Inverse Kinematic Analysis Of A Quadruped Robot

Muhammed Arif Sen, Veli Bakircioglu, Mete Kalyoncu

Abstract: This paper presents an inverse kinematics program of a quadruped robot. The kinematics analysis is main problem in the manipulators and robots. Dynamic and kinematic structures of quadruped robots are very complex compared to industrial and wheeled robots. In this study, inverse kinematics solutions for a quadruped robot with 3 degrees of freedom on each leg are presented. Denavit-Hartenberg (D-H) method are used for the forward kinematic. The inverse kinematic equations obtained by the geometrical and mathematical methods are coded in MATLAB. And thus, a program is obtained that calculate the legs joint angles corresponding to desired various orientations of robot and endpoints of legs. Also, the program provides the body orientations of robot in graphical form. The angular positions of joints obtained corresponding to desired different orientations of robot and endpoints of legs are given in this study.

Index Terms: Quadruped robot, D-H parameters, forward kinematic, inverse kinematic, MATLAB, kinematic program, simulation.

1 Introduction

Quadruped robots are an important place in robotic and their popularity are increasing. Quadruped robots are more complex in structure, more difficult to control than wheeled and crawler robots. They have a complex structure, which leads to a higher instructional in terms of robotics and control theory. Less energy consumption, good stability and locomotion on uneven and rough terrain are main advantages of quadruped robots. Low speed, difficult to build and control, need on-board power are limitations of quadruped robots. The number of study about quadruped robots has increased in recent years due to better performance in challenging terrain conditions. The example of current studies about quadruped robots; BigDog [1] is developed by Boston Dynamics, HyQ2Max [2] is developed by Semini et al., Jinpong [3] is developed by Cho et al. In quadruped robots, a good kinematic model is necessary to stability analysis and trajectory planning of system. There are two types of kinematic analysis: forward and inverse kinematics analysis. In the forward kinematic analysis, the joint variables are given to find the location of the body of the robot. In the inverse kinematic analysis, the location of the body is given to find the joint variables necessary to bring the body to the desired location. Detailed studies on the kinematic analysis of quadruped robots are available in the literature. Potts and Da Cruz [4], presented both the forward and inverse kinematics model of a complex quadruped robot named Kamambare with singularity analysis. Oak and Narwane [5], showed the kinematic modelling of a quadruped robot leg with four bar chain mechanism. Zhang et.al [6], propose a structure design of quadruped robot using a mammalian animal with the kinematics analysis, also optimized a leg of robot and simulated in ADAMs. Anand [7], provided insights into the kinematic analysis of a quadruped robot capable of both walking and functioning as a machining tool. Ganjare et.al [8], designed a quadruped robot with two joints of the leg, it enable to perform two basic motions: lifting and stepping at medium speed on flat terrain. Chenet.al [9], introduced a hydraulic quadruped robot and built kinematic model of the robot in part of their study. In this paper, a MATLAB program that calculates forward and inverse kinematics of a quadruped robot corresponding to desired different orientations of robot and endpoints of legs and demonstrate the 3-D robot form in graphical is developed. Kinematic equations are obtained by Denavit-Hartenberg method [10] and analytical solutions [11].

2 Kinematic Analysis

The quadruped robot is a robotic system that consists of a rigid body and four legs with three degrees of freedom (each leg has the same structure). The links of legs are connected to each other by rotary joints. The physical model of the quadruped robot is shown Fig. 1. The parameters of robot are given in Table 1.

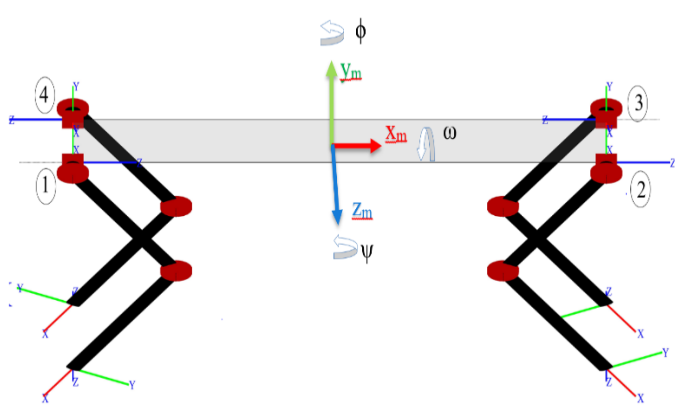


Fig. 1. *The physical model of the quadruped robot*

- *Muhammed Arif Sen, Department of Mechanical Engineering, Selçuk University, Konya, Turkey, marifsen@selcuk.edu.tr*
- *Veli Bakırcıoğlu, Technical Science Vocational Education and Training School, Aksaray University, Aksaray, Turkey, vbakircioglu@aksaray.edu.tr*
- *Mete Kalyoncu, Department of Mechanical Engineering, Selçuk University, Konya, Turkey, mkalyoncu@selcuk.edu.tr*

TABLE 1. *THE PARAMETERS OF ROBOT*

Physical Dimensions	The Length of Robot	L=1 [m]
	The Width of Robot	W=0.4 [m]
	The Length of Side Swing Joint	L1=0.1 [m]
	The Length of Hip Joint	L2=0.4 [m]
	The Length of Knee Joint	L3=0.4 [m]
Coordinate Systems	The Coordinate System of centre of Body	$[x_m, y_m, z_m]$
	The Main Coordinate System of Each Leg	$[x_0, y_0, z_0]$
	The Coordinate System of Side Swing Joint	$[x_1, y_1, z_1]$
	The Coordinate System of Hip Joint	$[x_2, y_2, z_2]$
	The Coordinate System of Knee Joint	$[x_3, y_3, z_3]$
	The Coordinate System of Endpoint of Leg	$[x_4, y_4, z_4]$
Variables	The Yaw Angle of Robot	ϕ
	The Pitch Angle of Robot	ψ
	The Roll Angle of Robot	ω
	The Angle of Side Swing Joint	θ_1
	The Angle of Hip Joint	θ_2
	The Angle of Knee Joint	θ_3

As shown in Figure 1, depending on the legs coordinates, the robot body can have different configurations. For this reason, the kinematic equation between the rotational movements (φ, ψ, ω) around the centre of body's coordinate system (x_m, y_m, z_m) and the coordinate system of each endpoint of leg (x_4, y_4, z_4) is investigated. Initially, to determine the position and orientation of the robot centre of body in the workspace, the transformation matrix is obtained in Eq.5 using the rotation matrices (Eq. 1, Eq. 2, Eq. 3, Eq. 4).

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\omega) & -\sin(\omega) & 0 \\ 0 & \sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos(\phi) & 0 & \sin(\phi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\phi) & 0 & \cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 & 0 \\ \sin(\psi) & \cos(\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{xyz} = R_x \, R_y \, R_z$$

$$\mathrm{T_M} = \mathrm{R_{xyz}} \times \begin{bmatrix} 1 & 0 & 0 & \mathrm{x_m} \\ 0 & 1 & 0 & \mathrm{y_m} \\ 0 & 0 & 1 & \mathrm{z_m} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The kinematic equation the centre of body's coordinate system (x_m, y_m, z_m) and The Main Coordinate System of each leg (x_0, y_0, z_0) is given by the transformation matrices given in Eq.6, Eq.7, Eq.8, Eq.9. The positions and orientations of each leg can be calculated according to the position and orientation of the robot's body.

$$T_{rightback} = T_M * \begin{bmatrix} \cos(\pi/2) & 0 & \sin(\pi/2) & -L/2 \\ 0 & 1 & 0 & 0 \\ -\sin(\pi/2) & 0 & \cos(\pi/2) & W/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{rigthfront} = T_M * \begin{bmatrix} \cos(\pi/2) & 0 & \sin(\pi/2) & L/2 \\ 0 & 1 & 0 & 0 \\ -\sin(\pi/2) & 0 & \cos(\pi/2) & W/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{leftfront} = T_M * \begin{bmatrix} \cos(-\pi/2) & 0 & \sin(-\pi/2) & L/2 \\ 0 & 1 & 0 & 0 \\ -\sin(-\pi/2) & 0 & \cos(-\pi/2) & -W/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{leftback} = T_M * \begin{bmatrix} \cos(-\pi/2) & 0 & \sin(-\pi/2) & -L/2 \\ 0 & 1 & 0 & 0 \\ -\sin(-\pi/2) & 0 & \cos(-\pi/2) & -W/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics of robot, deals with the relationship between the positions, velocities and accelerations of the robot links. Inverse kinematics is the process of finding the values of the joint variables according to the positions and orientations data of the endpoint of robot. In other words, in order to move the robot endpoint to the desired position, it is necessary to determine the rotational values of the joints with inverse kinematic analysis. The forward and inverse kinematics of analysis one leg of a quadruped robot are described in detail. The legs are in different orientations with each other but in the same structure, so it is sufficient to investigate the forward and inverse kinematics analysis of a single leg. Fig. 2 shows the coordinate systems and the angular positions of the right front leg joints. The Denavit-Hartenberg parameters for forward kinematics of the leg are given in Table 2.

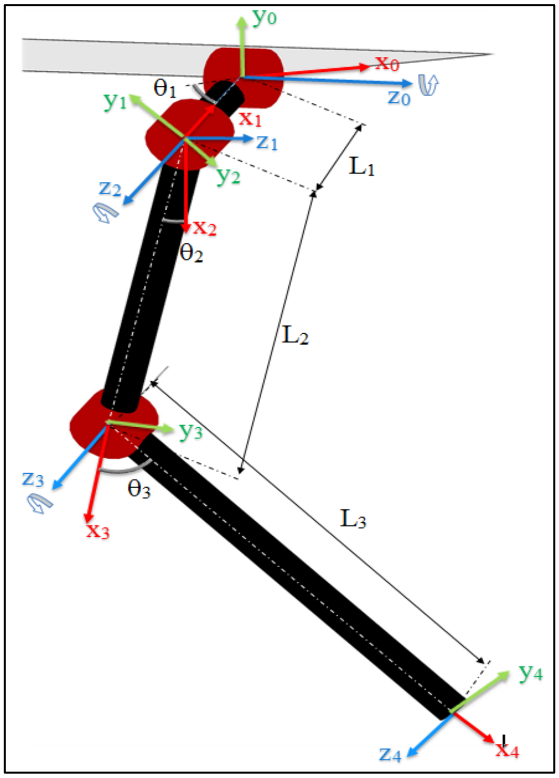


Fig. 2. *The Coordinate Systems of Leg Joints*

TABLE 2. *THE PARAMETERS OF DENAVIT-HARTENBERG*

Link	α i-1	a i-1	d_i	$θ_i$
0-1	0	L1	0	θ1
1-2	-π/2	0	0	-π/2
2-3	0	L2	0	θ2
3-4	0	L3	0	θ3

Each transformation matrices (T_0^1, $T_1^2\ T_2^3\ T_3^4$) are given in Eq. 10, Eq. 11, Eq. 12, Eq. 13 and the forward kinematic matrix (T_0^4) is given in Eq. 14. The elements of the forward kinematic matrix of a leg are given in Table 3.

$$T_0^1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & -L_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & -L_1\sin(\theta_1) \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & L_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^4 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_3\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & L_3\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^4 = T_0^1\, T_1^2\, T_2^3\, T_3^4 = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}$$

TABLE 3. *THE ELEMENTS OF THE FORWARD KINEMATIC MATRIX*

$m_{11} = \cos(\theta_2)\cos(\theta_3)\sin(\theta_1) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)$
$m_{12} = -\cos(\theta_2)\sin(\theta_1)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_1)\sin(\theta_2)$
$m_{13} = -\cos(\theta_1)$
$m_{14} = L_2\cos(\theta_2)\sin(\theta_1) - L_1\cos(\theta_1) + L_2\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) - L_3\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)$
$m_{21} = \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$
$m_{22} = \cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3)\sin(\theta_2)$
$m_{23} = -\sin(\theta_1)$
$m_{24} = L_3\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - L_2\cos(\theta_1)\cos(\theta_2) - L_3\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - L_1\sin(\theta_1)$
$m_{31} = \cos(\theta_2)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_2)$
$m_{32} = \cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3)$
$m_{33} = 0$
$m_{34} = L_2\sin(\theta_2) + L_3\cos(\theta_2)\sin(\theta_3) + L_3\cos(\theta_3)\sin(\theta_2)$
$m_{41} = 0$
$m_{42} = 0$
$m_{43} = 0$
$m_{44} = 1$

After obtained the transformation matrices and forward kinematic matrices required for the inverse kinematic solution of the Quadruped Robot, the inverse kinematic analysis is performed using analytical methods. Equations expressing the angular position of joints (θ_1, θ_2, θ_3) are obtained and given in Eq.15, Eq.16 and Eq.17. There are nonlinear equations in the solution of inverse kinematics problems. For every mathematical expression computed, there may not be a physical solution. Also, there may be more than one solution for the legs endpoint to go to the desired position. For this reason, the legs of the robot (1 and 3) and the leg of the robot (2 and 4) have been realized in the same kinematic structure but in different configurations.

$$\theta_1 = -atan2(-y_4, x_4) - atan2\left(\sqrt{x_4^{\,2} + y_4^{\,2} - L_1^{\,2}}, -L_1\right)$$

$$\theta_2 = atan2\left(z_4, \sqrt{x_4^{\,2} + y_4^{\,2} - L_1^{\,2}}\right) - atan2(L_3\sin(\theta_3), L_2 + L_3\cos(\theta_3))$$

$\theta_3 = atan2\left(-\sqrt{1-D^2}, D\right)$ (Legs for 1 and 3)

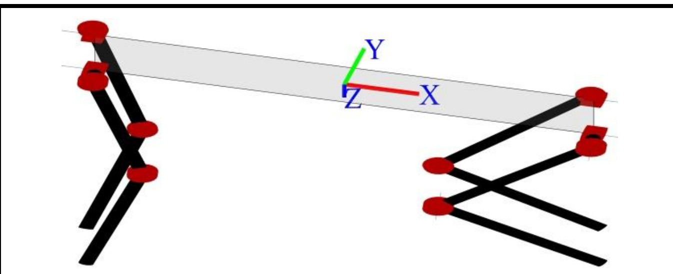
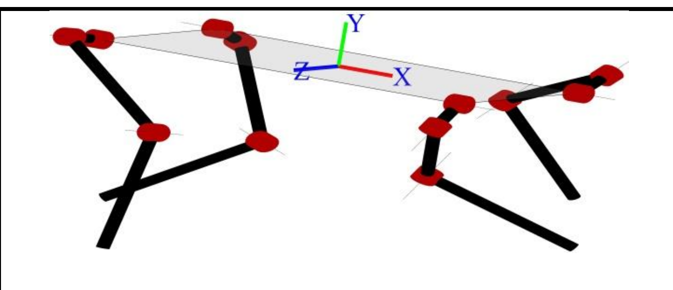
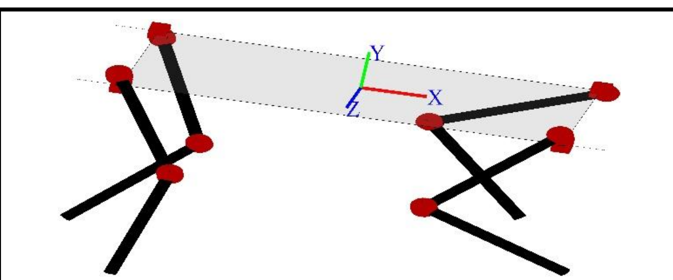
$$\theta_3 = atan2\left(\sqrt{1-D^2}, D\right) \quad \text{(Legs for 2 and 4)} \qquad (17)$$

$$D = \left({x_4}^2 + {y_4}^2 - {L_1}^2 + {z_4}^2 - {L_2}^2 - {L_3}^2\right)/(2\, L_2\, L_3)$$

3 CONCLUSION

In this study, forward and inverse kinematic analysis of a quadruped robot is investigated. Denavit-Hartenberg method is used for forward kinematics, analytical solutions is used for inverse kinematics. The obtained kinematic equations are transferred to MATLAB environment and a program is developed to obtain the angular position of joints of each legs corresponding to the different robot orientations and different coordinate systems of endpoint of legs. In order to test the program, 3 different inverse kinematic analyses are performed and the results obtained from the program are presented in graphical form. As a result, the program which developed in this study performs efficiently the kinetic analysis of a quadruped robot. Furthermore, the robot form can be display by using the program easily. In addition, this study will contribute to the work such as dynamic analysis, walking analysis on the four-legged robot. In addition, this study will contribute to the other studies about dynamic analysis, walking analysis on quadruped robots.

TABLE 3. *THE EXAMPLES RESULTS OBTAINED FROM THE PROGRAM*

$[x_4, y_4, z_4]=[0, -0.65, 0]$ $[x_m, y_m, z_m]= [0, 0, 0]$				θ_1	θ_2	θ_3
φ= 0° ψ=-15° ω=0°		Leg Number	1	7.5883°	28.7493°	-29.7695°
			2	11.5735°	-33.0804°	100.5692°
			3	11.5735°	33.0804°	-100.569°
			4	7.5883°	-28.7493°	29.7695°
$[x_4, y_4, z_4]=[-0.05, -0.55, 0]$ $[x_m, y_m, z_m]= [0, 0, 0]$				θ_1	θ_2	θ_3
φ= -45° ψ= 0° ω=-10°		Leg Number	1	-9.7298°	49.8269°	-53.8359°
			2	47.7890°	-30.1490°	67.8506°
			3	-31.9917°	59.6929°	-69.4310°
			4	31.8200°	-36.8724°	82.0530°
$[x_4, y_4, z_4]=[-0.15, -0.7, 0.05]$ $[x_m, y_m, z_m]= [0.1, 0.2, -0.3]$				θ_1	θ_2	θ_3
φ=-10° ψ=-10° ω=15°		Leg Number	1	-51.7965°	30.1317°	-35.2716°
			2	-48.0254°	-34.7341°	99.7991°
			3	34.9428°	62.5980°	-105.322°
			4	43.2869°	-25.3487°	59.5477°

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