1 INTRODUCTION

[1, 5] Extensively contemplated the ideas of partial lattices on outer measure and their attributes. By [2, 4, 5] the basic meanings of measure, lattice measures, lattice, partial lattice, Boolean lattice and quantifiable characteristics of a partial lattice in a countable Boolean lattice measure are summed up with the assistance of [3]. In 1963, [6] introduced a generalization of lattice measure concepts. He explained much about the reducibility and irreducibility of elements of a lattice. If L is a lattice, a ∈ L is said to be join reducible if there exist b, c ∈ L, such that b < a, c < a and b ∨ c = a. If a is not join reducible, a is said to be join irreducible in L. a is said to be meet reducible if there exist b, c ∈ L, such that a < b, a < c and b ∧ c = a. If a is not meet reducible, a is said to be meet irreducible in L. The least element and every atom of a lattice, bounded below is join irreducible. The greatest element and every dual atom of a lattice bounded above, is meet irreducible. Every element of a chain is meet irreducible as well as join irreducible; it may also happen that every element of a lattice is meet-reducible as well as join-reducible. All through the notations are given by Boolean lattice (L), countable Boolean lattice of partial lattices of L (L) and lattice measure (µ) characterized on L. H is a partial lattice, countable Boolean lattice of partial sublattices of H is (B) and measure on B is (µ). Section 2, provides meaning of locally-measurable lattice, complete-measure, saturated-lattice measure, and gives an outcome that In L, if (N_n) be any countable collection null partial lattices like N_n< N_n+1 then so their join is a null partial lattice in L.

Section 3, demonstrate that each lattice measure space can be incorporated into a complete-lattice measure space, and set up an outcome that If µ is lattice sigma-finite measure then it is saturated. Section 4, provides join (meet) irreducibility of an element of a partial lattice H, measurable function on a partial lattice and proves the join (meet) of two measurable functions defined on partial lattice is measurable, the set of all real-valued measurable functions is a vector space as well as lattice. The main object of this paper is to show that if {h_n} is an increasing (decreasing) sequence of join (meet) irreducible measurable functions on a partial measurable lattice space (H, B, µ) then their join (meet) is join (meet) irreducible measurable function.

2 Lattice measure space (L, L, µ)

Let (L, L, µ) be a lattice measure space In L, µ is said to be complete-measure if all partial lattices of sets of measure zero (i.e if N ∈ L, µ(N) = 0 and M ⊆ N implies M ∈ L) H ∈ L is said to be a locally-measurable lattice, if H ∧ N ∈ L for every N ∈ L with µ(N) < ∞. If every locally-measurable partial lattice is lattice measurable, then µ is called saturated. Observation 2.1 All locally-measurable partial lattices form a Sigma-algebra containing L. Result 2.1 In L, if {N_n} be any countable collection null partial lattices such that N_n< N_n+1 then so their join is a null partial lattice in L. Proof Evidence let N_1, N_2,…… be a succession of null partial lattices. i.e., for every positive whole number n, N_n ∈ L such that M_n ⊆ N_n Furthermore µ(N_n) = 0. Unmistakably \[ \bigvee_{n=1}^{\infty} M_n \subseteq \bigvee_{n=1}^{\infty} N_n \] Presently \[ \bigvee_{n=1}^{\infty} N_n \in L \] [clearly each N_n ∈ L by the meaning of L] Presently \[ p( \bigvee_{n=1}^{\infty} N_n ) \leq \sum_{i=1}^{\infty} p(N_i) = 0 \] [since \[ p(N_i) = 0 \] ∀ n] Along these lines the biggest partial lattice \[ \bigvee_{n=1}^{\infty} N_n \] whose lattice measure is zero Thus \[ \bigvee_{n=1}^{\infty} N_n \] is likewise a null partial lattice.
3 COMPLETE-MEASURE SPACE (L, L, μ)

Theorem 3.1 For (L, L, μ) to be lattice measure space, we locate (L, L₀, p₀) a complete-measure space such that L ⊆ L₀, if P ∈ L then p(P) = p₀(P), and also P ∈ L if and only if P = M ∨ N, where N ∈ L and M ⊆ T, T ∈ L, μ(T) = 0. Proof

Composition L₀ = (P ⊆ L / P = M ∨ N, N ∈ L, M ⊆ T, T ∈ L, μ(T) = 0). To prove L₀ is a sigma-algebra, let P ∈ L₀ implies P = M ∨ N, N ∈ L, M ⊆ T, T ∈ L, μ(T) = 0. Now P' = P' ∨ L = P' ∨ (M ∨ N) = (P' ∨ M) ∨ N = (M ∨ N) ⊆ L₀ (since P₀ is a Measurable function, then P₀ = M ∨ N, N ∈ L). Hence p₀ is a complete-measure function. Then P₀ = M₀ ∨ N₀, where N₀ ∈ L, M₀ ⊆ T, T₀ ∈ L, μ(T₀) = 0. Then P₀ = M₀ ∨ N₀ is a sequence of partial lattices from L₀. Also let {P₀} be a sequence of partial lattices from L₀.

Therefore P₀ is a measurable function. Then p₀ = M₀ ∨ N₀, where N₀ ∈ L, M₀ ⊆ T, T₀ ∈ L, μ(T₀) = 0. Hence P₀ = M₀ ∨ N₀ is a sequence of partial lattices from L₀.

Implies p₀(P₀) = p₀[M₀ ∨ N₀] = p₀{[M₀ ∨ N₀]} = {p₀(M₀ ∨ N₀)} = {p₀(M₀) ∨ p₀(N₀)} = {p₀(M₀) ∨ p₀(N₀)} = {p₀(M₀) ∨ p₀(N₀)} = {p₀(M₀) ∨ p₀(N₀)}

4 IRREDUCIBILITY OF AN ELEMENT IN A PARTIAL LATTICE

Definition 4.1 an element a in H is said to be join reducible in H if there exist c, b, c ≠ a, b ≠ a, b ∨ c exist in H and b ∨ c = a. If a is not join reducible in H, a is said to be join irreducible in H if there exist c, b, c ≠ a, b ≠ a, b ∨ c exist in H and b ∨ c = a. If a is not meet reducible in H, a is said to be meet irreducible in H. Example 4.1 The set of natural numbers is a lattice with a join b = l.c.m of {a, b} and a meet b = g.c.d of {a, b}. If n is prime, for n ≥ 1, then this lattice is irreducible. Example 4.2 In the partial lattice H of positive integers greater than 1 that are not prime nor for some n of the form n(n+1) in which a join b = l.c.m of {a, b} and a meet b = g.c.d of {a, b}. 4, 8, 9, 10, 14, 15 are join irreducible where as 36 or 49 is not. Definition 4.2 (measurable function on a partial lattice) If the set {l ∈ H : h(l) > a} = h₁(a, ∞) is measurable for all real numbers a, then we say that H is a real valued measurable function defined on a partial lattice H. Theorem 4.1 if H : H → R², then every a ∈ [0, ∞), the sets {l ∈ H : h(l) ≥ a}, {l ∈ H : h(l) < a}, {l ∈ H : h(l) ≤ a}, {l ∈ H : h(l) > a} are equivalent. Proof By [3] the above statement is equivalent. Definition 4.3 The extended real valued functions f and g are defined on partial lattice H then join, meet are defined by f ∨ g = max(f(l), g(l)) and f ∧ g = min(f(l), g(l)) for any l ∈ H. Theorem 4.2 [3] f : H → R, g : H → [0, ∞] are measurable then so are (1) f ∨ g (2) cf ∈ c ∈ R (3) f² (4) fg Proof By [3] the above statement is measurable.

5 IRREDUCIBILITY OF MEASURABLE FUNCTIONS ON A PARTIAL LATTICE

Let H be a partial lattice, B be a countable Boolean lattice of partial sublattices of H and μ be a measure on B. Theorem 5.1 If f : H → R, g : H → R are measurable then so is f ∨ g, f ∧ g Proof है रेखा (f (g) f (g) = (a) a ∈ H: max (f(l), g(l)) < a)} = {l ∈ H: f(l) < a and g(l) < a = {l ∈ H: (f(l) < a) ∧ (g(l) < a)} = f¹(a, ∞) ∧ g¹(a, ∞) Theorem 5.2 The set {h(μ) of all real-valued measurable functions is a vector space as well as lattice. Proof That μ(H) is a vector space over R follows from the fact that the zero function belongs to {h(μ)} and that μ(H) is a lattice follows from Theorem 5.1 Observation 5.1 If g and h are measurable and incomparable, g ∨ h as well as g ∧ h are measurable. If further g and h are both join (meet) irreducible, g ∨ h (g ∧ h) is clearly not joint (meet) irreducible. However in the case of infinite sequences, we have the following theorems 5.3 and 5.4. Theorem 5.3 If {hₙ} is an increasing sequence of joint irreducible measurable functions on a partial measurable lattice space (H, B, μ) then their joint is join irreducible measurable function. Proof Measurability: write h = ∫ hₙ x₁ hₙ

For all a ∈ R, h₁([a, ∞)) = h(l) ≥ a ↔ hₙ(l) ≥ a for some n implies a ∈ h₁([a, ∞)) ↔ a ∈ h₁([a, ∞)) Hence follows measurability of h. Irreducibility: To prove joint irreducibility of h, assume that h = g ∨ m where g, m are measurable functions on H. For any positive integer k, hₙₖ = hₙₖ ∨ hₙₖ ∧ (g ∨ m) = (hₙₖ ∨ g) ∨ (hₙₖ ∧ m). Since hₙₖ is joint irreducible and hₙₖ ∧ g and hₙₖ ∧ m are measurable either hₙₖ = hₙₖ ∧ g or hₙₖ = hₙₖ ∧ m is for every k, either g ≤ hₙₖ or m ≤ hₙₖ. Let A = k / hₙₖ ≤ g, B = k / hₙₖ ≤ m. Since every k must be either in A or B, one of A, B must be infinite. If A is infinite then for every n there exists kₙ ∈ A such that kₙ > n, kₙ > n implies hₙₖ, kₙ ∈
A \Rightarrow g \geq h. Hence g \geq h, for all n Hence g \geq \bigwedge_{n=1}^{\infty} h_n
= h g \vee m = h \Rightarrow g \leq h. Hence f = g. If B is infinite we get, by
a similar argument that h = m. h = g \vee m = f = g or h = m
Hence h is join irreducible. Theorem 5.4 If \{h_n\} is decreasing
sequence of measurable meet irreducible functions on a
partial measurable lattice space (H, \overline{B}, n) then $h = \bigwedge_{n=1}^{\infty} h_n$
movable and irreducible. Proof Measurability: Let a \in B.

If $h(l) \geq a$ then $h_n(l) \geq a$ for all n. Implies \{l \in H : f(l) \geq a\} \subseteq \bigwedge_{n=1}^{\infty} h_n(l) \geq a
\forall n. Hence \bigwedge_{n=1}^{\infty} \{l \in H : h_n(l) \geq a\} \subseteq \{l \in H : h(l) \geq a\}

(1) If $h_n(l) \geq a$ for all n then $h(l) \geq a$

(2) From (1) and (2) \{l \in H : h(l) \geq a\} = \bigwedge_{n=1}^{\infty} \{l
\in H : h_n(l) \geq a\} Since $h_n$ is measurable for all n, \{l \in H : h_n(l) \geq a\} is measurable, hence \{l \in H : h(l) \geq a\} is measurable,
hence h is measurable. Irreducibility: Suppose h = g \wedge m.

Then for all n, $h_n \leq h = h \leq h_n \vee (g \wedge m) = (h_n \vee g) \wedge (h \vee m)$. Since $h_n$ is meet irreducible, either $h_n \geq h = h \vee g$ or $h_n \leq h = h \vee m$ That is; either $h_n \geq h$ or $h \geq m$. Let A = \{n \in H : h_n \geq g\}, B = \{n \in H : h_n \geq m\}. As in theorem 5.3 either
A or B must be infinite. If A is infinite then for every n there
exists $k_n \geq n$ in A so that $h_n \geq k_n \geq g$. Since $h_n \geq g$ for all n,
h = \bigwedge_{n=1}^{\infty} h_n \geq g$. If B is infinite, a similar argument yields h \geq m. Thus either h \geq g or h \geq m Since h = g \wedge m, h \leq g and h \leq m. Hence h = g or h = m Hence h is meet irreducible.

CONCLUSION
We proved that each lattice measure space can be
incorporated into a complete-lattice measure space, and
established a result that if p is lattice sigma-finite measure
then it is saturated also provided the definitions of
measurable function on a partial lattice and verified the
equivalent conditions of measurable functions, the
measurability of sum (difference) functions, constant
multiplication of function, square of function, product of
functions, join (meet) of two functions, the set \eta(H) of all
real valued measurable functions is a vector as well as
lattice, finally proved that if \{h_n\} is an increasing
(decreasing) sequence of join (meet) irreducible
measurable functions on partial lattice (H, \overline{B}, n) then their
join (meet) is join (meet) irreducible measurable function.

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