Availability And Cost Analysis Of A Two Unit Cold Standby Repairable System Subject To Two Types Of Critical Errors.

Surbhi Gupta, Yachika Sharma, Anjali Nathani

Abstract: This paper deals with a 2-state repairable system with two kinds of failures. In this mathematical model we deal with different state probabilities which have been evaluated using Laplace transform. Here we have considered a cold standby system involving two units- one unit is in operating condition /state and other unit in standby state. These two units are connected through an imperfect switching mechanism. We can say that on failure of operating unit, the system may fail completely either due to failure of software or due to human error. All failures are repairable. In the end, some numerical illustrations are given to explain the behaviour of reliability, M.T.S.F and of the two parameters involved in cost analysis procedure.

Keywords : cost analysis, M.T.S.F, repairable system, reliability.

1. INTRODUCTION

THIS paper deals with two unit cold standby repairable system with two types of failure which can occur in our system due to software mechanism and due to human error In this system we have considered a cold standby system comprising of two identical units and that is the main operating unit and the standby unit which is connected through a imperfect switching mechanism which is not hundred percent reliable. After failure of main unit the standby unit starts functioning. If the switching is not perfect, the system goes to failed state till the repair of mechanism is carried out. In all these operative situations, the system may suffer break down due to human error. We also calculate various state probabilities in the form of Laplace Transform. All necessary graphical illustrations are given at the end of the paper to show the practical utility of the model. M.T.T.F and cost functions are plotted.

2 PRINCIPLE ASSUMPTIONS

Some general principle assumptions required in the system. These are:-

1. Initially the system is in good condition.
2. Each unit has only two states of operation that is either good or failure.
3. Only one repair facility repairs all the failures and the repair is done the units work as a new one with normal efficiency.
4. Switching over mechanism is a good option but environmental conditions may become an obstacle.
5. Repair rates of the units of standby system are and also of switching mechanism are constant here but the repairs of failed states due to human error or because of software failure follow the time distributions.

- *Surbhi Gupta is Assistant professor in Amity university, Noida India, PH-*
- *7835941930. E-mail: sgupta11@amity.edu*
- *Yachika Sharma is student in Amity University Noida India, PH-8130182376. . E-mail: yachika.ys@gmail.com*
- *Anjali Nathani is Assistant Professor in Amity university, Noida India, PH-7982508038. E-mail: anathani@amity.edu*

3 NOTATIONS AND SYMBOLS INVOLVED IN THE SYSTEM

Some of the notations which are required for the system are:-

$R_{0,s}(t)$ – Both operative unit and standby unit ae in good state at any time t.

$R_{f,s}(t)$ - main unit is failed but standby is still in working condition.

$R_i(t)$ - failed state due to switching mechanism.

$R_f(n, t)$ - failed state due to software failure of standby unit repair, elapsed repair time is n.

$R_h(m, t)$ - failed state due to critical human error and here elapsed time is m.

Symbols Involved In the system

The symbols which are required for the system are :-

η- failed rate of main unit.

η_h - Constant failure rate due to critical human error occurs.

η_s– Constant failure rate due to failure of standby rate.

ν^1– represents the constant repair rate.

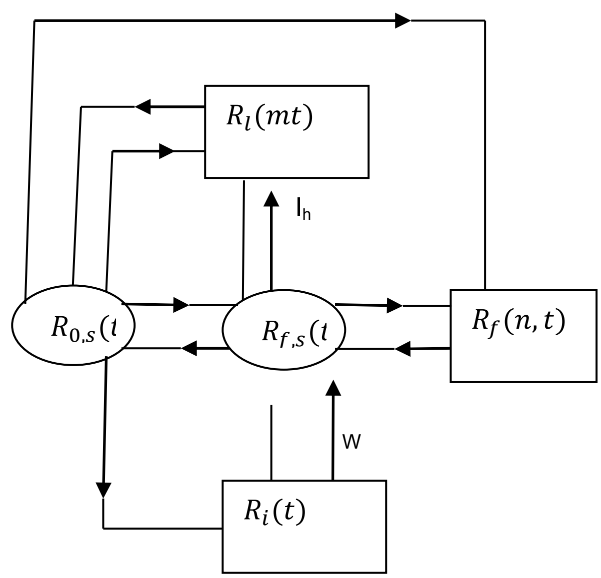
ω^1- represents the constant repair rate of switching over device.

ρ , q – represents the probability of switching mechanism.

ϕ (x) - Transition repair rate required in the state

$P_{f,s}$ (y) - Transition repair rate required in the state P_h

4 PICTORIAL REPRESENTATION



5 PROBABILITIES REQUIRED

The probabilities which are going to be used in our system are as follows :-

At any instant 't' we require :-

P_0(t)- Probability of system required in the state $R_{0,s}$(t).

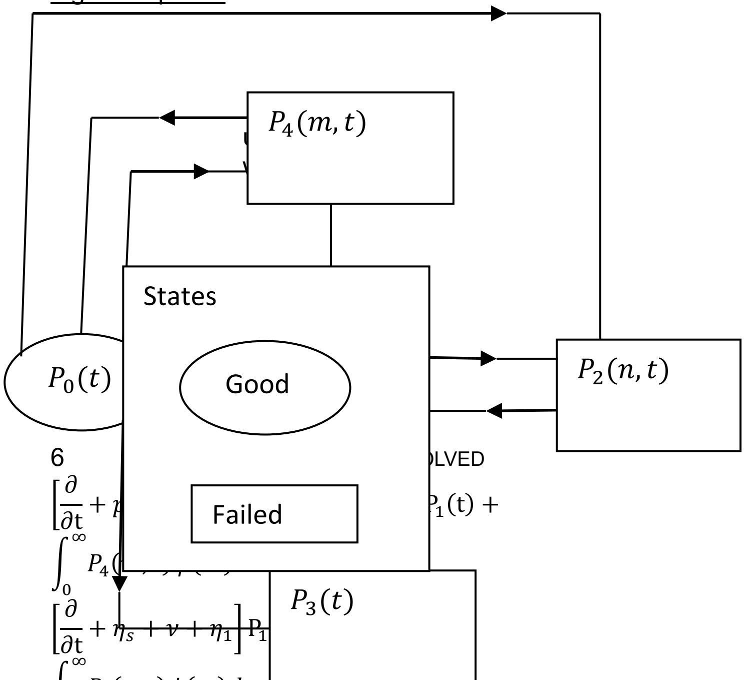
P_1(t)- Probability of system required in the state $R_{f,s}$(t).

P_i(t)- Probability of system required in the state R_i(t).

P_h(m, t)- Probability of system required in the state R_h(m, t).

P_f(n, t)- Probability of system required in the state R_f(n, t)

Figure required.



6 ... LVED

$\left[\frac{\partial}{\partial t} + \rho \ldots \right.$ ... $P_1(t) +$

$\int_0^{\infty} P_4(\ldots$

$\left[\frac{\partial}{\partial t} + \eta_s + \nu + \eta_1\right] P_1 \ldots$

$\int_0^{\infty} P_2(n,t)\phi(m)dn > eq(2)$

$$\left[\frac{\partial}{\partial n} + \frac{\partial}{\partial t} + \phi(x)\right] P_2(n,t) = 0 > eq(3)$$

$$\left[\frac{\partial}{\partial t} + \omega\right] P_3(t) = q\eta.P_0(t) > eq(4)$$

$$\left[\frac{\partial}{\partial m} + \frac{\partial}{\partial t} + \psi(y)\right] P_4(m,t) = 0 > eq(5)$$

Boundary Conditions Used.

$P_2(0, t) = \eta_s \{P_1(t) + P_0(s)\} > (6).$

$P_4(0, t) = \eta_l \{P_1(t) + P_0(t)\} > (7).$

Initial Conditions Used.

$$P_k(t)\begin{cases}1 \text{ if } k = 0 \text{ Otherwise}\\ 0\end{cases} > (8)$$

7. SOLUTION REQUIRED FOR THE MODEL

Using Laplace transform above equations that is from equation 1 to 7 and using equation 8 as well one may get following solution :-

Solutions of the mathematical model are –

$$(s + \rho\eta + \eta_h + q\eta + \eta_s)\overline{P_0}(s) = 1 + \nu.\overline{P_1}(s) + \int_0^{\infty} \overline{P_4}(m,s)\psi(y)dy > (9)$$

$$(s + \eta_s + \upsilon + \eta_h)\overline{P_1}(s) = \omega\overline{P_3}(s) + \int_0^{\infty} \overline{P_2}(n,s)\phi(x)dx > (10)$$

$$\left[s + \frac{\partial}{\partial n} + \phi(x)\right]\overline{P_2}(x,s) = 0 > (11)$$

$$[s + w]\overline{P_3}(s) = q\eta\overline{P_0}(s) > (12)$$

$$\left[s + \frac{\partial}{\partial m} + \psi(y)\right]\overline{P_4}(m,s) = 0 > 13$$

$$\overline{P}_2(o,s) = \eta_s\{\overline{P_1}(s) + \overline{P_0}(s)\} > (14)$$

$$\overline{P}_4(o,s) = \eta_h\{\overline{P_1}(s) + \overline{P_0}(s)\} > (15)$$

$$\overline{P}_2(n,s) = \eta_s\{\overline{P_1}(s) + \overline{P_0}(s)\}e^{-sn-\int_0^x \phi(x)dx} > eq(16)$$

$$\overline{P}_2(s) = F_\phi(s)\eta_s\{\overline{P_0}(s) + \overline{P_1}(s)\} > (17)$$

$$\overline{P}_3(s) = \frac{q\eta\overline{P_0}(s)}{(s+w)} > (18)$$

$$\overline{P}_4(m,s) = \eta_h\{\overline{P_1}(s) + \overline{P_1}(s)\}e^{-sm-\int_0 \psi(m)dm} > (19)$$

$$\overline{P}_4(s) = \eta_h\{\overline{P_1}(s) + \overline{P_0}(s)\}F\psi(S) > (20)$$

Now using equation 10 and 16 we get –

$$\overline{P}_1(s) = \overline{P_0}(s).K_1 > (21)$$

$$\overline{P}_0(s) = \frac{1}{Z_s} > eq(22)$$

Where

$$Z(s) = [S + \rho\eta + \eta_h + q\eta + \eta_s - \vartheta.k1 - \eta_h(1 + k_1)F\psi(s)]$$

$$K_1 = \frac{\omega q\eta + \eta_s}{(s+w)[s + \eta_s + \nu + \eta_h - \eta_s\overline{S_\phi}(s)]}$$

$$\overline{P}_1(s) = \frac{K_1}{Z(s)}$$

$$\overline{P_2}(s) = \eta_s F\phi(S)\frac{K_1 + 1}{Z(s)} > (23)$$

$$\overline{P_4}(s) = \frac{q\eta}{(s+w)}\frac{K_1}{Z(s)} > (24)$$

$$\overline{P_4}(s) = \frac{\eta_h(1 + K_1)F\psi(s)}{Z(s)} > (25)$$

$$\sum_{i=0}^{4} \overline{P_i}(s) = \frac{1}{s} \rightarrow (26)$$

$$\lim_{s\to 0} SF(s) = \lim_{s\to 0} F(t) = F(say)$$

$$P_0 = \frac{1}{z'(s)} > (27)$$

$$P_1 = \frac{H}{z'(0)} > (28)$$

$$P_2 = \eta_S F_\phi \frac{(H+1)}{z'(0)} > (29)$$

$$P_3 = \frac{q\eta}{w}\frac{H}{z'(0)} > (30)$$

$$P_4 = \eta_h \frac{(1+H)}{z'(0)} F_\psi > (31)$$

$$Where\ H = \frac{(\omega q\eta)}{w[\eta_s + \nu + \eta_h - \eta\overline{S_\psi}(0)]}$$

$$\overline{S_y}(s) = \frac{\gamma}{s+\gamma}\ Where\ \gamma = \phi, \psi$$

$$\overline{P_0}(s) = \frac{1}{B'(s)} > (32)$$

$$\overline{P_1}(s) = \frac{T}{B'(s)} > (33)$$

$$\overline{P_2}(s) = \eta_s \frac{(1+T)}{(s+\phi)B'(s)} > (34)$$

$$\overline{P_3}(s) = \frac{q\eta}{(s+w)}\frac{T}{B'(s)} > (35)$$

$$\overline{P_2}(s) = \eta_s \frac{(1+T)}{B'(s)(s+\psi)} > (36)$$

Where: -

$$B'(s) = \left[s + \rho\eta + \eta_h + q\eta + \eta_s - \nu.T - (1-T)\eta_h\frac{1}{(s+\psi)}\right]$$

$$T = \frac{(s+\psi)(\omega.q\eta + \eta s)}{(s+w)[(s+\psi)(s+\eta_s - \nu + \eta_h) - \eta_s\psi]}$$

$$P_{up}(s) = \frac{(1+T_1')}{B_1(s)} > (37)$$

$$\overline{P_{down}}(s) = \frac{1}{s} - \overline{P_{up}}(s)$$

$$B_1(s) = \left[s + \rho\eta + \eta_h + q\eta + \eta_s - (1+T_1')\frac{\eta_h}{s}\right]$$

$$T_1' = \frac{\eta_s}{s(s+\eta_s+\eta_h)}$$

8. Some Numerical Illustrations.

When the repair is not feasible then using above Calculations we obtain equations 38 and 39 respectively

$$R_e(t) = \exp\{-(\rho\eta + \eta_h + q\eta + \eta_s)t\} \ldots\ldots\ldots (38)$$

$$\text{M.T.S.F (Mean Time to System Failure)} = \int_0^\infty R(t)dt.$$

$$= \frac{1}{(\rho\eta + \eta_h + q\eta + \eta_s)} \ldots\ldots..(39)$$

Now calculating the different values of reliability according to the increase in software failure rate and in human failure rate we get :-

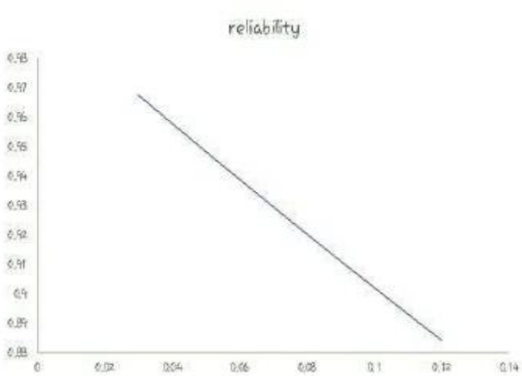
9. Results and Graphs.

Table – 1

Putting the below stated values in equation 38 we get –

$\eta = 0.001,\quad p = 0.5,\quad q = 0.5,\quad \eta_h = 0.002,\quad t = 1$

η_s	R_e (t)
0.03	0.967538559
0.04	0.95791139
0.05	0.948380012
0.06	0.938943473
0.07	0.92960083
0.08	0.920351147
0.09	0.9111935
0.10	0.90212697
0.11	0.89315066
0.12	0.884263662



From table 1 we get that the value of reliability decreases when taken with the software failure rate

Table – 2

$\eta = 0.001,\quad \eta_h = 0.002,\quad p = q = 0.5$

η_s	M. T. S. F
0.02	43.47826087
0.03	30.3030303
0.04	23.25581395
0.05	18.86792453
0.06	15.87301587
0.07	13.69863014
0.08	12.04819277
0.09	9.70873578
0.10	8.849557522
0.11	8.849557522

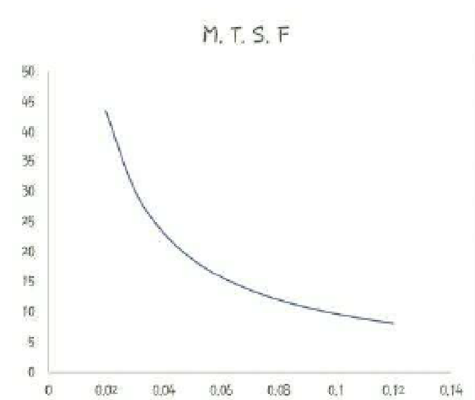
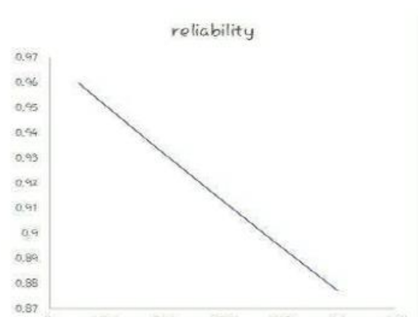
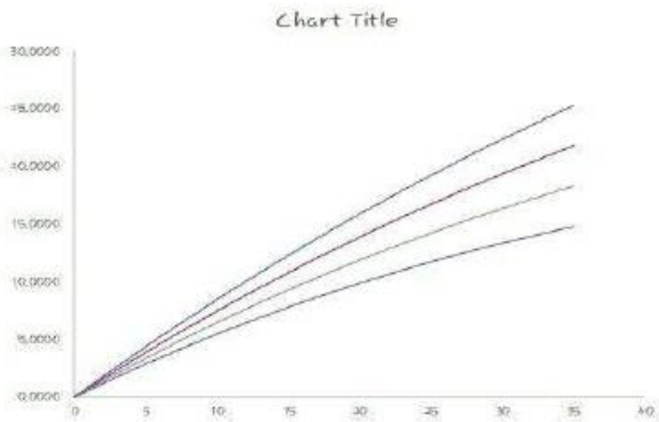


Table – 3

η_h	R_e (t)
0.01	0.9598829129
0.02	0.95027867
0.03	0.940823239
0.04	0.931461892
0.05	0.922193691
0.06	0.91301771
0.07	0.9039330
0.08	0.894938748
0.09	0.886033959
0.10	0.877217774





we can say that the concept of cost analysis refers to the measure of cost – output relationship Many economists have defined it that the cost incurring in hiring the inputs and how better or well these inputs can be rearranged to increase the level or outputs of the firm. Cost–benefit analysis (CBA), sometimes called benefit costs analysis (BCA), is a systematic approach to estimating the strengths and weaknesses of alternatives used to determine options which provide the best approach to achieving benefits while preserving savings.
cost analysis of the system is mathematically defined as :-

$$H(t) = C_1 \int_0^t P_{up}(t)dt - C_2 t.$$

Where: –
C_1 represents the revenue per unit time
C_2represents the service cost per unit time.
Where H(t) represents the expected profit during the time interval (0,t].

RESULT OF COST ANALYSIS PROCEDURE

Time	C = 0.1	C = 0.2	C = 0.3	C = 0.4
0	0	0	0	0
5	4.3632	3.863	3.3632	2.8632
10	8.4515	7.4515	6.4515	5.4515
15	12.2791	10.7791	9.2791	7.7791
20	15.8606	13.8606	11.86066	9.8606
25	19.2102	16.7102	14.2102	11.7102
30	22.3411	19.3411	16.3411	13.3411
35	25.2655	21.7655	18.2655	14.7655

10 CONCLUSION

So far from the information collected and from the analysis we conclude that the reliability of the system decreases with the increase in software failure rate, and in human failure rate but it remains a little high for some time. Not only the reliability , we have also seen that the values of M. T. S. F starts decreasing as the failure rate increases also when we take the component of cost analysis procedure with respect to time we see increasing nature of values in it.

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