + (1+\beta\gamma')s\right]J_1(\varepsilon_n)}$$

Applying Laplace Inverse Transform, we get

$$u_f = 2\sum_{n=1}^{\infty}\left[\frac{c(1+\beta\gamma')}{\varepsilon_n^3}\right]\left[\frac{J_0(\varepsilon_n r)}{J_1(\varepsilon_n)}\right] e^{-\left[\frac{a^2\varepsilon_n^2 + \frac{Ha^2}{\gamma}}{1+\beta\gamma'}\right]t} \qquad (29)$$

The required velocity of the clean air from(12), using(17)&(29)take the for

$$u_f = \frac{c}{4}(1-r^2) + 2\sum_{n=1}^{\infty}\left[\frac{c(1+\beta\gamma')}{\varepsilon_n^3}\right]\left[\frac{J_0(\varepsilon_n r)}{J_1(\varepsilon_n)}\right] e^{-\left[\frac{a^2\varepsilon_n^2 + \frac{Ha^2}{\gamma}}{1+\beta\gamma'}\right]t} \qquad (30)$$

2.4 Coarse Dust Particle:

$$\gamma' \geq {}^{L}/_{u}\ \ , v\ is\ negligible$$

$$\frac{\partial u_a}{\partial t} = c + a\left(\frac{\partial^2 u_a}{\partial r^2} + \frac{1}{r}\frac{\partial u_a}{\partial r}\right) - \beta u_a - \frac{Ha^2}{\gamma}u_a \qquad (31)$$

$u_a = u_c$ *is the velocity of coarse dust particle*

$$\frac{\partial u_c}{\partial t} = c + a\left(\frac{\partial^2 u_c}{\partial r^2} + \frac{1}{r}\frac{\partial u_c}{\partial r}\right) - \beta u_c - \frac{Ha^2}{\gamma}u_c \qquad (32)$$

To find the solution of (12) as before we decompose

$$u_c(r,t) = u_{cs}(r) + u_{ct}(r,t)$$

Where

$u_{cs}(r)$ is the steady state equation of coarse dust particle.
$u_{ct}(r,t)$ is the unsteady state equation of coarse dust particle

The steady state equation (31) is

$$o = c + a\left(\frac{d^2u_{cs}}{d} + \frac{1}{r}\frac{du_{cs}}{dr}\right) - \beta u_{cs} - \frac{Ha^2}{\gamma}u_{cs} \quad (33)$$

Using B.C $\frac{du_{cs}}{dr} = 0,\ at\ r = 0$; $u_{cs}(r) = 0\ , at\ r = 1$;
$u_{cs} = v_{cs} = \frac{c}{4}(1 - r^2)$ at t = 0

Applying Hankel Transform of equation (33),

$$\left(\beta + \frac{Ha^2}{v}\right)\int_0^1 u_{cs}\, rJ_0(\varepsilon r)dr = a\int_0^1\left(\frac{d^2u_{cs}}{d} + \frac{1}{r}\frac{du_{cs}}{dr}\right)rJ_0(\varepsilon r)dr + c\int_0^1 rJ_0(\varepsilon r)dr$$

$$\widetilde{u_{cs}} = c\frac{J_1(\varepsilon_1)}{\varepsilon\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon^2\right)} \quad (34)$$

Applying Inverse Hankel Transform of (34),we get

$$u_{cs}(r) = 2c\sum_{n=1}^{\infty}\left(\frac{J_0(\varepsilon_n r)}{\varepsilon_n\left[\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right]J_1(\varepsilon_n)}\right) \quad (35)$$

The unsteady state of equation (31) is

$$\frac{\partial u_{ct}}{\partial t} = c + a\left(\frac{\partial^2 u_{ct}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{ct}}{\partial r}\right) - \beta u_{ct} - \frac{Ha^2}{v}u_{ct} \quad (36)$$

Boundary and initial conditions $\frac{du_{ct}}{dr} = 0,\ at\ r = 0$;
$u_{ct}(r) = 0\ , at\ r = 1\ u_{cs} = u_{ct}$ at t = 0

Applying Laplace Transform, we get,

$$\left[s + \beta + \frac{Ha^2}{v}\right]\widetilde{u_{ct}} = a\left[\frac{\partial^2\widetilde{u_{ct}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{u_{ct}}}{\partial r}\right] + 2c\sum_{n=1}^{\infty}\left(\frac{J_0(\varepsilon_n r)}{\left[\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right]J_1(\varepsilon_n)}\right)$$

Applying Hankel Transform,we get

$$\left[s + \beta + \frac{Ha^2}{v}\right]\overline{\overline{u_{ct}}} = -a^2\varepsilon_n^2\overline{\overline{u_{ct}}}(\varepsilon)\int_0^1 2c\sum_{n=1}^{\infty}\frac{J_0(\varepsilon_n r)}{J_1(\varepsilon_n)}\frac{rJ_0(\varepsilon_n r)dr}{\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right)}$$

$$\therefore \overline{\overline{u_{ct}}} = \frac{cJ_1(\varepsilon_n)}{\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right)\left[s + \beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right]}$$

Applying Inverse Hankel Transform, we get

$$\widetilde{u_{ct}} = \sum_{n=1}^{\infty}\frac{2cJ_0(\varepsilon_n r)}{\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right)\left[s + \beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right]J_1(\varepsilon_n)}$$

Applying Inverse Laplace Transform,

$$u_{ct} = 2\sum_{n=1}^{\infty}\frac{cJ_0(\varepsilon_n r)}{\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right)J_1(\varepsilon_n)}e^{-\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right)t} \quad (37)$$

The velocity of coarse dust summing equations(35) & (37) becomes,

$$u_c = 2c\sum_{n=1}^{\infty}\frac{J_0(\varepsilon_n r)}{\varepsilon_n\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right)J_1(\varepsilon_n)}\left[1 + e^{-\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right)t}\right] \quad (38)$$

The mass flow rate of clean air is

$$Q_a(t) = \frac{2\pi R_o^2 V_a}{L}\int_0^1 u_a r\, dr = \frac{2\pi R_o^2 V_a}{L}\left[\frac{c}{4}\left(\frac{r^2}{2} - \frac{r^4}{4}\right) + 2\sum_{n=1}^{\infty}c\frac{e^{-\left(a^2\varepsilon_n^2 + \frac{Ha^2}{v}\right)t}}{\varepsilon_n^3 J_1(\varepsilon_n)}\frac{rJ_1(\varepsilon_n)}{\varepsilon_n}\right]_0^1$$

$$Q_a(t) = \frac{2\pi R_o^2 V_a}{L}\left[\frac{c}{16} + 2\sum_{n=1}^{\infty}c\frac{e^{-\left(a^2\varepsilon_n^2 + \frac{Ha^2}{v}\right)t}}{\varepsilon_n^4}\right] \quad (39)$$

Similarly The mass flow rate of fine dusty air is

$$Q_f(t) = \frac{2\pi R_o^2 V_a}{L}\int_0^1 u_f\, r\, dr = \frac{2\pi R_o^2 V_a}{L}\left[\frac{c}{4}\left(\frac{r^2}{2} - \frac{r^4}{4}\right) + 2\sum_{n=1}^{\infty}\frac{c(1+\beta v)}{\varepsilon_n^3 J_1(\varepsilon_n)}e^{-\left(\frac{a^2\varepsilon_n^2 + \frac{Ha^2}{v}}{(1+\beta\gamma')}\right)t}r\frac{J_1(\varepsilon_n r)}{\varepsilon_n}\right]_0^1$$

$$Q_f(t) = \frac{2\pi R_o^2 V_a}{L}\, c\left[\frac{1}{16} + 2\sum_{n=1}^{\infty}\frac{c(1+\beta v)}{\varepsilon_n^4}\, e^{-\left(\frac{a^2\varepsilon_n^2 + \frac{Ha^2}{v}}{(1+\beta\gamma')}\right)t}\right] \quad (40)$$

The mass flow rate of coarse dusty air is

$$Q_c(t) = \frac{2\pi R_o^2 V_a}{L}\int_0^1 u_c r\, dr = \frac{4\pi R_o^2 V_a c}{L}\frac{1}{\varepsilon_n J_1(\varepsilon_n)\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right)}\left[\left[\frac{rJ_1(\varepsilon_n r)}{\varepsilon_n} + e^{-\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right)t}\right]\frac{rJ_1(\varepsilon_n r)}{\varepsilon_n}\right]_0^1$$

$$Q_c(t) = \frac{4\pi R_o^2 V_a c}{L}\frac{\left[1 + e^{-\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right)t}\right]}{\varepsilon_n^2\left(\beta + \frac{Ha^2}{v} + a^2\varepsilon_n^2\right)} \quad (41)$$

The wall shear stress also called skin friction for clean air is,

$$\tau_a = \left[-\mu_a\rho v_0^2\frac{\partial u_a}{\partial r}\right]_{r=1}$$

$$\tau_a = \left\{-\mu_a \rho v_0^2 \frac{\partial}{\partial r}\left[\frac{c}{4}(1-r^2) + 2\sum_{n=1}^{\infty}\frac{cJ_0(\varepsilon_n r)}{\varepsilon_n^3 J_1(\varepsilon_n)} e^{-\left(a^2\varepsilon_n^2 + \frac{Ha^2}{v}\right)t}\right]\right\}_{r=1}$$

$$\therefore \tau_a = \mu_a \rho v_0^2 c\left[{}^1\!/_2 + 2\sum_{n=1}^{\infty}\frac{e^{-\left(a^2\varepsilon_n^2+\frac{Ha^2}{v}\right)t}}{\varepsilon_n^2}\right] \qquad (42)$$

Similarly for fine dusty air

$$\tau_f(t) = \left[-\mu_f \rho v_0^2 \frac{\partial u_f}{\partial r}\right]_{r=1}$$

$$\tau_f(t) =$$

$$-\mu_f \rho v_0^2 \frac{\partial}{\partial r}\left\{\frac{c}{4}(1-r^2) + 2c\sum_{n=1}^{\infty}\frac{c(1+\beta\gamma')}{\varepsilon_n^3} e^{-\left(\frac{a^2\varepsilon_n^2+\frac{Ha^2}{v}}{(1+\beta\gamma')}\right)t}\frac{J_0(\varepsilon_n r)}{J_1(\varepsilon_n)}\right\}_{r=1}$$

$$\tau_f(t) = \mu_f \rho v_0^2 c \frac{\partial}{\partial r}\left\{{}^1\!/_2 + 2\sum_{n=1}^{\infty}\frac{(1+\beta\gamma')}{\varepsilon_n^2} e^{-\left(\frac{a^2\varepsilon_n^2+\frac{Ha^2}{v}}{(1+\beta\gamma')}\right)t}\right\} \qquad (43)$$

Also the wall shear stress for coarse dusty air is

$$\tau_c(t) = \left[-\mu_a \rho v_0^2 \frac{\partial u_c}{\partial r}\right]_{r=1}$$

$$\tau_c(t) = -\mu_a \rho v_0^2 \frac{\partial}{\partial r}\left\{\frac{2cJ_0(\varepsilon_n r)}{\varepsilon_n J_1(\varepsilon_n)\left(\beta+\frac{Ha^2}{v}+a^2\varepsilon_n^2\right)}\left[1+e^{-\left(\beta+\frac{Ha^2}{v}+a^2\varepsilon_n^2\right)t}\right]\right\}_{r=1}$$

$$\tau_c(t) = \frac{2\mu_a \rho v_0^2 c}{\left(\beta+\frac{Ha^2}{v}+a^2\varepsilon_n^2\right)}\left[1+e^{-\left(\beta+\frac{Ha^2}{v}+a^2\varepsilon_n^2\right)t}\right] \qquad (44)$$

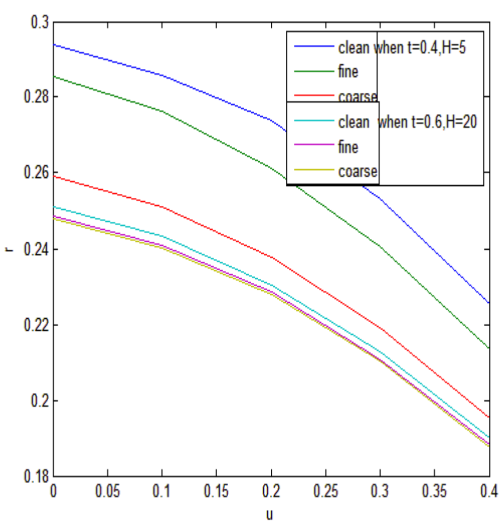

Fig. 1 The Effect of the magnetic field H on the velocity profile 'u' with radial position of 'r' for the varying of time t, β=0.2, γ'=0.1

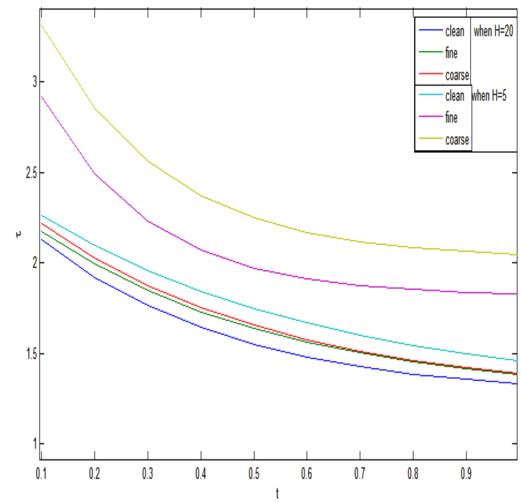

Fig. 2 The Effects of the magnetic field H on the wall shear stress 'τ' With time 't' for different values of H, β=0.2, γ'=0.1

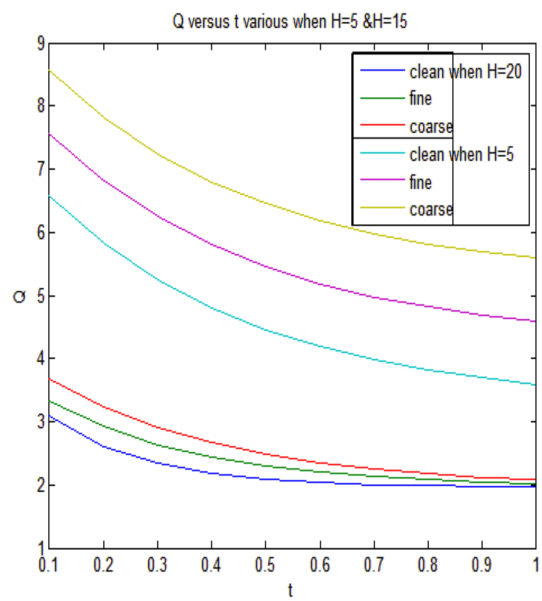

Fig. 3 The effects of the magnetic field H on the flow rate 'Q' With time 't' for different values of H, β=0.2, γ'=0.1

3.Results and Discussion

The changes in the velocities, wall shear stress and flow rate of clean air, fine dust particle and coarse dust particle with varying magnetic field H and time t due to dust parameter β and γ' are computed and the results are shown graphically in Figures 1 to 3. The velocity for the clean air, fine dust particles and coarse dust particles for varying time are drawn in Figure 1. This figure shows that an increase in magnetic field H and time t decrease the velocities, it is clear that the velocities are parabolic in nature and reveals that

u_a(r,t)> u_f(r,t)> u_c(r,t), for 0<r<1.

The wall shear stress and flow rate for the clean air, fine dust particles and coarse dust particles of the airways varying with time t for different values of magnetic field H also computed and the results are drawn in Figure 2 and 3.. The Figures 2 & 3 shows that an increase in magnetic field H decrease the wall shear stress as in the case of flow rates. we conclude that

$$\tau_c(t) > \tau_f(t) > \tau_a(t), \text{ for } 0<t<1$$

$$Q_c(t) > Q_f(t) > Q_a(t). \text{ for } 0<t<1$$

If the mass of the dust particles are small, their influence and the fluid flow is reduced, and in the limit as the fluid becomes ordinary viscous. Hence it may be concluded that the velocity depends more on the mass concentration of the dust particles than on their size. The tendency of a radial field to concentrate particles at its center could be useful for designing a magnetic field to retain particles in the airways. The results of this study indicated that magnetic field H, mass concentration of dust β and relaxation time γ' is the most significant parameter in determining whether magnetic particles can be retained by a magnetic field at a target site in the airways. We conclude that the effect of magnetic field can decrease the work of breathing and optimize the spontaneous breathing of patient during respiratory disease.

4. References

[1] Saffman, P.G. On the stability of laminar flow of a dusty gas. J.Fluid Mech.,13,120, 1961.

[2] Srivastava. P. N.. On the unsteady flow of a viscous incompressible fluid in a pipe or channel. Math. Seminar. Delhi, 3, pp,44-53. 1963

[3] Micheal. D. H.. Kelvin-Helmholtz instability of a dusty gas. Proc. Camb. Phil. Soc., 61, pp,569-71. 1965

[4] Michael, D.H., and Miller, D. A. Plane parallel flow of a dusty gas. Mathematic, 13, pp,97-109. 1966.

[5] Michael, D. H., and Norey, P. W.. The laminar flow of a dusty gas between rotating cylinders. QJMAM, 21, pp,375-88. 1968

[6] Sambasiva Rao, P. Unsteady flow of a dusty viscous liquid through circular cylinders. Def. Sci. J., 19, pp.135-38. 1969.

[7] Tewari, V. D., and Bhattacharjee. Dusty viscous fluid flow through a rotating circular cylinder, J. Scient. Res. B.H.U., 23,pp. 145-50. 1972.

[8] Gupta R.K and Guptha,S.C :Flow of a dusty gas through a channel with arbitrary time varying pressure gradient,ZAMP,27,pp 119(1976)

[9] Ghaly,M., Teplitz ,D, the Biological effects of grounding the human body during sleep, as measured by cortisol levels and subjective reporting of sleep, pain and stress. The Journal of Alternative and Complimentry Medicine 10(5),pp767-776(2004)

[10] Nirmala P.Ratchagar and Chitra.M, Effects of fine and coarse dust particle on the transport of air in the trachea,journal of Indian acad.Math,vol.29,No.2,pp 551-570,2007.