Students Strategies In Solving Problem Of Patterns Generalization

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Abstract: This study describes the strategies used by students when faced with the problem of pattern generalization. The 5th and 6th grade elementary school students were given problem of pattern and asked to solve it. Based on the results of think aloud, written works and interviews are classified strategies used by students in solving problems.From the results of data analysis it can be concluded that most students use recursive strategy, few students using explicit strategy and one student using image which make it easier to determine nth rule .

1. Introduction

Research in mathematics education has explored the use of patterning activities to introduce algebraic thinking [1], [2]. Patterning activities has been considered as one way to introduce algebra to students [3]. Two central themes of algebraic thinking are appropriate for young students; (a) making generalizations and (b) using symbols to represent mathematical ideas and to represent and solving problems [4]. Research on strategies used by students in solving problems of patterns has been done by researchers [5]–[12]. The results reveal that students often see the same patterns differently and usually use a trial and error strategy to determine the nth rule. [10] have pointed out that the initial stage in generalization involves "grasping a commonality or regularity which [3] said," seeing the general through the particular. According to [13] that students in grade 3-5 should investigate numerical and geometric patterns and express them mathematically in words or symbols, students should analyze the structure of the pattern and how it changes, organize this information systematically, and use the analysis to develop generalizations about the mathematical relationships in the pattern. In Indonesia, especially in elementary school students, pattern recognition has begun since grade 1 through jump number and simple geometric patterns, such as the following:

1, 3, 5, …, …, ….	Jump number ... →

Figure 1. *Jump Number by 2 [14]*

Figure 2. *Simple Geometric Pattern [15]*

But problem such as the stickers cube and upside-down T patterns have not been explored. Problems such as stickers cube and upside-down T patterns will help students in generalization that is to determine the nth rule. The results of previous studies [1], [6], [14] have shown that students tend to seek recursive similarities, by discovering what changed in each term p_{k+1} from previous terms p_k Their research shows that students tend to use recursive strategies (the next pattern is based on the previous pattern) to describe generalization, rather than looking for functional relationships among variables. [1] emphasizes that recursive strategies make students predict the next term, but do not encourage students' ability to perceive structural relationships between two data to find the rule or not to view data as the domain and codomain function. According to [15], awareness of structural understanding is a very important aspect for the emergence of algebraic thinking. Generalization is considered an essential component of algebraic activity. [16] indicates that developing generalizations is considered as one of the important goals in mathematics learning.

2. Method

This study was conducted to explore the strategy that students use when solving a problem of pattern. The sample of the study was the students of elementary school with the total number of 18 students (age 10-12). The qualitative data were obtained from the result of "think aloud" and from the students' notes during their attempt to solve the problems and from the result of in-depth interviews. The results were then analyzed and classified into three strategy: (a) most of students using recursive strategy, (b) few of

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students using explicit strategy, and (c) one student using image which make it easier to determine nth rule.

3. Result and Discussion

This section explains the strategy that students used when solving a problem of pattern generalization. From 18 students who are given a problem of pattern generalization can be classified as follows:

Table 1. *Classification Strategies used by Students*

Recursively strategy	Explicitly strategy	Using image
11 students	6 students	1 students

From the table can be seen that most of the students use recursive strategy i.e. 11 students, 6 students use explicit strategy, and just 1 student use image to help her find the nth rule. When solving the problem in Figure 3 students tend to see the pictures shown in the sequences of 5, 9, 13,... And can quickly obtain the number of square in the next figure. Students can easily complete near generalization tasks, but tend to have difficulty when asked to determine the number of squares in Fig. 49 because it requires them to construct the number of squares in Fig. 48.

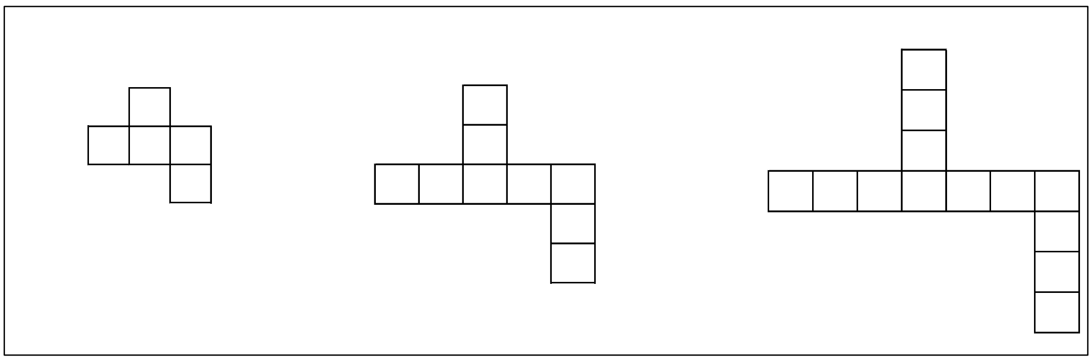


Fig.1 *Fig.2* *Fig.3*

Figure 3: *Problem of patterns generalization*

Based on the figure above:

a) Determine the number of squares in Fig.7, Fig.10, Fig.12, and Fig. 49. Explain your answer!

b) How to determine the number of square in Fig.n? Explain your answer!

According to [5], in near generalization task such as obtaining the number of squares in Fig. 7, the most common method for obtaining an answer involves listing, that is, extending the last figures number by listing successive cases until the desired figure number has been reached. So when asked to determine the number of square in Fig.7, Amy said:" add 4, so Figure 4 has 17 squares, Figure 5 has 21 squares, Figure 6 has 25 squares, and Fig.7 has 29 squares. A similar strategy is shown by Nabila. When asked to determine the number of squares in Fig.10 and Fig.12, she can obtain it, she says: **"**there are 21 squares horizontally for Fig.10, 10 squares upwards, 10 squares downwards, so there are 41 squares. For Fig.12, there are 25 squares horizontally, 12 squares upwards, 12 squares downwards**,** so there are 49 squares. But, when asked to determine the number of squares in Fig.49, she has difficulty to obtain it, she try to make a list, she writes**:**

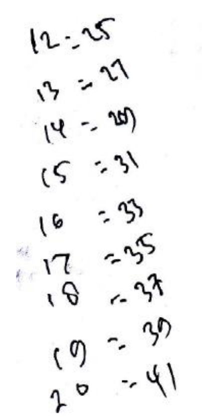
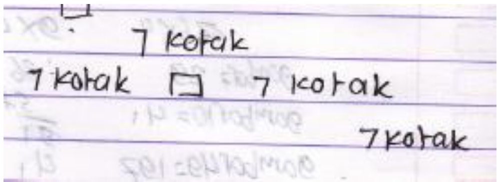


Figure 4. *Nabila's list of Fig.49 of Figure 3.a*

She stops her work, and says," add 2 until Fig.49 (the number of squares horizontally)**.** Doing with far generalization tasks is difficult for students. Students who see Fig.1, Fig.2, and Fig.3 as a set of squares and describe them as sequence 5, 9, 13, ... would be difficult to determine the number of square in Fig.n because they have to construct the Fig. $n-1$ first. On the contrary, when students look at the Fig.1, Fig.2 ,and Fig.3 as meaningful forms, tends to be easier for them in solving far generalization tasks, such as Ammar does in determining the number of squares in Fig.7, he said," for Fig.7 you have 7 squares here (make 7 squares), 1 fix square in the middle**,** 7 squares left, 7 squares right, and 7 squares downwards, so there will be $7 \times 4 + 1 = 29$ squares. When asked to determine the square in Fig.49, he said," 49 × 4 + 1 = 197 squares. A similar strategy is shown by Tasbitha, when asked to determine the number of square in Fig. 7, she writes:



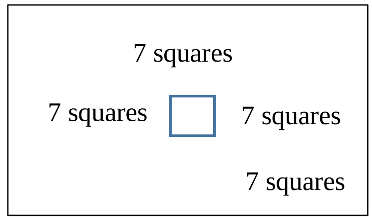


Figure 6: *Tasbitha's stage of Fig.7*

She creates an image to represent the requested square, and then says," so in Fig.7 there are $7 + 7 + 7 + 7 + 1$ so you have 29 squares. When asked to determine the number of square in Fig. 49, in the same way, she creates an image to represent 49 squares, and then writes." $49 + 49 + 49 + 49 + 1$, so you have 197 squares. According to [17], there are two perception that are very important when students perceive the pattern i.e. sensory perception and cognitive perception. Sensory (or object) perception is when individuals see an object as being a mere object in itself. Cognitive perception goes beyond the sensory when individuals see or recognize a fact or a property in relation to the object. For Amy and Nabila, they see Figure 3 only as a set of square just like sequence 5, 9, 13, ..., and often such students will have difficulty in determining the n^{th} rule, on the contrary for Ammar and Tasbitha Figure 3 has meaning, they recognize that there is an addition of the same square on the left side, top side, right side, and bottom side of the fixed square, so when they ask to determine any figure, they do it easily.

Conclusion

By giving students problems such as Figure.3 may introduce students to think algebraically. By encouraging students to reflect their thinking about what they have done can help students develop different ways of thinking about the problem. Two important characteristics of the problem given before facilitate generalization. First, the problem requires students to find the number of squares for different figure before asking them to construct a general rule. This progression helps students identify which is varied and which remain the same when calculating the number of squares. Second, by requiring students to find near generalization tasks, followed by far generalization tasks, the problem forces students to move beyond using drawing and counting strategies toward identifying a general relationships that exist in the problem.

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