Global Stability For Divorce In Arrange/Love Marriage Due To Extra Marital Affair

Nita H. Shah, Purvi M. Pandya, Moksha H. Satia

Abstract: Every person in this world is deemed to find his or her partner sooner or later. It can be love or arranged marriage and some may seek for compassion as an extra marital affair. Due to which they may face separation leading to divorce in few cases. Certain examples of negotiations are also reported where couples reunite and give their marriage a second possible chance. This scenario is formulated by a mathematical model using the system of non-linear ordinary differential equations. Also, using next generation matrix method, the basic reproduction number is computed which gives the rate of susceptible individual getting divorce. Stability analysis for equilibrium points and numerical simulations are carried out to validate the data.

Index Terms: Affair, Basic reproduction number, Bifurcation, Divorce, Marriage, Mathematical model, Stability.

1 Introduction

Marriage is a traditional custom which is followed by every religion to make a relationship between a woman and a man (generally), officially as well as socially accredited. People involve themselves in a wedlock for varied reasons such as, satisfying emotional and physical needs, raising children, family and societal enforcements etc. Over the years, various matrimonial trends have been observed, ranging from bizarre customs of child marriage, popularly known arranged marriage, a social taboo of love marriage, to now legally accepted same sex marriages. But, as on one side, humans have evolved in various dimensions, on the other side, the stability and faith in such an impregnable system is decreasing. Males and females are more in a competing role, than a complementary role, which is a core foundation of a successful marriage. Nowadays, more and more people are opting to get divorced or seek for extramarital pleasures (infidelity), leading to a shattered family. This is attributed to decreased persistence and perseverance levels. Thus, there arises a need to address this socially debatable issue with a structured methodical data analysis.

Many surveys are done to study this life cycle from marriages to divorce and remarriage. In Becker (1973) article, a theory of marriage was analyzed and argued that the gain from marriage as compared to remaining single is positive related to daily incomes and expenses and thereafter the in next article was extended analyze some circumstances of marriage, separation, divorce and remarriage (Becker, 1974). Also, an analysis of first and second marriages for women of age 15-44 was done in the United States in 1995 which presents the probability of divorce and remarriage using certain methods (Bramlett & Mosher, 2001). Even a book was written explaining marriage, divorce and remarriage (Cherlin, 2009). Along with theorical explanations, a mathematical model explaining a social epidemiology model of divorce which was propagated by women in Spain was formulated (Duato & Jódar, 2013). Also, a divorce transmission model was designed using mathematical model (Gambrah et al., 2018). A discussion about what predicts divorce and the relationship between marital processes and marital outcomes was done in a book using different methods such as videotapes of couples' conversation, brief interviews with couple, a few questionnaires etc. (Gottman, 2014). An article of theory of marriage timings argue related to trends and differentials in marriage timing, imbalance in which sometimes lead to divorce (Oppenheimer, 1988). Also, book entitled "Divorce" which is considered to be a major life transition was discussed (Price & McKenry, 1988).

In this research paper, mathematical model is formulated in Section 2 along with stability analysis for all equilibrium points in Section 3. Backward bifurcation is obtained in Section 4 and numerical simulation to validate the data is carried out in Section 5. Section 6 concludes the observations for the proposed problem.

2 Mathematical Model

Here, as shown in Fig. 1 the model consists of six compartments namely susceptible individuals (S_I) for marriage who either opts for arrange marriage (A_M) or love marriage (L_M). Some of them may have extra marital affair (E_M) which sometimes leads to partition (P) or in some cases divorce (D). After getting divorce they again become susceptible for any kind of marriage. The model parameters and state variables are depicted in Table 1.

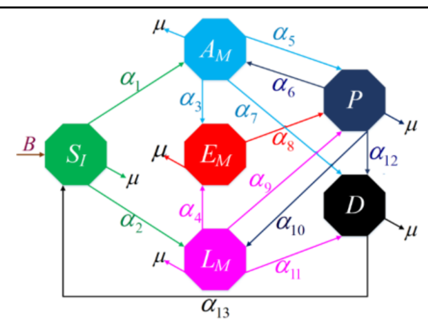

Fig. 1. Mathematical model

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TABLE 1
MODEL PARAMETERS AND STATE VARIABLES

Notation	Description	Parametric values	Source
B	New Recruitment rate	0.4	Case study
α_1	Arranged marriage rate of individuals	0.6	Case study
α_2	Love marriage rate of individuals	0.05	Case study
α_3	Rate of infidelity among arranged married couples	0.01	Case study
α_4	Rate of infidelity among love married couples	0.01	Case study
α_5	Rate at which individuals take a step to separate	0.1	Case study
α_6	Rate at which arranged married couples reunite	0.02	Case study
α_7	Divorce rate of arranged married couples	0.05	Case study
α_8	Separation rate of individuals having extra marital affair	0.3	Case study
α_9	Separation rate of love married couples	0.1	Case study
α_{10}	Rate at which love married couples reunite	0.02	Case study
α_{11}	Divorce rate of love married couples	0.15	Case study
α_{12}	Divorce rate of separated individuals	0.3	Case study
α_{13}	Rate of divorced individuals again becomes susceptible for marriage	0.2	Case study
μ	Natural Death rate	0.1	Assumed

$$\frac{dS_I}{dt} = B - \alpha_1 S_I A_M - \alpha_2 S_I L_M + \alpha_{13} D - \mu S_I$$

$$\frac{dA_M}{dt} = \alpha_1 S_I A_M + \alpha_6 P - x_1 A_M$$

$$\frac{dE_M}{dt} = \alpha_3 A_M + \alpha_4 L_M - x_2 E_M \qquad (1)$$

$$\frac{dL_M}{dt} = \alpha_2 S_I L_M + \alpha_{10} P - x_3 L_M D$$

$$\frac{dP}{dt} = \alpha_5 A_M + \alpha_8 E_M + \alpha_9 L_M - x_4 P$$

$$\frac{dD}{dt} = \alpha_7 A_M + \alpha_{11} L_M + \alpha_{12} P - x_5 D$$

where,

$x_1 = \alpha_3 + \alpha_5 + \alpha_7 + \mu, x_2 = \alpha_8 + \mu, x_3 = \alpha_4 + \alpha_9 + \alpha_{11} + \mu,$

$x_4 = \alpha_6 + \alpha_{10} + \alpha_{12} + \mu, x_5 = \alpha_{13} + \mu$

and $S_I + A_M + E_M + L_M + P + D \le N$.

Also, $S_I > 0; A_M, E_M, L_M, P, D \ge 0$.

Adding the above differential equations of system (1), we have

$$\frac{d}{dt}\left(S_I + A_M + E_M + L_M + P + D\right) = B - \mu\left(S_I + A_M + E_M + L_M + P + D\right) \ge 0$$

which implies that $\limsup_{t \to \infty}\left(S_I + A_M + E_M + L_M + P + D\right) \le \frac{B}{\mu}$.

Therefore, the feasible region of the model is

$$\Lambda = \left\{ (S_I, A_M, E_M, L_M, P, D) \in R^6 : S_I + A_M + E_M + L_M + P + D \le \frac{B}{\mu} \right\}.$$

Now, the divorce-free equilibrium point of the model is $Y_0 = \left(\frac{B}{\mu}, 0, 0, 0, 0, 0\right)$ and endemic point is

$$Y^* = \left(S_I^*, A_M^*, E_M^*, L_M^*, P^*, D^*\right)$$

where,

$S_I^* = r$,

$$A_M^* = \frac{\alpha_5\alpha_6(\alpha_4\alpha_8 + \alpha_9 x_2)(B - r\mu)}{\begin{array}{l}(\alpha_4\alpha_8 + \alpha_9 x_2)(r\alpha_1(\alpha_{12}\alpha_{13} + \alpha_{10}x_5) \\ \quad + x_5(r\alpha_1\alpha_6 - \alpha_{10}x_1) - \alpha_{13}(\alpha_6\alpha_7 + \alpha_{12}x_1)) \\ \quad\quad + (\alpha_{11}\alpha_{13} - x_3x_5)(\alpha_6(\alpha_3\alpha_8 + \alpha_5x_2) + x_2x_4(r\alpha_1 - x_1))\end{array}},$$

$$E_M^* = \frac{\begin{array}{l}\alpha_5((\alpha_4\alpha_8 + \alpha_9x_2)(\alpha_{10}\alpha_4\mu x_1 - r\mu(\alpha_1\alpha_{10}\alpha_4 + \alpha_2\alpha_3\alpha_6)) \\ \quad + x_2x_4\alpha_4(B\alpha_2 - \mu x_3)(x_1 - r\alpha_1) + \alpha_6(\alpha_4\mu x_3(\alpha_3\alpha_8 + \alpha_5x_2) \\ \quad - B\alpha_2x_2(\alpha_4\alpha_5 - \alpha_3\alpha_9)))\end{array}}{\begin{array}{l}\alpha_2x_2[(\alpha_4\alpha_8 + \alpha_9x_2)((r\alpha_1 - x_1)(\alpha_{10}x_5 + \alpha_{12}\alpha_{13}) \\ \quad + \alpha_6(r\alpha_1\alpha_5 - \alpha_7\alpha_{13})) + (\alpha_{11}\alpha_{13} - x_3x_5)(\alpha_6(\alpha_3\alpha_8 + \alpha_5x_2) \\ \quad + x_2x_4(r\alpha_1 - x_1))]\end{array}},$$

$$L_M^* = \frac{\begin{array}{l}x_5(\alpha_{10}\mu(\alpha_4\alpha_8 + \alpha_9x_2)(x_1 - \alpha_1r) + (x_3\mu - B\alpha_2)(\alpha_6(\alpha_3\alpha_8 + \alpha_5x_2) \\ \quad + x_2x_4(r\alpha_1 - x_1)))\end{array}}{\begin{array}{l}\alpha_2[(\alpha_4\alpha_8 + \alpha_9x_2)((r\alpha_1 - x_1)(\alpha_{10}x_5 + \alpha_{12}\alpha_{13}) + \alpha_6(r\alpha_1x_5 - \alpha_7\alpha_{13})) \\ \quad + (\alpha_{11}\alpha_{13} - x_3x_5)(\alpha_6(\alpha_3\alpha_8 + \alpha_5x_2) + x_2x_4(r\alpha_1 - x_1))]\end{array}},$$

$$P^* = \frac{\begin{array}{l}(r\mu - B)x_5[(\alpha_3\alpha_8 + \alpha_5x_2)\alpha_1x_2x_3x_4(r\alpha_2 - x_3) + (\alpha_4\alpha_8 + \alpha_9x_2) \\ \quad ((\alpha_3\alpha_8 + \alpha_5x_2)(\alpha_{10} + \alpha_6)(\alpha_1x_3 - \alpha_2x_1) + \alpha_2x_1x_2x_4(x_1 - r\alpha_1))]\end{array}}{\begin{array}{l}x_2x_4(\alpha_3\alpha_8 + \alpha_5x_2)[\alpha_2x_1(x_3x_5 - \alpha_{13}\alpha_{11})(2\alpha_6 + \alpha_{10}) + \alpha_1(\alpha_{13} \\ (\alpha_{10}\alpha_{11}(r\alpha_2 - x_3) + r\alpha_2(\alpha_6\alpha_{11} + \alpha_{12}x_3)) - x_3^2(\alpha_{12}\alpha_{13} + \alpha_6x_5))] \\ + (\alpha_6 + \alpha_{10})[\alpha_2\alpha_6(\alpha_{11}\alpha_{13} - x_3x_5)(\alpha_3\alpha_8 + \alpha_5x_2)^2 + \alpha_1\alpha_{10} \\ (x_1x_5 - \alpha_{13}\alpha_7)(\alpha_4\alpha_8 + \alpha_9x_2)^2] + (\alpha_4\alpha_8 + \alpha_9x_2)[(\alpha_{10} + \alpha_6) \\ \{(\alpha_3\alpha_8 + \alpha_5x_2)(\alpha_1\alpha_{13}(\alpha_{10}\alpha_{11} + \alpha_{12}x_3) + x_5(\alpha_1\alpha_6x_3 - \alpha_{10}\alpha_2x_1) \\ - \alpha_{13}\alpha_2(\alpha_6\alpha_7 + \alpha_{12}x_1)) + \alpha_1x_2x_4(\alpha_{13}\alpha_7(\alpha_{13}x_3 - r\alpha_2) - x_1x_3x_5)\} \\ + x_2x_4(\alpha_1\alpha_{10}x_3(\alpha_{13}\alpha_7 - x_1x_5) + \alpha_2x_1^2(\alpha_{10}x_5 + \alpha_{12}\alpha_{13}) + \alpha_{13}\alpha_2x_1 \\ (\alpha_6\alpha_7 - r\alpha_1\alpha_{12}))] + x_2^2x_4^2[\alpha_1(r\alpha_{13}\alpha_2(\alpha_7x_3 - \alpha_{11}x_1) \\ + x_3^2(x_1x_5 - \alpha_{13}\alpha_7)) + \alpha_2x_1^2(\alpha_{11}\alpha_{13} - x_3x_5)]\end{array}},$$

$$D^* = \frac{\begin{array}{l}\alpha_{11}x_2x_4(\mu x_3 - B\alpha_2)(x_2x_4(\alpha_1r - x_1) + \alpha_6(\alpha_3\alpha_8 + \alpha_5x_2)) \\ \quad - (\alpha_4\alpha_8 + \alpha_9x_2)(x_2x_4(\alpha_1\alpha_{12}r(B\alpha_2 - \mu x_3) + \alpha_{10}\alpha_{11}\mu \\ \quad (\alpha_1r - x_1)) - \alpha_{12}\mu(\alpha_2\alpha_6r(\alpha_3\alpha_8 + \alpha_5x_2) - x_2x_3(\alpha_5\alpha_6 - x_1x_4)) \\ \quad - \alpha_{12}(\alpha_{10}\mu(\alpha_1r - x_1)(\alpha_4\alpha_8 + \alpha_9x_2) - B\alpha_2x_1x_2x_4 - \alpha_3\alpha_6\alpha_8\mu x_3) \\ \quad + \alpha_2\alpha_6\alpha_7x_2x_4(B - \mu r))\end{array}}{\begin{array}{l}\alpha_2x_2x_4[(\alpha_4\alpha_8 + \alpha_9x_2)(r\alpha_1(\alpha_{10}x_5 + \alpha_{12}\alpha_{13}) + x_5(r\alpha_1\alpha_6 - \alpha_{10}x_1) \\ -\alpha_{13}(\alpha_{12}x_1 - \alpha_6\alpha_7)) + (\alpha_{11}\alpha_{13} - x_3x_5)(x_2x_4(r\alpha_1 - x_1) \\ +\alpha_6(\alpha_3\alpha_8 + \alpha_5x_2))]\end{array}}$$

and

$$r = \frac{1}{2\alpha_1\alpha_2 x_2 x_4}[x_2x_4(\alpha_1\alpha_3+\alpha_2x_1)-\alpha_1\alpha_{10}(\alpha_4\alpha_8+\alpha_9x_2)-\alpha_2\alpha_6$$
$$(\alpha_3\alpha_8+\alpha_5x_2)+\{\alpha_1^2\alpha_{10}^2(\alpha_4\alpha_8+\alpha_9x_2)^2+x_2^2x_4^2(\alpha_1\alpha_3-\alpha_2x_1)^2$$
$$+2\alpha_1\alpha_{10}(\alpha_4\alpha_8+\alpha_9x_2)(\alpha_2\alpha_6(\alpha_3\alpha_8+\alpha_5x_2)-x_2x_4(\alpha_1x_3-\alpha_2x_1))$$
$$+\alpha_2\alpha_6(\alpha_3\alpha_8+\alpha_5x_2)(\alpha_2\alpha_6(\alpha_3\alpha_8+\alpha_5x_2)+2x_2x_4(\alpha_1x_3-\alpha_2x_1))\}^{\frac{1}{2}}]$$

Now, we evaluate the basic reproduction number known as threshold R_0 using next generation matrix method (Diekmann et. al., 2009).

Let $X' = (S_I, A_M, E_M, L_M, P, D)'$ and $X' = \frac{dX}{dt} = F(X) - V(X)$ where $F(X)$ denotes the rate of arrival of new individual in the compartment and $V(X)$ denotes the rate of divorce due to extra marital affair which are given by

$$F(X) = \begin{bmatrix} \alpha_1 S_I A_M \\ 0 \\ \alpha_2 S_I L_M \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and}$$

$$V(X) = \begin{bmatrix} -\alpha_6 P + x_1 A_M \\ -\alpha_3 A_M - \alpha_4 L_M + x_2 E_M \\ -\alpha_{10} P + x_3 L_M \\ -\alpha_5 A_M - \alpha_8 E_M - \alpha_9 L_M + x_4 P \\ -\alpha_7 A_M - \alpha_{11} L_M - \alpha_{12} P + x_5 D \\ -B + \alpha_1 S_I A_M + \alpha_2 S_I L_M - \alpha_{13} D + \mu S_I \end{bmatrix}$$

Now, $DF(X_0) = \begin{bmatrix} f & 0 \\ 0 & 0 \end{bmatrix}$ and $DV(X_0) = \begin{bmatrix} v & 0 \\ J_1 & J_2 \end{bmatrix}$, where f and v are 6×6 matrices defined as $f = \left[\frac{\partial F_i(X_0)}{\partial X_j}\right]$ and $v = \left[\frac{\partial V_i(X_0)}{\partial X_j}\right]$.

Finding f and v, we get

$$f = \begin{bmatrix} \alpha_1 S_I & 0 & 0 & 0 & 0 & \alpha_1 A_M \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 S_I & 0 & 0 & \alpha_2 L_M \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$v = \begin{bmatrix} x_1 & 0 & 0 & -\alpha_6 & 0 & 0 \\ -\alpha_3 & x_2 & -\alpha_4 & 0 & 0 & 0 \\ 0 & 0 & x_3 & -\alpha_{10} & 0 & 0 \\ -\alpha_5 & -\alpha_8 & -\alpha_9 & x_4 & 0 & 0 \\ -\alpha_7 & 0 & -\alpha_{11} & -\alpha_{12} & x_5 & 0 \\ \alpha_1 S_I & 0 & \alpha_2 S_I & 0 & -\alpha_{13} & \alpha_1 A_M + \alpha_2 L_M + \mu \end{bmatrix}$$

Here, v is non-singular matrix.

For this model, the basic reproduction (threshold) at is obtained as the spectral radius of matrix fv^{-1}.

$$R_0 = \frac{B(\alpha_1\alpha_6(\alpha_4\alpha_8+\alpha_9x_2)+\alpha_2\alpha_{10}(\alpha_3\alpha_8+\alpha_5x_2))}{\mu(x_1x_2x_3x_4-\alpha_{10}x_1(\alpha_4\alpha_8+\alpha_9x_2)-\alpha_6x_3(\alpha_3\alpha_8+\alpha_5x_2))} \quad (2)$$

3 STABILITY

Now, let us discuss the global stability behavior of the equilibrium points using Lyapunov's function.

Theorem 3.1 (Stability at Y_0): *The disease-free equilibrium point is globally asymptotically stable.*

Proof. Consider the Lyapunov's function

$$L_1(t) = A_M(t) + E_M(t) + L_M(t) + P(t) + D(t)$$

then, $L_1'(t) = A_M'(t) + E_M'(t) + L_M'(t) + P'(t) + D'(t)$

$$= S_I(\alpha_1 A_M + \alpha_2 L_M) - \mu(A_M + E_M + L_M + P + D) - \alpha_{13} D$$

we get $\frac{dL_1}{dt} < 0$ if $\alpha_1 B < \mu^2$ and $\alpha_2 B < \mu^2$ whereas $\frac{dL_1}{dt} = 0$ only if $A_M, E_M, L_M, P, D = 0$.

Therefore, Y_0 is globally asymptotically stable (LaSalle, 1976).

Theorem 3.2 (Stability at Y^*): *The endemic equilibrium point Y^* is globally asymptotically stable.*

Proof. Consider the Lyapunov function,

$$L(t) = \frac{1}{2}\begin{bmatrix} (S_I - S_I^*) + (A_M - A_M^*) + (E_M - E_M^*) \\ + (L_M - L_M^*) + (P - P^*) + (D - D^*) \end{bmatrix}^2$$

then,

$$L'(t) = \begin{bmatrix} (S_I - S_I^*) + (A_M - A_M^*) + (E_M - E_M^*) \\ + (L_M - L_M^*) + (P - P^*) + (D - D^*) \end{bmatrix}\begin{pmatrix} S_I' + A_M' + E_M' \\ + L_M' + P' + D' \end{pmatrix}$$

$$= -\mu\begin{bmatrix} (S_I - S_I^*) + (A_M - A_M^*) + (E_M - E_M^*) \\ + (L_M - L_M^*) + (P - P^*) + (D - D^*) \end{bmatrix}^2 \le 0$$

Here, we denote $B = \mu(S_I^* + A_M^* + E_M^* + L_M^* + P^* + D^*)$.

Therefore, Y^* is globally stable.

4 BIFURCATION

To analyze the backward bifurcation, let $\alpha_6 = 0.32$ and susceptible individual for marriage compartment should be at least non zero. Solving system (1) we get,

$$F(S_I^*) = aS_I^{*2} + bS_I^* + c = 0 \quad (3)$$

where,

$$a = \alpha_1\alpha_2 x_2 x_4$$

$$b = \alpha_1\alpha_{10}(\alpha_4\alpha_8 + \alpha_9 x_2) + \alpha_2\alpha_6(\alpha_3\alpha_8 + \alpha_5 x_2) - x_2 x_4(\alpha_1 x_3 + \alpha_2 x_1)$$

$$c = x_1 x_2 x_3 x_4 - \alpha_{10} x_1(\alpha_4\alpha_8 + \alpha_9 x_2) - \alpha_6 x_3(\alpha_3\alpha_8 + \alpha_5 x_2)$$

The coefficient a must always be positive and c should depend upon the value of R_0, if $R_0 < 1$ then c is positive and if $R_0 > 1$ then c is negative. For $a > 0$, the positive result depends on the sign of b and c. The equation (3) has two roots; one is positive and other is negative for $R_0 > 1$. Now, if $R_0 = 1$ then $c = 0$ and we obtain a non-zero solution of equation (3) as $-\frac{b}{a}$ which is positive if and only if $b < 0$. For $b < 0$, there exists a positive interior equilibrium point for $R_0 = 1$ that means the equilibria continuously depends upon R_0, indicating that there exists an interval for R_0 which have two positive equilibria $\beta_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$, $\beta_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$. For backward bifurcation, putting the discriminant $b^2 - 4ac = 0$ and then solving for the critical points of R_0 gives

$$R_C = 1 - \frac{b^2((\alpha_4\alpha_8 + \alpha_9 x_2)(\mu\alpha_{10} x_1 + B\alpha_1\alpha_6) + (\alpha_3\alpha_8 + \alpha_5 x_2)(\mu\alpha_6 x_3 + B\alpha_{10}\alpha_2) - \mu x_1 x_2 x_3 x_4)}{4a(-\mu(\alpha_{10} x_1(\alpha_4\alpha_8 + \alpha_9 x_2) + \alpha_6 x_3(\alpha_3\alpha_8 + \alpha_5 x_2) - x_1 x_2 x_3 x_4)^2)}$$

If $R_C < R_0$, then $b^2 - 4ac > 0$ and for the point of R_0 backward bifurcation exists such that $R_C < R_0 < 1$ (Khan et al. (2014), Wangari et al. (2016)).

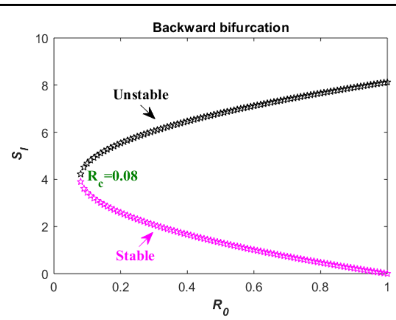

Fig. 2. Backward bifurcation

5 Numerical Simulation

To validate the results, numerical simulation is carried out using parametric values given in Table 1 with some new values of $\alpha_3 = 0.3, \alpha_4 = 0.03, \alpha_5 = 0.4$ and $\alpha_{10} = 0.2$.

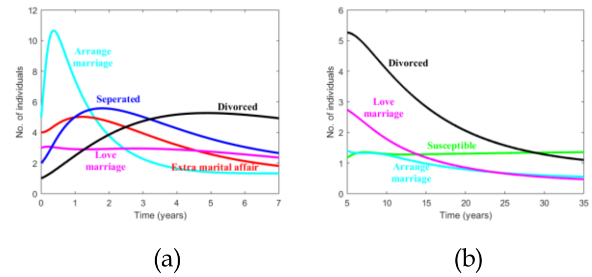

Fig. 3. Transmission pattern of individual in respective compartment

As shown in Fig. 3(a), approximately 5 individual who opt for arrange marriage gets separated, out of which some have extra marital affair which leads to divorce. Also, approximately 3 individual who go for love marriage gets separated which then leads to divorce. Due to some family or maybe some different reasons they opt for arrange marriage which sometimes leads to infidelity. Even after getting divorced, some are again susceptible for any kind of marriages as shown in Fig. 3(b).

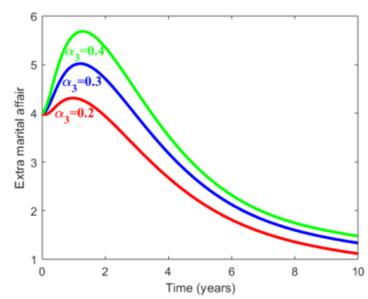

Fig. 4. Impact of α_3 on extra marital affair

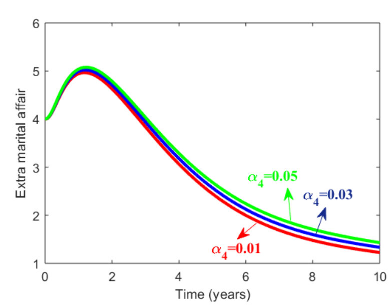

Fig. 5. Effect of α_4 on extra marital affair

As shown in Fig. 4, increase in α_3 by 20% results in increase of individual having extra marital affair by 24.87%. Also, Fig. 5 shows that 2% increase of extra marital affair will be observed if there is an increase in rate of α_4 by 4%.

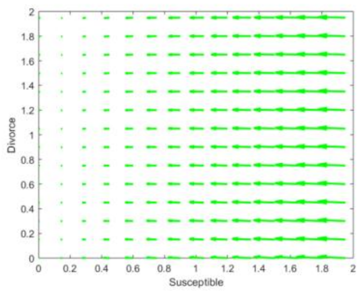

Fig. 6. Behavior of Divorce among susceptible individual

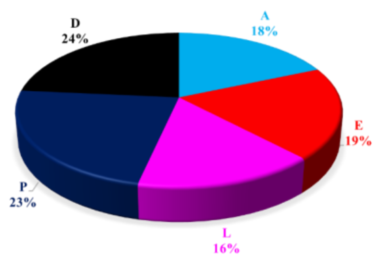

Fig. 7. Percentage of partition and divorce among arranged and love married couples due to extra marital affair

The direction in Fig. 6 shows that the intensity of getting divorce gradually decreases among susceptible individual. The pie chart in Fig. 7 connotes that 18% individuals opt for arrange marriage and 16% for love marriage, out of which 19% have extra marital affair. From these many individuals 23% gets separated and overall 24% gets divorced.

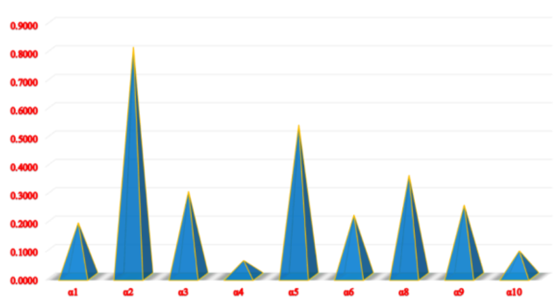

Fig. 8. Sensitivity analysis

In this sensitivity analysis as obtained in Fig. 8, impact of α_2 on divorce is the highest which is the rate of love marriage individuals. Also, there is no effect of $\alpha_7, \alpha_{11}, \alpha_{12}, \alpha_{13}$ on divorce.

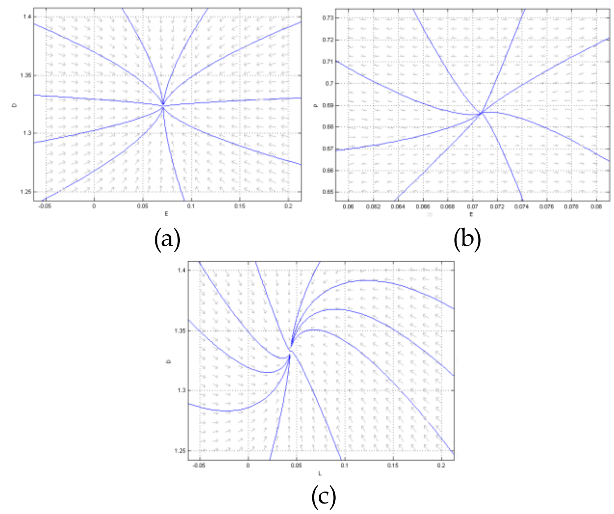

Fig. 9. Trajectory field and solution curve

For dynamical system (1), Fig. 9 shows the trajectory around the endemic equilibrium point. The stability can be obtained if the direction of the curves is moving towards equilibrium point. So as shown in fig., the solution is asymptotically stable as it eventually converges to the equilibrium point.

6 Conclusion

To scrutinize the spread of the issue of divorce among married couples due to extra marital affair, a mathematical model is formulated. System of nonlinear differential equations is solved to find equilibrium points and stability analysis for these points connotes that the model is locally as well as globally asymptomatically stable. Using Next generation matrix method, a basic reproduction number, also known as threshold has been evaluated which is 0.14 which connotes that approximately 14% susceptible individuals are likely to go for divorce. It also shows that the issue is endemic in nature and also, bifurcation curve depicts that equilibrium is globally stable as basic reproduction number is less than critical threshold. To justify the results, numerical simulations have been carried out which graphically represents this social issue. And the stability analysis by using trajectory field is computed. Also, the sensitivity analysis and quiver graph show the behavior of each parameters and compartments respectively. In future, one can add factors which leads to divorce/extra marital affairs prevailing in the society. Control on extra marital affair and divorce will be more practical to counsel for happy and healthy arranged/love marriage life.

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