Development Of Two Stages Variable Sampling Plans (2S-VSP) With Measures Of Mean And Variance For Continuous Production Process

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Abstract: In modern quality control sections, the variable sampling plans are widely used in deciding the acceptance or rejection of the products. In industries variable quality characteristics play a vital role in making the decision about the process or batches. Many quality control practitioners insist that in a variable sampling plan, if a batch is not accepted based on first sample then the decision should be based on more strict criteria. Indeed variation should be monitored in the process. Hence a special type of two stages variable sampling plans is being developed. In the first stage variable criteria, the measures of Location (mean) is considered, the second stage is dealt with measures of dispersion(Variance). The performance measures such as Operating characteristics function and its related measures are derived. A new designing methodology for determining the parameters of the sampling plans are given. Tables are constructed to facilitate easy application of sampling plans in the Quality Control Section of production industries.

Index Terms: Variable Sampling, Two stages, Variable criteria, Mean and Variance.

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1. INTRODUCTION

In sampling inspection by the method of variables, numerical quality characteristics, X, is measured on a continuous scale for each item in the sample. Most of the times it is assumed that the quality characteristics X follow normal distribution and that a product is turned to be defective when x falls beyond a specification limit or outside an specification interval. The basic idea is that from the sample mean ($\bar{x}$) and standard deviation (σ), an estimate of the percent defective in the lot can be obtained and this estimate is compared with the standard quality level. Acceptance sampling plans for variable such as those in Statistical Research Group (SRG) (1947), Bowker and Goode (1952) and MIL-STD-414 (1957) assume that the population from which the sample drawn is normal. John H Curtiss (1947) has developed variable single sampling plan (nσ, kσ) with known standard deviation (σ) and upper specification limit (U), the proportion non-conforming is given by $p = 1 - F(v)$. Liebermann GJ and Resnikoff GJ (1955) have contributed to variable sampling plans. Schilling (1957) has developed sampling plans by combining variables and attributes characteristics.

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Hamaker(1979) has given a method of constructing single sampling variable inspection plans such that the OC curve of a given single sampling attribute inspection plan and the OC curve of the variable plan have the same indifference quality level po and the same relative slope h0. Bender(1975) has given a table of single sampling variable inspection plans matched at the points AQL and LQL with attribute inspection plans given in table II-A of MIL-STD-105D(1963). Bender (1975) achieves this matching by means of an iterative procedure involving the non-central t-distribution. Collani E.V. (1991), has identified few pitfalls in Variable Sampling Plans. Devaarul(2004) has contributed to special type of variable and mixed sampling plans. The two stages variable sampling plans comprises of mean target and dispersion measures to control the lot quality. In the first stage of variable sampling plans the upper specification limit or lower specification are considered as the criteria. According to the specifications, the target is being set and the production process continues. The lots comprises of units or products subjected to inspection for quality audit. In the quality control section variable sampling plans are widely used to sentence the lot. If the first stage variable sampling does not lead to the acceptance of the lot then the second stage is dealt with more rigorous condition. In modern quality control systems, variable sampling plans are widely applied in various stages of production. In industries, mean and variance criteria play a vital role in controlling the target level and process variance. If only mean a criterion is used for making a decision, then it cannot control the variance of the products. Hence to offset the disadvantage a new two stage variable sampling plans is developed. This paper presents a new algorithm to make a unique decision on the lot for variable sampling plans with mean and variance combination. The Operating characteristics function and other related measures of the two stage variable sampling plans are derived. A new designing methodology for determining the parameters of the sampling plans are given. Comparisons are made between two and three stages of sampling plans. Tables are constructed to facilitate easy application in the Quality Control Section of production industries.

2. FORMULATION OF 2S- VARIABLE SAMPLING PLANS

Let

N : Lot size

n_1 : First Sample size

n_2 : Second Sample size

k_1 = Variable factor such that a lot is accepted if $\bar{x} \leq U - k_1\sigma$

k_2 = Variable factor such that a lot is accepted if $s^2 \leq k_2$

β = Probability of acceptance

β_1 = Probability of acceptance for P_i

β_1' = Probability of acceptance assigned to () stage for percent defective P_i

σ = Population standard deviation

p = Fraction defective

p_i = Specified value of fraction defective

P_a = Probability of acceptance

$\bar{x}$ = Sample mean

Z_A = z value for acceptance limit for $\bar{x}$

μ = Population mean (process)

Z_u = z value of upper specification limit

Z_((W)) = Standard normal deviate such that

$$\frac{1}{\sqrt{2\pi\, z\,(w)}}\int_{e}^{\infty -\frac{1}{2}t^2\, dt} = W$$

χ_(α,n_2-1)^2= Chi-Square value with n2-1 degrees of freedom

3. ALGORITHM FOR TWO STAGES VARIABLE SAMPLING PLANS

Stage I:

Step 1: Draw a random sample of size n1.

Step 2: Determine the sample mean x ̅

Step 3: If (x) ̅≤ U-k_1 σ , accept the lot.

Step 4: If x ̅>U-k_1 σ , go to next stage.

Stage II:

Step 5: Draw another sample of size n_2.

Step 6: Determine the sample variance s2.

Step 7: If 〖s^2≤ k〗_2, accept the lot otherwise reject the lot.

4. ADVANTAGES OF NEW VSP.

1. In two stage variable sampling plans, the variable characteristics are given more importance.
2. In the first stage if the variable criterion is satisfied then the lot is accepted. This shows if the quality is maintained based on the average, the advantage is for the producer.
3. The algorithm moves to the second stage and checks the variable criteria on variance if the first stage fails to accept the lot. This makes the consumer satisfied with the sampling inspection.
4. So, in these two stages, based on the variable criteria of mean and variance the lot is accepted once the process average and dispersion are satisfied in the first and second stage respectively.
5. This pressurizes the producer to make and maintain the products within the required standard. Hence consumers are satisfied with the new variable sampling plans.

5. OPERATING CHARACTERISTICS AND ASSOCIATED MEASURES OF MIXED PLANS

The four principal curves, which describe the performance of an acceptance sampling plan for various percent defective, are the Operating Characteristic curves, the ASN curves, the AOQ and the ATI curves. The operation of the mixed plans cannot be properly assessed until formulae for the ordinates of each of these measures are defined for the known values of percent defective.

5.1 Probability of Acceptance

The Operating Characteristics function is defined as

$$P_a(p) = P_{n_1}[\bar{x} \leq U - k_1\sigma] + P_{n_1}[\bar{x} > U - k_1\sigma]P_{n_2}[s^2 \leq k_2]$$

Proof:

The lot will be accepted in the following cases

Case (i)

From the sample of size n_1 ifx ̅≤U-k_1 σ , the lot will be accepted

Case (ii)

If case (i) fails, then from another sample of size n_2 if 〖s^2≤ k〗_2, the lot will be accepted

Case (i) & (ii) are mutually exclusive cases.

By the law of addition theorem on probability we get

$$P_a(p) = P(i) + P(ii)$$

$$P_a(p) = P_{n_1}[\bar{x} \leq U - k_1\sigma] + P_{n_1}[\bar{x} > U - k_1\sigma]P_{n_2}[s^2 \leq k_2]$$

Where, x ̅ follows normal distribution with mean μ and variance σ^2/n.

The probability distribution of (n2 - l)s^2/σ^2depends only on 'n2' and not on μ or σ, and follows chi-squaredistribution with (n¬_2-1) degrees of freedom.

5.2. Average Sample Number ASN:

$$ASN = n_1 + \{n_1 P[\bar{x} \leq U - k_1\sigma]\}\{n_2 P[s^2 \leq k_2]\}$$

5.3 Average Outgoing Quality AOQ:

AOQ = p.Pa(p)

6. DESIGNING 2S-VSP THROUGH AQL:

Step1: Let the first stage probability acceptance be β_1^'

Let the second stage probability acceptance be β_1^''

Step2: Determine the sample size n1 for the known β_1^'

Step3: Calculate the acceptance limit k1 for the existing process average AQL= p1

$$k_1 = z\,(p_1) + \frac{z(\beta_1')}{\sqrt{n_1}}$$

Where z(w) is the standard normal variate corresponding to w such that

$$\frac{1}{\sqrt{2\pi}}\int_{z\,(w)}^{\infty} e^{-\frac{1}{2}t^2} dz$$

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Step4: Determine β_1^'' the second stage probability of acceptance such that

$$\beta_1'' = \frac{\beta_1 - \beta_1'}{1 - \beta_1'}$$

Step5: Determine the second sample size n2 and Calculate the acceptance limit k2for the known β_1^"

Now calculate the acceptance limit k2 for the known σ=σ_0

$$P[s^2 \le k_2 | \sigma = \sigma_0] \ge 1 - \alpha = \beta_1^{''}$$

The probability distribution of (n2 - l)s^2/σ^2depends only on 'n2' and not on μ or σ, and is known to follow chi-square distribution with (n¬_2-1) degrees of freedom

We know that,

$$(n_2 - 1)k_2 = \sigma_o^2\{\chi^2_{\alpha,n_2-1}\}$$

$$n_2 - 1 = \frac{\sigma_o^2}{k_2}\{\chi^2_{\alpha,n_2-1}\}$$

Where, {χ_(α,n_ -square variate with n2 - 1 degrees o

Hence n2 an abulated for various values of probab

$$n_2 = 1 + \frac{\sigma_o^2}{k_2}\{\chi^2_{\alpha,n_2-1}\}$$

Two Stage Va Mean and Variance Criteria indexed

σ^2

Table 1: Values of the parameters k_1 and k_2 for the known n_1 and n_2 for the known AQL and by assuming first stage probability of acceptance is 65% having total probability of acceptance with 95%

AQL	Known	$n_1=n_2=$	50	$n_1=n_2=$	100	$n_1=n_2=$	150	$n_1=n_2=$	200
p_1	σ^2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2
0.001	10	3.14472	12.16157	3.12876	11.52182	3.12169	11.24041	3.11748	11.07318
0.002	20	2.93265	24.32314	2.91669	23.04363	2.90962	22.48082	2.90541	22.14637
0.003	30	2.80227	36.48471	2.78631	34.56545	2.77924	33.72123	2.77503	33.21955
0.004	40	2.70656	48.64628	2.69060	46.08726	2.68353	44.96164	2.67932	44.29273
0.005	50	2.63032	60.80785	2.61436	57.60908	2.60729	56.20205	2.60308	55.36592
0.006	60	2.56664	72.96941	2.55068	69.13089	2.54361	67.44246	2.53939	66.43910
0.007	70	2.51176	85.13098	2.49580	80.65271	2.48872	78.68288	2.48451	77.51229
0.008	80	2.46341	97.29255	2.44745	92.17452	2.44038	89.92329	2.43616	88.58547
0.009	90	2.42011	109.45412	2.40415	103.69634	2.39708	101.16370	2.39286	99.65865
0.01	100	2.38084	121.61569	2.36488	115.21815	2.35781	112.40411	2.35359	110.73184

7. EXAMPLE

A production process is known to have 0.1% process fraction defective. Determine 2 stage variable sampling plans for 95% probability of acceptance.

Solution:

It is given that AQL = 0.1%. From table 1, one can find the required parameters for the known 95% probability acceptance.

For practical reasons let the first stage probability of acceptance be 65%.

Let n1=n2=100.

From table 1 when p1 = 0.001, we get k1=3.12876, k2=11.52182.

Operating procedure:

Step 1: Draw a random sample of size n1= 100.

Step 2: Determine x ̅_1.

Step 3: If x ̅_1≤ U-3.12876σ , accept the lot.

Step 4: If x ̅_1≥U-3.12876σ , go to next step.

Step 5: Draw another sample of size n_2.= 100.

Step 6: Determine sample variance s2

Step 7: If s^2≤11.52182, accept the lot otherwise reject the lot.

8. DESIGNING THROUGH LQL

Step 1: Let the first stage probability acceptance be β_2^'

Let the second stage probability acceptance be β_2^"

Step 2: Determine the sample size n1 for the known β_2^'

Step 3: Calculate the acceptance limit k1 for the existing process average LQL= p2

$$k_1 = z(p_2) + \frac{z(\beta_2')}{\sqrt{n_1}}$$

Where z(w) is the standard normal variate .

Step 4: Now determine β_2^" the second stage probability of acceptance such that

$$\beta_2'' = \frac{\beta_2 - \beta_2'}{1 - \beta_2'}$$

Step 5: Determine the second sample size n2 and Calculate the acceptance limit k2 for the known β_2^"

Now calculate the acceptance limit k2 for the known σ=σ_0

$$P[s^2 \le k_2 | \sigma = \sigma_0] \ge 1 - \alpha = \beta_2^{''}$$

The probability distribution of (n2 - l)s^2/σ^2depends only on 'n2' and not on μ or σ, and is known as a chi-square distribution with (n¬_2-1) degrees of freedom

We know that,

$$(n_2 - 1)k_2 = \sigma_o^2\{\chi^2_{\alpha,n_2-1}\}$$

$$n_2 - 1 = \frac{\sigma_o^2}{k_2}\{\chi^2_{\alpha,n_2-1}\}$$

$$n_2 = 1 + \frac{\sigma_o^2}{k_2}\{\chi^2_{\alpha,n_2-1}\}$$

$$k_2 = \frac{\sigma_o^2}{n_2 - 1}\{\chi^2_{\alpha,n_2-1}\}$$

Where,{χ_(α,ı -square variate with n2 - 1 degrees of freedom.

Two Stage Sampling Plan with Mean and Variance Criteria - LQL

Table 2: Values of the parameters k_1 and k_2 for the known n_1 and n_2 for the known LQL and by assuming first stage probability of rejection is 10% having total probability of rejection with 5%

LQL	Known	$n_1=n_2=$	50	$n_1=n_2=$	100	$n_1=n_2=$	150	$n_1=n_2=$	200
p_2	σ^2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2
0.01	10	2.55897	6.96405	2.49083	7.81252	2.46065	8.19895	2.44266	8.43244
0.02	20	2.28637	13.92810	2.21823	15.62504	2.18805	16.39790	2.17006	16.86487
0.03	30	2.11341	20.89215	2.04528	23.43757	2.01510	24.59686	1.99710	25.29731
0.04	40	1.98330	27.85621	1.91517	31.25009	1.88499	32.79581	1.86699	33.72974
0.05	50	1.87747	34.82026	1.80934	39.06261	1.77916	40.99476	1.76116	42.16218
0.06	60	1.78739	41.78431	1.71926	46.87513	1.68908	49.19371	1.67108	50.59461
0.07	70	1.70841	48.74836	1.64028	54.68765	1.61009	57.39267	1.59210	59.02705
0.08	80	1.63769	55.71241	1.56956	62.50018	1.53937	65.59162	1.52138	67.45949
0.09	90	1.57337	62.67646	1.50524	70.31270	1.47506	73.79057	1.45706	75.89192
0.1	100	1.51417	69.64051	1.44604	78.12522	1.41585	81.98952	1.39786	84.32436

8. EXAMPLE BASED ON LQL:

A production process known to have 3% process average fraction defective. Determine 2 stage mixed sampling plan for the known LQL.

Solution:

It is given that LQL = 3%.

Let the probability of acceptance at LQL is 3%. From table 2, one can find the required parameters for 10% probability acceptance. For practical reasons if n1=n2=200.

From table 2 when p2 = 0.03, k1=1.99710, k2=25.29731.

Operating procedure:
Step 1: Draw a random sample of size n1 = 200.
Step 2: Determine x ̅_1.
Step 3: If x ̅_1≤ U-1.99710σ , accept the lot.
Step 4: If x ̅_1≥U-1.99710σ , go to next step.
Step 5: Draw another sample of size n_2.= 200.
Step 6: Determine sample variance s2
Step 7: If s^2≤25.29731, accept the lot otherwise reject the lot.

CONCLUSION

In this research article, a new 2 stage variable sampling plan is developed. Since the final decision is based on variable criteria the inspection result will lead to exact decision. The new algorithm is easy to operate in quality control section. It is found that probability of acceptance in higher when the quality of the lots is maintained. But if the quality deteriorates then the probability of acceptance also decreased rapidly. Based on the designing procedure tables are constructed to facilitate quality control engineers in order to implement the new sampling plan. It is found that the 2-stage variable sampling plan is more sensitive towards deterioration of the quality. Hence this sampling plan pressurizes the producer to maintain the standard quality so that customer is satisfied. The comparison between 2stage and 3stage variable sampling plan shows that the average sample number is more economical in case of our newly developed sampling plans.

REFERENCES

[1] Bender, Jr. A. (1975), ‘ Sampling by Variables to control fraction defective’, Journal of Quality Technology, Vol.17, No.3, pp.139-143.

[2] 2. Bowker, A.H. And Goode, H.P.(1952), ‘ Sampling Inspection by Variables’, McGraw – Hill Inc., NewYork.

[3] Adeyemo, I., Fakolujo, O., & Adepoju, G. (2015). Ant colony optimisation approach To selective harmonic elimination in multilevel Inverter. IMPACT: International Journal of Research in Engineering & Technology, 3, 31-42.

[4] Collani E.V.(1991), “ The Pitfall of Acceptance Sampling by Variables”, Frontiers in Statistical Quality Control, Vol.4. pp.91-99.

[5] Fazal, A., & Bashir, S. (2017). Family of Poisson Distribution and its Application. International Journal of Applied Mathematics & Statistical Sciences (IJAMSS), 6(4), 1-18.

[6] Devaarul, S (2004), “ Certain Studies relating to Mixed Sampling plans and Reliability based Sampling Plans”, Ph.D., Thesis, Department of Statistics, Bharathiar University, Coimbatore, Tamilnadu, India.

[7] Geetha, S. Variable Sampling Plan based on CPMK involving Minimum Sum of Risk.

[8] Hamaker H.C.(1950), ‘ The Theory of Sampling Inspection Plans’, Philips technical review, Vol 11, No.9, pp.260-270.

[9] Mahalakshmi, P. Predictive Mathematical Optimal Location Model based on Multi Criteria Evaluation: Validation by Aquaculture.

[10] John H Curtiss (1947), “Acceptance Sampling by Variables, with special reference to the case in which quality is measured by average or dispersion”, Journal of Research, USA, Vol.39, pp.271-290.

[11] Mondal, T. (2018). Participation in SHG and the Level of Empowerment of the Poor Women: An Empirical Study between Pre-SHG and Post-SHG Situations. IMPACT: International Journal of Research in Humanities, Arts and Literature, 6(01), 239-250.

[12] Liebermann Gj and Resnikoff Gj (1955), “ Sampling Plans for Inspection by Variables”, Journal of the American Statistical association, vol.50, pp457-516.

[13] Ozoh, F. O., Uma, K. E., & ndubuisi, P. (2016). The investigation of trade openness and resources utilization on economic growth: Nigeria in focus. International Journal of Humanities, Arts, Medicine and Sciences (BEST: IJHAMS), 4(9), 135-148.

[14] Savage I.R. (1955), “Mixed Variables and Attributes Plans, The Exponential Case”, Technical Report No.23, Applied Mathematics and Statistics Laboratory, Stanford University, Stanford, California.

[15] Schilling, E.G.(1967) , ‘ A general method for determining the OC of Mixed variable-attributes Sampling plans, Single sided specifications, SD known’, Ph.D. Thesis, Rutgers – The state university, New Brunswick, New Jersey.

[16] Padmanaban, S., & William, M. L. (2016). A Nonparametric Discriminant Variable-Elimination Algorithm for Classification to Two Populations. International Journal of Applied Mathematics and Statistical Sciences, 5(6), 7-16.

[17] Schilling E.G. and Dodge H.F (1969), “Procedure and Tables for Evaluating Dependent Mixed Acceptance Sampling Plans”, Technometrics, pp. 341-372.