An EPQ Model For Variable Production Rate With Exponential Demand And Unsteady Deterioration

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Abstract: The focus of research is to study the nature of economic production quantity model with unsteady deterioration rate is considered. In practical situation it is discover that the starting of any production activity, the rate of production in many firm remain less up to certain time, but after some time the production rate increase with time. Due to this reason the model consider variable production rate. Demand rate is associated with exponential function depend on time. Under this situation, mathematical form is formulated to enlarge the total profit. Numerical part is provide for respective solution to support the model and sensitivity analysis shows changes in some parameters.

Key Words: EPQ model, unsteady deterioration, exponential demand, variable production rate

1. INTRODUCTION:
One common misconception in industry is that production and inventory control are separate functions. Inventory control writes the orders while production control gets them made in the plant. However, the basic truth is that inventories in a manufacturing plant are maintained to support production or are themselves the result of production. Only where inventories are purchased and then resold without requiring further work can inventory control have meaning apart from production control. Wide range of literature written on the EOQ and EPQ model with deterioration studied in past. For making best inventory policy demand play crucial role in main part of inventory structure like production, distribution of items and retailing. Researchers are busy to develop the inventory models imagine the different types of demand like deterministic and probabilistic. Deteriorating commodity are commonly characterized in duration of their existence or usefulness are correlate with time as available stock. Originally inventory models for deteriorating commodity was attention by Whitin(1957). Ghare and Schrader (1963) initially create a form with equable rate of deterioration Balkhi [2001] develop a model on finite horizon production lot size. Goyal (2003) provide solution for production inventory about time varying demand and deterioration rates. A production inventory model with price and stock dependent demand was studied by Teng and Chang(2005) developed. Production model given by Bansal (2012) was based on assumption that demand as a form of price dependent. Ghasemi (2015) developed EPQ models for rapid deteriorating items. Raafat (1991) and Ruxian (2010) present study on deteriorating items inventory models. Roy and Chaudhary (2009) develop production inventory model over stock dependent demand. Samanta and Roy (2004) established a production inventory model with deteriorating commodity and shortages. Patel and Patel (2013) analyzed production inventory model for Weibull deteriorating item on price and quantity based demand under differ holding cost.

Sheikh and Patel (2017) provide a production model with desperate deterioration rates. Saha and Chakrabarti (2018) study an EPQ model for deteriorating commodity among probabilistic demand and movable production rate. Production inventory model using varying deterioration rates under inflation and delay in payments was develop by Patel (2017). This study is relate with EPQ model with unsteady deterioration rate is taken. Demand function should be consider as an exponential with time. Due to production escalate rate of variable production taken. Under this situation, a total profit of the mathematical form has been elaborated. Numerical case given to represent the model. Affectability investigation is likewise done for parameters.

2. ASSUMPTIONS AND SYLLABARY:
The subsequent series of written symbols are used to develop the mathematical form: d(t): Demand is a growing form of time i.e. exponential (ae^st), a>0, 0<b<1)k_o: Manufacturing rate per unit time A: Manufacturing setup charge c: Pickup charge per unit p: Selling price per unit h: carrying charge per unit time T: Total period of inventory l(t): Inventory measure at time t Q_o: Inventory measure initially Q_o: Inventory measure at time t Deterioration rate P: Total profit Below belief are taken as expansion of the model. The demand consider to be exponential function based on time. Replenishment rate is extent and immediate. Initially starting time is nil. Shortages are not taken into consideration. Deteriorated units neither be renovate nor put away throughout the period of inventory. Variable production rate K is taken as; K = \begin{cases} k_0, & 0 \leq t \leq t_l \\ k_0e^{\mu(t-t_l)}, & t_l \leq t \leq T \end{cases} 
Where \mu is a constant (0 < \mu < 1) (stated by Patra and Mondal (2015). In this paper production rate assumed to be variable. Starting of any production process first the process is in slow nature but after some time production increase due to some improvement factors i.e. some efficient parameter goes to production gradually increase. Based on this concept manufacture maintain the process due to some extra cost consider as development cost i.e. \alpha k_o. (\alpha is a constant)
3. CONCEPT OF THE MATHEMATICAL ANALYSIS:

Below diagram shows behaviour of the inventory at time 0

To T. The production rate $k_n$ remains fixed at $0 \leq t \leq t_1$, then production rate increase due to some improvement in production method in interval $t_1 \leq t \leq t_2$. The inventory sets are expressed as differential equations.

$$\frac{d}{dt} I_n(t) = k_n \cdot d(t)$$

(1)

$$\frac{d}{dt} I_1(t) + 0 \cdot I_2(t) = e^{-\theta t} I_n \cdot d(t), \quad t_1 \leq t \leq t_2$$

(2)

$$\frac{d}{dt} I_1(t) + 0 \cdot I_2(t) = -d(t), \quad t_1 \leq t \leq T$$

(3) with initial orders $I(0) = 0$, $I(t_1) = Q_1$, $I(t_2) = Q_2$, $I(T) = 0$. Solving differential equations (1) to (3) we have,

$$I_1(t) = \frac{k_n \cdot a e^{-\theta t}}{b} + \frac{a}{b}$$

(4)

$$I_1(t) = \left( \frac{t^2}{8} + \frac{1}{2} \right) \left( -\mu k_n - ab \right) t^2 + \frac{1}{6} (1 + \mu t_1) k_n \cdot a \cdot t^2$$

$$+ \left( (1 + \mu t_1) + t_1 - a + Q_t + \frac{1}{2} (Q, 0 - \mu k_n + ab) \right) t_1$$

$$- \left( -\frac{1}{2} \cdot 0 \cdot k_n \cdot 8 \cdot 0 \cdot ab \right) t^2 - \frac{1}{6} (k_n \cdot a) \cdot t^2$$

$$- \frac{1}{2} \cdot 0 \cdot t^2 - \frac{1}{6} (k_n \cdot a) \cdot t^2$$

(5)

$$I_1(t) = -a \left( t + \frac{1}{8} b \cdot t^3 + \frac{1}{6} b \cdot t^2 \right)$$

$$+ \frac{1}{24} a T \left( 24 + 3 b T + 40 T^2 + 12 b T \right)$$

(6)

$$t_1$$ in solution of differential equation (4), we obtain

$$Q_1 = k_n \cdot t_1 \cdot \frac{a e^{-\theta t}}{b} + \frac{a}{b}$$

(7)

Putting $t = t_2$ in solution of differential equation (5) and (6), we acquire

$$I_1(t_2) = \left( \frac{t^2}{8} + \frac{1}{2} \right) \left( -\mu k_n - ab \right) t^2 + \frac{1}{6} (1 + \mu t_1) k_n \cdot a \cdot t^2$$

$$+ \left( (1 + \mu t_1) + t_1 - a + Q_t + \frac{1}{2} (Q, 0 - \mu k_n + ab) \right) t_1$$

$$- \left( -\frac{1}{2} \cdot 0 \cdot k_n \cdot 8 \cdot 0 \cdot ab \right) t^2 - \frac{1}{6} (k_n \cdot a) \cdot t^2$$

$$- \frac{1}{2} \cdot 0 \cdot t^2 - \frac{1}{6} (k_n \cdot a) \cdot t^2$$

(8)

(8) and (9), we have $t_2 = \frac{a T}{k_n (1 + \mu t_1)}$

(10)

The answer of result (10) produced that $t_2$ is in form of $t_1$ and $T$, so $t_2$ is not a determination variable. Similarly, putting value of $Q_1$ in result (5), we get

$$I_2(t) = \frac{1}{8} (\mu k_n + ab) \cdot t^4 + \frac{1}{3} (1 + \mu t_1) k_n + a) \cdot t^3$$

$$+ \frac{1}{24} (b \cdot k_n \cdot a + 2 \cdot b - \mu k_n + ab) \cdot t^2$$

(9)

(9)

Putting $t = t_2$ in result (6), we have

$$Q_2 = + \frac{1}{24} a T \left( 24 + 3 b T + 40 T^2 + 12 b T \right)$$

(11)

(12)

(not consider the higher powers of $\theta$) The EPQ model have the following cost which are the part of the total profit (P) function: (a) Set-up cost (SeC) = A

$$H_C = b \left( \int_{t_1}^{t_2} \frac{t_1}{I_1(t_1)} dt + \int_{t_1}^{t_2} I_1(t_1) dt + \int_{t_1}^{t_2} I_2(t_2) dt \right)$$

(13)

$$C_D = c \left( \int_{t_1}^{t_2} 0 \cdot t_1(t_1) dt + \int_{t_1}^{t_2} 0 \cdot t_1(t_2) dt \right)$$

(14)

(15)
Development Cost = αk₀  

The total profit (P), through an inventory cycle made up of the following: 

\[ P(T, t) = \frac{1}{T} \left( SR \cdot SeC + HC + DC - \alpha k_o \right) \]  

(17)

Fill result (13) to (17) in result (18), we get overall profit per unit in terms of \( t_1 \) and \( T \). Putting \( t_1 = v \cdot t_2 \), \( 0 < v < 1 \) and value of \( t_2 \) from result (10) and (18) we get profit in expression of \( T \).

\[ P(T) = \frac{1}{T} \left( SR \cdot SeC + HC + DC - \alpha k_o \right) \]  

(19)

Differentiating equation (19) regarding \( T \) and equate it to zero, we get optimal value of \( T \). 

\[ \frac{\partial P(T)}{\partial T} = 0 \]  

(20)

Optimal value of \( T \) satisfies the second order condition given below. 

\[ \frac{\partial^2 P(T)}{\partial T^2} \geq 0 \]  

(21)

4. NUMERICAL EXAMPLE:
Examine model for the below given values in appropriate units, we present a numerical example to support the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% Change</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>+20</td>
<td>5.356</td>
</tr>
<tr>
<td></td>
<td>+10</td>
<td>5.249</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>5.209</td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>5.171</td>
</tr>
<tr>
<td>( \theta )</td>
<td>+20</td>
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<td></td>
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<td></td>
<td>-20</td>
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<tr>
<td>( h )</td>
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<tr>
<td></td>
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<td></td>
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<tr>
<td>( B )</td>
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<td>0.548</td>
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<tr>
<td></td>
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</tr>
<tr>
<td>( p )</td>
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<td>0.526</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td>( k_o )</td>
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<tr>
<td></td>
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<td>0.542</td>
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<tr>
<td></td>
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<td>0.580</td>
</tr>
</tbody>
</table>

From the conclusion of the above parametric analysis, the following consideration can be made. For the parameter \( a \) and \( p \) when we change decrease to increase the profit will increase. For other parameters \( \theta \), \( h \), \( B \) and \( k_o \) are less sensitive as increase to decrease the profit will decrease.

5. PARAMETRIC ANALYSIS:
Now we observe the achievement of the parameters on profit function. Moving included parameter by ±20% and ±10% hold one parameter at once and keep rest of parameters are same.

6. CONCLUSION:
In this study of EPQ model with unsteady deterioration rate is examine. The model justly taken as demand as exponential function of time. Due to some circumstance during production process, the rate of production is less up to certain time, but after some time the production rate increase with time. Due to this reason the model consider variable production rate. Numerical part proves the applicability of the derive model and parametric analysis shows the effect in some parameter. The derived model can expand for items having different types of demand and deterioration.

REFERENCES


