# Optimal Bit Energy for IR-UWB Signaling over AWGN Channels

Nasr Rashid, Mohamed Shehata

**Abstract**— This paper discusses the impact of tuning the temporal width of an information carrying waveform on the bit error rate performance of an impulse radio ultrawide band (IR-UWB) communication system. Theoretical expressions are developed for the energy per bit in terms of the pulse width of its encoding waveform. Two of the most widely adopted types of IR-UWB signaling waveforms are considered. It is shown that, under spectral constraints, only a discrete set of pulse widths should be possessed by the IR-UWB signaling waveforms such that the minimum bit error rate performance is achieved. The accuracy of the analytically obtained expressions is confirmed by the excellent agreement with the results obtained via numerical simulations.

**Index Terms**—AWGN channels, Bit error rate, Impulse Radio Ultra wideband communications

## 1 Introduction

Since 2002, a huge bandwidth of 7.5 GHz (from 3.1 to 10.6 GHz) has been dedicated by the US Federal Communication Commission (FCC) to the development of the ultrawide band (UWB) wireless technologies [1]. However, the power spectral density (PSD) of a UWB signal emitted over this band, usually called the useful UWB band, is not allowed to exceed -41.3 dBm/MHz. Accordingly, a UWB signal that fully occupies the entire 7.5 GHz band with the maximum admissible PSD possesses a total power of only 0.56 mW. Unless carefully reshaped, the spectrum of a UWB signal does not fully occupy the useful UWB band. Moreover, in impulse radio - ultrawide band (IR-UWB) communications, each information symbol carries only a small fraction of the UWB signal power. Therefore, the spectrum of a UWB signal should be carefully reshaped such that the energy carried by each information symbol is maximized, while respecting the FCC spectral constraints at each part of the spectrum. The efficient utilization of the weak energy carried by each symbol is crucial to the improvement of the bit error rate (BER) performance of IR-UWB systems.

According to [2], the spectrum of a UWB signal can be reshaped by varying the derivative order and/or the temporal width of a basis function. Throughout the literature, various approaches have been reported to demonstrate the design, generation and transmission of IR-UWB waveforms derived from time and/or amplitude scaled versions of Gaussian (e.g., [3-4]) or hyperbolic secant (sech) (e.g., [5-6]) basis functions. To the best of the authors' knowledge, almost all of the reported approaches have investigated the variation of the BER performance with the electrical/optical power of the generated IR-UWB waveform. However, no study has been reported on the impact of varying the temporal width of the generated waveform on the BER performance.


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## 2 Impulse Radio Ultrawide Band Signaling Waveforms

The bit error probability, denoted by $P_e$, of an on-off keying (OOK) modulated signal over an additive while Gaussian noise (AWGN) channel is given by

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{4N_0}}\right) \quad (1)$$

where $E_b$ is the energy per bit, $N_0$ is the PSD of the AWGN process and $erfc(z)$ is the complementary Gaussian error function, defined as follows:

$$erfc(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} \exp\left(-y^2\right) dy \quad (2)$$

Throughout the literature, IR-UWB waveforms are usually time/amplitude scaled derivatives of either a Gaussian or a sech pulse. Therefore, throughout the rest of this paper, $\phi(t,\tau)$ is defined as follows:

$$\underset{-\infty<t<\infty}{\phi^{(m)}}(t,\tau) = \frac{d^m}{dt^m} = \begin{cases} \exp\left(\frac{-t^2}{\tau_g^2}\right) & ;\ \text{for a Gaussian pulse} \\ \operatorname{sech}\left(\frac{t}{\tau_s}\right) & ;\ \text{for a sech pulse} \end{cases} \quad (3)$$

$\tau_g \triangleq \tau / 2\sqrt{\log(2)}$ and $\tau_s \triangleq \tau / 2\sec h^{-1}(0.5)$ , $\tau_g$ is the Gaussian pulse width, defined as the $e^{-1}$ point of the Gaussian basis function, and $\tau_s$ is the pulse width of the sech basis function. The relationships $\tau_g$ and $\tau_s$ to $\tau$ guarantees that the Gaussian and the sech basis functions have a common FWHM pulse width of $\tau$. The energy per bit in (1) is calculated by applying Parseval's theorem to the waveforms defined in (3) as follows:

$$E_b = \chi_m \int_{-\infty}^{\infty} \left|\phi^{(m)}(t,\tau)\right|^2 dt = \frac{\chi_m}{\pi} \int_{-\infty}^{\infty} \left|\Phi_m(f,\tau)\right|^2 df \approx \frac{\chi_m}{\pi} \int_F \left|\Phi_m(f,\tau)\right|^2 df \quad (4)$$

where $F \triangleq [f_L, f_H]$, $f_L$ and $f_H$ are the lower and the upper bounds of the useful UWB band, respectively, $\left|\Phi_m(f,\tau)\right|$ is the Fourier transform of $\phi^{(m)}(t,\tau)$ and $\chi_m$ is a scaling constant that adapts the PSD of the IR-UWB waveform to the maximum emission level admissible by the FCC spectral mask, defined as follows:

$$\chi_m = \frac{\max\{S_{FCC}(f)\}}{\max\left\{\left|\Phi_m(f,\tau)\right|^2\right\}} \quad (5)$$

where $S_{FCC}(f)$ is the FCC spectral mask. The Fourier transform of (3) is expressed as follows:

$$\underset{-\infty<t<\infty}{\Phi_m}(f,\tau) = (j2\pi f)^{(m)} \begin{cases} \sqrt{\pi}\tau_g \exp\left(-\left(\pi f \tau_g\right)^2\right) \\ 4\pi\tau_s \operatorname{sech}\left(4\pi^2 f \tau_s\right) \end{cases} \quad (6)$$

The value of $\chi_m$ is obtained by determining the proper basis function from (6) and solving $\frac{\partial\left|\Phi_m(f,\tau)\right|^2}{\partial f} = 0$ for $f$. Considering Gaussian-based IR-UWB waveforms [7], the peak PSD frequency, denoted by $f_{p,g}$ ,is given by:

$$f_{p,g} = \sqrt{\frac{m}{2}}\left(\frac{\pi}{\tau_g}\right) \quad (7)$$

The peak PSD frequency of an $m^{th}$ order sech-based IR-UWB waveform, denoted by $f_{p,s}$ results from the numerical solution of the following transcendal equation:

$$4\pi^2 f_{p,s}\tau_s \tanh(4\pi^2 f_{p,s}\tau_s) = m \quad (8)$$

Accordingly, the corresponding scaling constant ($\chi_m = \chi_{m,g}$ or $\chi_m = \chi_{m,s}$ for Gaussian or sech-based IR-UWB waveforms, respectively) is expressed as follows:

$$\chi_{m,g} = \max\{S_{FCC}(f)\} \frac{\exp\left(-\frac{\left(\omega_{p,g}\tau_g\right)^2}{2}\right)}{\omega_{p,g}^{2m}\pi\tau_g^2} \quad (9)$$

$$\chi_{m,s} = \max\{S_{FCC}(f)\} \frac{\cosh^2\left(2\pi\omega_{p,s}\tau_s\right)}{\omega_{p,s}^{2m}\left(4\pi\tau_s\right)^2} \quad (10)$$

where $\omega = 2\pi f$ , $\omega_{p,g} = 2\pi f_{p,g}$ and $\omega_{p,s} = 2\pi f_{p,s}$ In contrast to what is widely adopted in most of the literature reported on the design of IR-UWB waveforms, it is clear from (9) and (10) that, tuning the pulse width of the input basis function should be accompanied by a corresponding variation in its amplitude, with the pulse width as the key controllable parameter, such that a maximum PSD of $\max\{S_{FCC}(f)\}$ is always guaranteed for the considered IR-UWB waveform. The energy of a bit encoded by a Gaussian-based IR-UWB waveform, denoted by $E_{g,m}$ is obtained as follows:

$$E_{g,m} = \max\{S_{FCC}(f)\} \frac{\left(\pi\tau_g\right)^2}{G_p} \int_{\omega_L}^{\omega_H} \omega^{2m} \exp\left(-\frac{\left(\omega\tau_g\right)^2}{2}\right) d\omega = \max\{S_{FCC}(f)\}\left(\frac{\pi^2 2^{\frac{m+1}{2}}}{G_p \tau_g^{m-1}}\right) \times \left(\Upsilon\left(\sqrt{2}\pi f_H \tau_g\right) - \Upsilon\left(\sqrt{2}\pi f_L \tau_g\right)\right) \quad (11)$$

where $G_p = \left(\omega_{p,g}^{2m}\pi\tau_g^2\right)\exp\left(-\frac{\left(\omega_{p,g}\tau_g\right)^2}{2}\right)$ and $\Upsilon(u)$ is given by

$$\Upsilon(u) = (1-\zeta)\Gamma\left(\frac{m+\zeta+1}{2}\right) erf(u) - \exp(u^2)\sum_{n=0}^{L-1} \frac{\Gamma\left(\frac{m+1}{2}\right)}{\Gamma\left(\frac{m+1}{2} - n\right)} u^{3-2n-1}$$

where $L = \frac{m+\zeta}{2}$; $\zeta = 0$ if $m$ is odd and $\zeta = 1$ if $m$ is even, $erf(z) = 1 - erfc(z)$ is the Gaussian error function, and $\Gamma(z)$ the gamma function, defined as follows:

$$\Gamma(z) = \int_0^{\infty} y^{z-1}\exp(-y)dy$$

For sech-based IR-UWB waveforms, the bit energy, denoted by $E_{s,m}$ is given by

$$E_{s,m} = \max\{S_{FCC}(f)\} \frac{\left(4\pi\tau_s\right)^2}{S_p} \int_{\omega_L}^{\omega_H} \omega^{2m} \operatorname{sech}^2\left(2\pi\omega\tau_s\right) d\omega = \max\{S_{FCC}(f)\}\left(\frac{m!\pi^{m-1}2^{6-m}}{S_p \tau_s^{m-1}}\right) \times \left(\Xi\left(2\pi\omega_H\tau_s\right) - \Xi\left(2\pi\omega_L\tau_s\right)\right) \quad (12)$$

where $s_p = \left(4\omega_{p,s}^m \pi\tau_s\right)\operatorname{sech}^2\left(2\pi\omega_{p,s}\tau_s\right)$ and $\Xi(u)$ is given by:

$$\Xi(u) = \sum_{l=0}^{K}\sum_{q=0}^{m} \frac{(-1)^{l+q}(l+1)}{(m-q)!(-2(l+1))^{q+1}} \times u^{m-q}\exp(-2(l+1)u) \; ;K \gg 1$$

It is important to highlight that, generally speaking, the accuracy of $\Xi(u)$ depends on the particular value of $K$. However, due to the rapid convergence of $\Xi(u)$ with $K$, this accuracy is essentially constant for $K > 10$. The BER performance of OOK modulated Gaussian and sech-based IR-UWB waveforms is evaluated by substituting (11) and (12), respectively, into (1).

## 3 Simulation Results and Discussion

Three different approaches are employed to calculate the bit error probability corresponding to each waveform type, order $m$ and pulse width $\tau$. The three approaches start by calculating the angular peak emission frequencies in (7) and (8) for $m \in \{1,2,\ldots,7\}$ and $0 \le \tau \le 500\,ps$. In addition, according to [1], $\max\{S_{FCC}(f)\} = -41.3$ dBm/MHz, $\omega_L = 6.2$ Grad/s and $\omega_H = 21.2$ Grad/s. In the first approach (analytical approach), the values of the afore-mentioned parameters are directly substituted in the closed form solutions of (11) and (12). The evaluated bit energies are directly substituted into (1) to evaluate the bit error probability, with a noise PSD of $N_0 = k_B T_0$, where $k_B$ = 1.23 $\times 10^{-38}$J/K is the Boltzmann constant and $T_0 = 300$ K is the nominal receiver noise temperature. In the second approach (a semi-analytical approach), the integrals in (11) and (12) are evaluated by using numerical integration techniques (the trapezoidal method) before substituting the resulting values of the bit energies into (1). The third approach is based on the Monte Carlo simulation technique using MATLAB. In this approach, a pseudo-random binary sequence (PRBS) of $2^{31}-1$ bits length is generated with equal probabilities of 1's and 0's. Each of the "1" bits is encoded by a waveform of order $m$ and pulse width $\tau$. It is important to indicate that, the duration of the encoding waveform, denoted by $T_b$, is an integer multiple of $\tau$ such that the energy per bit is $\ge 0.99E_b$. A noise vector, having a PSD of $N_0$, is generated and is superimposed to the modulated waveform stream before being processed by an integrate-and-dump operation over the bit/waveform duration. The samples resulting from the integrate-and-dump operator are compared to a threshold level of $\sqrt{E_b}/2$ to decide upon the estimated value of the transmitted bit. For each waveform type, order $m$ and pulse width $\tau$, the aforementioned procedure is repeated for $10^3$ iterations. The cumulative transmission errors are counted and are divided by the PRBS length in each iteration, and the resulting BER values are averaged over all iterations.

The results obtained from the three approaches are plotted in Figs. 1 (a) and (b), which depict the impact of tuning the FWHM pulse width $\tau$ of the input basis function on the BER performance of OOK modulation using Gaussian and sech-based IR-UWB waveforms, respectively. Obviously, there is an excellent agreement between the analytically derived expressions and those obtained from numerical computations.

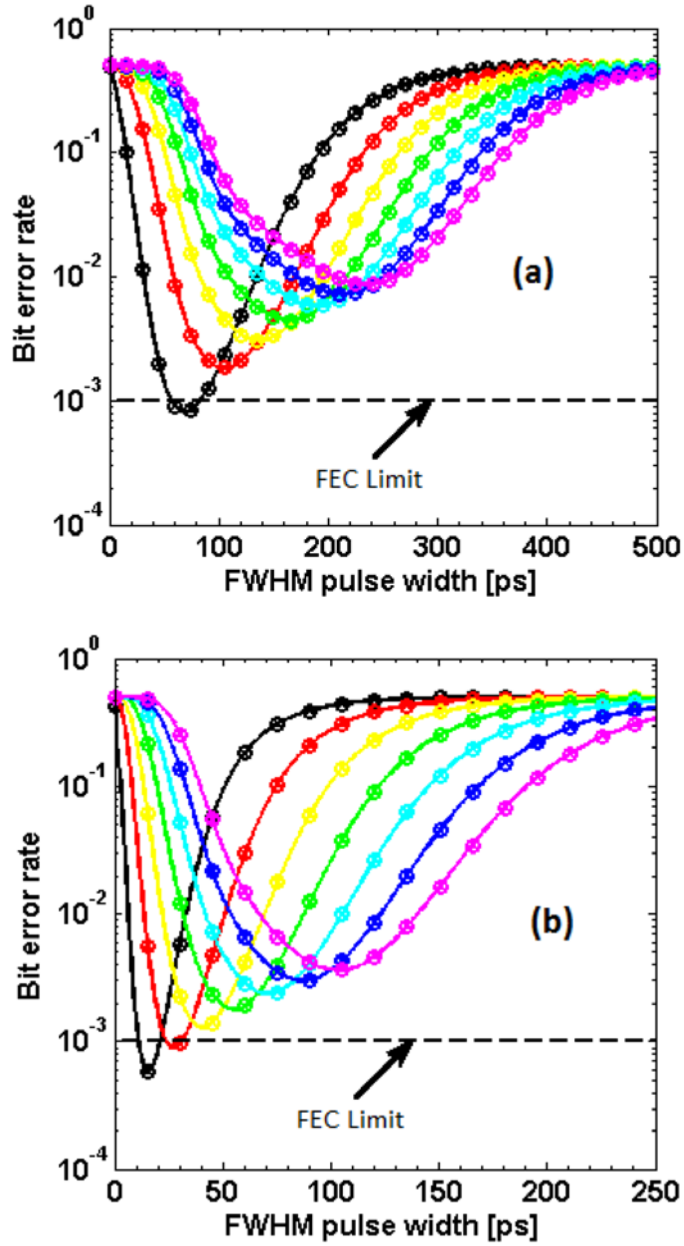


Fig. 1 Average bit error rate performance of OOK modulation versus the FWHM pulse width of (a): Gaussian-based and (b): sech-based IR-UWB signaling waveforms. Solid lines: Numerical evaluation of the theoretically obtained expressions. Cross Markers: numerical values obtained by numerical integration techniques. Circle Markers: values obtained by simulations.

As clear from both figures, each waveform type of order $m$, is characterized by an optimal FWHM pulse width at which the waveform attains a global maximum PE. Since the considered range of $\tau$ includes the global minimum BER points of both waveform types, the direct search algorithm (DSA) is applied over all values of $\tau$ until the corresponding optimum FWHM pulse widths are obtained. Generally, the BER shows a global minimum at an optimum pulse width that depends on the selection of the type basis function as well as its derivative order. However, both waveform types, the FEC limit of $10^{-3}$ is achieved only by the first order pulse shapes, usually called monocycles [8]. For the Gaussian-based monocycle, the minimum BER performance is achieved at a pulse width that is twice as much as the pulse width of a corresponding sech-based monocycle. This implies that the transmission bit rate can be doubled by using a sech-based monocycle instead of its Gaussian-based counterpart. However, on the other hand, a Gaussian monocycle is more robust against the variations in the pulse width of its basis function. The preference of a particular waveform over another depends on the design criteria as well as the advantages to be sacrificed by the system designer.

## 4 Conclusion

This paper investigates the sensitivity of the bit error rate performance to the variations of the temporal width of the signaling waveform in impulse radio ultrawide band communication systems.
Analytical expressions for the energy per bit, in terms of the pulse width of the signaling waveform, are derived. The validity of the derived expressions is confirmed by three different approaches. Simulation results show perfect agreement with the theoretically predicted performance.

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