Mathematical Modeling Of Roll Motion Of Ships: New Approach Of Homotopy Perturbation Method

D. Gowthaman, P. Balagnesan, L. Rajendran

Abstract: In this paper, a mathematical model of nonlinear roll motion of ships is discussed. The model relies on second-order nonlinear differential equation containing a nonlinear term associated with damping moment. The closed-form analytical expressions of roll angle and velocity are obtained using the Homotopy perturbation method in terms of the time variable. In this work, the nonlinear damping and restoring moment are also presented. Our analytical results are compared with the numerical results, and satisfactory agreement is noted.

Keywords: Ship rolling motion, Nonlinear differential equation, Damping moment, Restoring moment, Homotopy perturbation method, Mathematical modeling, Ship dynamics.

1. INTRODUCTION

An unpredictable ocean surrounding is one in all the most essential factors moving a vessel's safe operation baffled. Severe weather, giant irregular waves, and high winds will cause vessel shipwreck with ensuing loss of life and property. Nonlinear roll motion is one in all the most reasons resulting in ship stability failures or maybe wreck of the vessels. The matter of roll motion in a very random ocean has been a troublesome challenge within the past decades (Shang-Rou Hsieh et al. 1994). Roll damping contains each linear (wave generated) and nonlinear (viscous generated, usually quadratic) elements. Three variables will form the basis of the nonlinear ship rolling model: random excitation, nonlinear stiffness, and nonlinear damping. There are several methods available to predict the roll damping, such as numerical method (Xian-Rui Houa et al. 2018), semi-empirical method (Kawahara et al. 2012), nonparametric identification method (Xian-Rui Houa et al. 2018) and wavelet-based spectral algorithm method (Hariharan et al. 2016). Recently Ranjan Kumar Mitra et al. (2018) studied the nonlinear roll oscillation of semisubmersible system theoretically. Abbas Dashtimanesh et al. (2019) discusses hydrodynamic forces and moments in the movement of planning hulls in conjunction with sway, roll, and yaw. And the roll motion of ships of single degree of freedom was modeled by a nonlinear second-order differential equation.

But the exact solution of the nonlinear second-order differential equations is complicated to find in roll motion of ships. Recently some of the advanced analytical methods such as a homotopy perturbation method (Saranya et al. 2018), homotopy analysis method, and the Adomin decomposition method (Praveen et al. 2014) are used to solve nonlinear problems in physical and chemical sciences. In this paper, this nonlinear model has been analyzed by using homotopy perturbation method. Numerical simulations were carried out to investigate the features of coupled ship motions and to validate the proposed mathematical model. The damping and restoring moments are reported, which are the significant parameters of ship dynamics.

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2. MATHEMATICAL FORMULATION OF THE PROBLEM

The ship's schematic diagram is shown in Fig. 1. The roll motion of a ship at sea for a single degree of freedom can be defined by a second nonlinear ordinary differential equation of the form (Xian-Rui Houa et al. 2018) is characterized by

$$(I_{XX}+J_{XX})\ddot{\theta}+D(\dot{\theta})+R(\theta)=M(t) \quad (1)$$

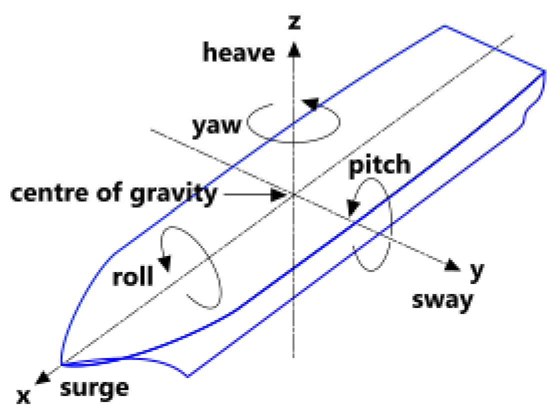


***Fig. 1** Ship schematic diagram*

Where θ is the roll angle and $\dot{\theta}$ is the roll velocity. Equation (1) represents the general equation of roll motion of ships. I_{xx} is the mass moment of inertia $(kg\,m^2)$; J_{xx} is the added mass moment of inertia $(kg\,m^2)$; D is the nonlinear damping moment $(N\,m)$; R is the restoring moment $(N\,m)$; M is the wave exciting moment $(N\,m)$. Dividing equation (1) by the total mass moment of inertia $I_{xx}+J_{xx}$, the normalized roll motion equation is obtained as follows:

$$\ddot{\theta}+d(\dot{\theta})+r(\theta)=k(t) \quad (2)$$

Where

$$d(\dot{\theta}) = D(\dot{\theta})/(I_{xx} + J_{xx});\ r(\theta) = R(\theta)/(I_{xx} + J_{xx});$$
$$K(t) = M(t)/(I_{xx} + J_{xx}). \quad (3)$$

Various mathematical models to predict the rolling motion of a ship in waves have been developed (Bass et al. 1988; Malta et al. 2010; de Oliveira et al. 2012, 2014). The nonlinear damping moment is a function of roll rate. Damping moments can be written in the form of a linear term and a nonlinear term. The model considers herein is composed of two nonlinear forms, i.e., quadratic form and cubic form. The general form of damping moment of the roll angle is expressed as follows:

$$d(\dot{\theta}) = d_1\dot{\theta} + f_1(\dot{\theta}) \quad (4)$$

Where d_1 is the damping coefficient and $f_1(\dot{\theta})$ is a nonlinear function of $\dot{\theta}$. The general form of restoring moment of the roll angle is expressed by Taylor series

$$r(\theta) = c_1\theta + c_3\theta^3 + c_5\theta^5 \quad (5)$$

Where $c_{1,}\, c_2$ and c_3 are restoring moment coefficients.

2.1 Basic concept of homotopy perturbation method.

Let's consider the following nonlinear equation to illustrate t his form.

$$L(u) + N(u) - f(r) = 0 \quad (6)$$

when $L(u)$ is linear part $N(u)$ is nonlinear part and $f(r)$ is known as analytical part. The boundary conditions are

$$u(0) = A \text{ and } \dot{u}(0) = 0 \quad (7)$$

To explain this form, let's consider the following nonlinear fo rmula.

$$H(v,p) = (1-p)[L(u) - L(u_0)] + p\left[L(u) + N(u) - f(r)\right] \quad (8)$$

Where $p \in [0,1]$ is an embedding parameter, and u_0 is an initial approximation of equation (6), that satisfies the boundary conditions. When $p=0$, equation (8) becomes

$$H(v,0) = L(v) - L(u_0) = 0 \quad (9)$$

When $p=0$ the equation (8) become an equation linear. When $p=1$ this transform to nonlinear equation. The process of changing p from zero to unity is that of $L(v) - L(u_0) = 0$ to $L(u) + N(u) - f(r) = 0$. The embedding parameter p is first used as a "small parameter" and conclude that equation solutions (6) can be written as power series in

$$p: v = v_0 + pv_1 + p^2v_2 + \ldots \quad (10)$$

Substituting equation (10) in equation (8), we get the following system of linear equations.

$$L(v_0) = 0 \quad (11)$$

$$L(v_1) + N(v_0) - f(r) = 0 \quad (12)$$

Setting $p=1$ results in the approximate solution of the equation (10) $u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \ldots$

3. ESTIMATION OF ROLL ANGLE, VELOCITY, NONLINEAR RESTORING AND DAMPING MOMENT FOR VARIOUS NONLINEAR PROBLEMS VIA HPM

Nonlinear damping is used in microspeakers, vibration isolation systems, vibration energy harvesters, and demonstrates its practical application by using this method to expand the dynamic range of these devices. The HPM is particularly well suited to the analysis of nonlinear damping in roll motion of ships. In this paper, two nonlinear roll motion of ships such as linear plus cubic damping (Wright et al. 1979) and linear plus quadratic damping (Chan et al. 1995; Morrall et al. 1980; Sathyaseelan et al. 2017) are discussed as follows:

3.1 Linear plus cubic damping (Wright et al. 1979)

The nonlinear roll motion of ships equation (2) with linear plus cubic damping moment can be written as follows:

$$\frac{d^2\theta}{dt^2} + \mu_1\frac{d\theta}{dt} + 24\theta + \mu_3\left(\frac{d\theta}{dt}\right)^3 - 24\theta^3 = 0 \quad (13)$$

Where $\mu_1 = 0.2$ *and* $\mu_3 = 0.1$ which are obtained by experimentally. The initial conditions are

$$\theta(0) = 0.2 \text{ and } \dot{\theta}(0) = 0 \quad (14)$$

It is challenging to find the exact solution of the nonlinear differential equation (6) in the roll motion of ships. In this paper, the nonlinear equation is solved using the homotopy perturbation method (HPM). Appendix A presents the basic concept of HPM. The homotopy perturbation method is already proposed by Ji-Haun He, and also this same method has been followed by many scientists, engineers, and mathematicians for solving various nonlinear problems. This method is very easy, effective, and accurate. Also, one or two iterations lead to highly accurate solutions (Rasi et al. 2015; He 2000, 2005, 2006). By applying the HPM to the equation (6), the roll angle is obtained as follows:The homotopy for the equation (13) can be written as follows:

$$(1-p)\left[\ddot{\theta} + \mu_1\dot{\theta} + \mu_3(\dot{\theta}(t=0))^3 + 24\theta - 24(\theta(t=0))^3\right] + p\left[\ddot{\theta} + \mu_1\dot{\theta} + \mu_3\dot{\theta}^3 + 24\theta - 24\theta^3\right] = 0 \quad (15)$$

The approximate solution of the equation (15) is

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + \ldots \quad (16)$$

The experimental values of the parameters are

$$\mu_1 = 0.2, \mu_3 = 0.1 \quad (17)$$

The following equation is obtained by substituting the equations (16) and (17) in equation (15) and equating the p powers.

$$p^0: \ddot{\theta}_0 + 0.2\dot{\theta}_0 + 24\theta_0 - 0.192 = 0 \quad (18)$$

The boundary conditions for the above equation (18) is

$$\theta_0(0) = 0.2,\ \dot{\theta}_0(0) = 0 \quad (19)$$

The solution of the equation (18) by using the initial conditions (19) yields

$$\theta_0(t) = 0.008 + e^{(-0.1t)}\begin{bmatrix} 0.192\cos(4.8979t) \\ +0.0039\sin(4.8979t) \end{bmatrix} \quad (20)$$

Considering the first iteration of equation (16) when p= 1, we get $\theta(t) = \theta_0(t)$ Equation (20) can be rewritten as

$$\theta(t) = 0.008\, e^{(-0.1t)} [e^{(0.1\,t)} + 24.005\,\sin(4.8979\,t + 1.55049)] \quad (21)$$

The envelope for roll free decay curve is $0.19204\;\; e^{-0.01\,t}$ it is the instantaneous amplitude of the free decay curve. Using equation (21), the velocity can be obtain as follows:

$$\dot{\theta}(t) = 0.940789 e^{-0.1t}\sin(4.8979t + 3.1417) \quad (22)$$

The restoring moment and the damping moment are obtained using by roll angle and velocity.

$$r(\theta) = 24\begin{bmatrix} 0.008\, e^{-0.1t}[e^{-0.1\,t} + 24.005\,\sin(4.8979\,t \\ +1.55049)] \end{bmatrix} \\ 24\begin{bmatrix} 0.008\, e^{-0.1t}[e^{-0.1\,t} + 24.005\,\sin(4.8979\,t \\ +1.55049)] \end{bmatrix}_3 \quad (23)$$

$$d(\dot{\theta}) = \mu_1\begin{bmatrix} 0.9408\; e^{-0.1t}\sin(4.8979\;t \\ +3.40136\times 10^{-9}) \end{bmatrix} \\ +\mu_3\begin{bmatrix} 0.9408\; e^{-0.1t}\sin(4.8979\;t \\ +3.40136\times 10^{-9}) \end{bmatrix}_3 \quad (24)$$

3.2 Linear plus quadratic damping (Chan et al. 1995; Morrall et al. 1980; Sathyaseelan et al. 2017)

The nonlinear roll motion of ships (equation (2)) with quadratic damping moment can be written as follows:

$$\ddot{\theta} + \mu_1\dot{\theta} + \mu_2\,|\dot{\theta}|\dot{\theta} + 2.25\theta - \theta^3 = 0 \quad (25)$$

Where $\mu_1 = 0.15\;\; and\;\; \mu_2 = 0.2$ which are obtained by experimentally. The initial conditions are

$$\theta(0) = 0.2\; and\; \dot{\theta}(0) = 0 \quad (26)$$

Solving the equation (12) using HPM, we can obtain the roll angle for all values of time as follows:

The homotopy for the equation (8) can be written as follows:

$$(1-p)\left[\ddot{\theta} + \mu_1\dot{\theta} + \mu_2(\dot{\theta}(t=0))^2 + 2.25\theta - (\theta(t=0))^3\right] \\ + p\left[\ddot{\theta} + \mu_1\dot{\theta} + \mu_2\dot{\theta}^2 + 2.25\theta - \theta^3\right] = 0 \quad (27)$$

The approximate solution of the equation (27) is

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + \ldots \quad (28)$$

The experimental values of the parameters are

$$\mu_1 = 0.15\;\; and\;\; \mu_2 = 0.2 \quad (29)$$

Through substituting equation (28) and (29) in equation (27) and equating like powers of p , the following equation is obtained.

$$p^0 : \ddot{\theta}_0 + 0.15\dot{\theta}_0 + 2.25\theta_0 - 0.008 = 0 \quad (30)$$

The initial conditions of the equation (30) is

$$\theta_0(0) = 0.2 \text{ and } \dot{\theta}_0(0) = 0 \quad (31)$$

Solving the equation (30) by using (31) the we get

$$\theta_0(t) = 0.0036 + e^{(-0.75\;t)} \\ [0.1964\cos(1.4981t) + 0.0098\sin(1.4981t)] \quad (32)$$

Considering the first iteration of equation (28) when $p = 1$, we get $\theta(t) = \theta_0(t)$.

Equation (32) can be rewritten as

$$\theta(t) = 0.0036\, e^{(-0.075\;t)} \\ (e^{(0.075\;t)} + 54.6234\sin(1.4981t + 1.52094)) \quad (33)$$

The envelope for the roll free decay curve is $0.19664424\;\; e^{-0.075\;t}$.

From equation (33) we get velocity as

$$\dot{\theta}(t) = 0.294962\, e^{-0.075t}\sin(1.4981t + 3.14176) \quad (34)$$

The restoring moment and the damping moment are obtained using the roll angle and velocity.

$$r(\theta) = 2.25\begin{bmatrix} 0.0036\, e^{-0.075\,t}\,(e^{0.075\,t} + 54.6234\sin(1.4981t \\ +1.52094)) \end{bmatrix} \\ -\begin{bmatrix} 0.0036\, e^{-0.075\,t}\,(e^{0.075\,t} + 54.6234\sin(1.4981t \\ +1.52094)) \end{bmatrix}_3 \quad (35)$$

$$d(\dot{\theta}) = \mu_1\begin{bmatrix} -0.025034\, e^{-0.075\,t}\sin(1.4981\,t \\ +3.55421\times 10^{-7}) \end{bmatrix} \\ +\mu_2\begin{bmatrix} -0.025034\, e^{-0.075\,t}\sin(1.4981\,t \\ +3.55421\times 10^{-7}) \end{bmatrix}_2 \quad (36)$$

4. Discussion

The second-order nonlinear equations (13) and (25) symbolize the mathematical models for the nonlinear roll motion of ships. It is very tough to discover the exact analytical solution of nonlinear equations. Only the approximate analytical fallouts are attained using modified HPM. The new analytical expressions of roll angle for linear plus cubic damping and linear plus quadratic damping model are indicated in the equations (21) and (33). The rolling of ships is described in the following four ways: (1) a decreasing roll amplitude, (2) an increasing roll amplitude, (3) a constant roll amplitude, and (4) an alternation of increasing and decreasing amplitudes. The variation of the roll amplitude mainly depends on the wave period and wave height. In linear plus cubic damping and quadratic damping model the roll angle is always in decreasing amplitude. Figures 2a and 2b show that the comparison between numerical and analytical results for linear plus cubic damping and quadratic damping. The close correspondence between the analytical and simulation results validates the analytical solution. From the chassis, it is remarked that the roll angle always decreases for any increment of the time. From the Figures, it is also observed that the peak amplitudes, which is the main aspect foremost to a ship's capsizing depending upon the initial conditions. The amplitude of the roll angle step by step decreases with time because of damping. But the period of the cubic damping model is very less than the quadratic damping model. And the period for the linear plus cubic damping model between 0 to 1.6. And also, the period for the linear plus quadratic damping model is between 0 to 5.26. And the period of linear plus cubic damping model is less than the linear plus quadratic damping model.

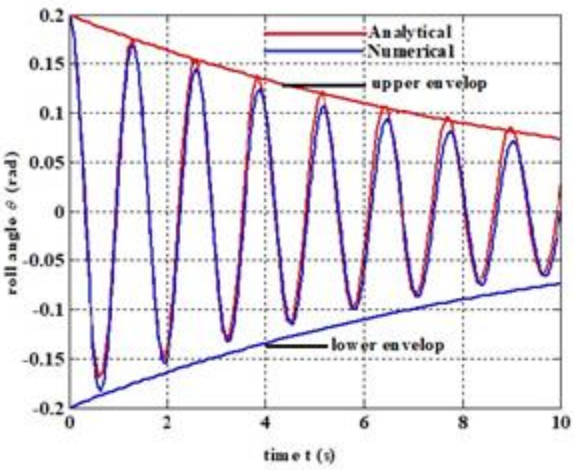


Fig. 2a *Comparison of analytical expression of roll angle (21) and numerical results of linear plus cubic damping model.*

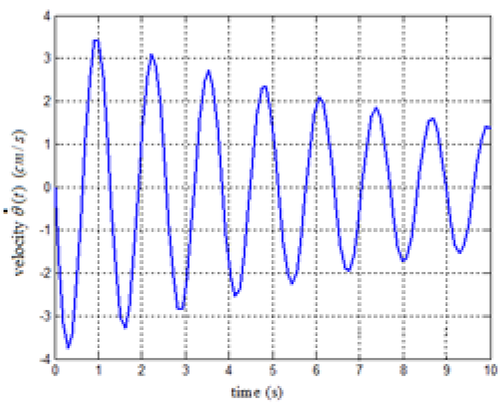


Fig. 3a *Velocity vs time using equation (22)*

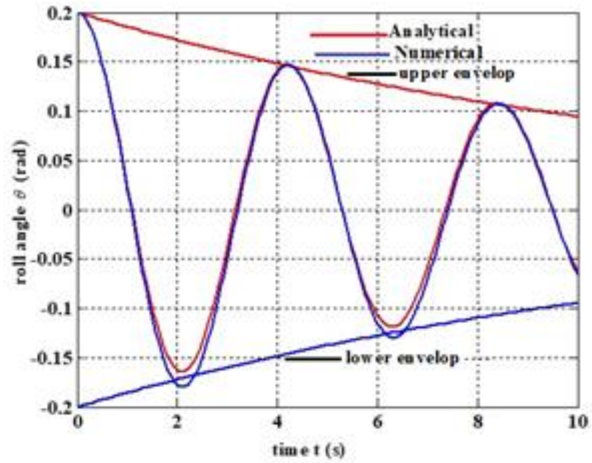


Fig. 2b *Comparison of analytical expression of roll angle (33) and numerical results for linear for quadratic damping model.*

Figures 3a and 3b denote the velocity of roll motion for the equations (22) and (34). Figures 4a and 4b designates the restoring moment of the roll motion for the equations (23) and (35), finally to end with Figures 5a and 5b labels the damping moment of the roll motion for equations (24) and (36), and also observed the amplitude from the Figures 5a and 5b the linear plus cubic damping is 0.222 and linear plus quadratic damping is 0.03752. Finally, Figures 6a and 6b denote the roll decal expressions for various initial conditions.

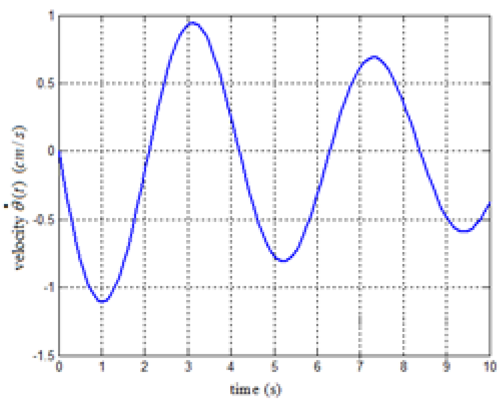
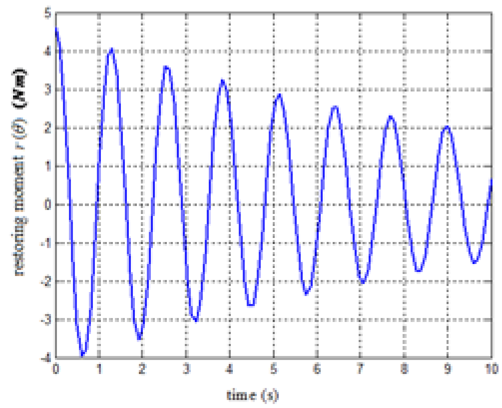


Fig.3b *Velocity vs time using equation (34).*



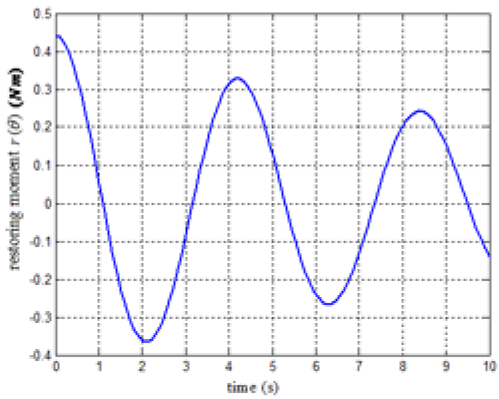


Fig. 4a *Restoring moment vs time using equation (23).*

Fig. 4b *Restoring moment vs time using equation (35).*

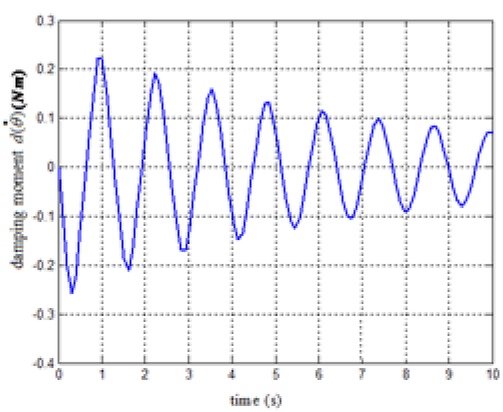


Fig. 5a *Damping moment vs time using equation (24).*

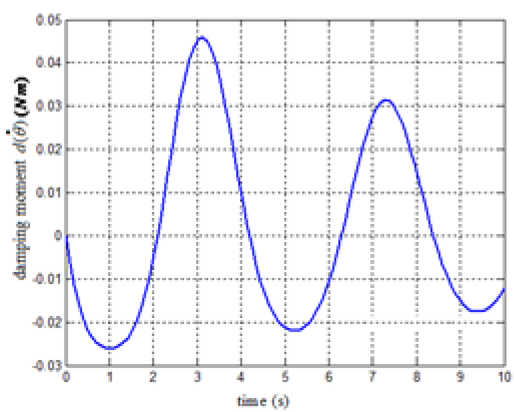


Fig. 5b *Damping moment vs time using equation (36).*

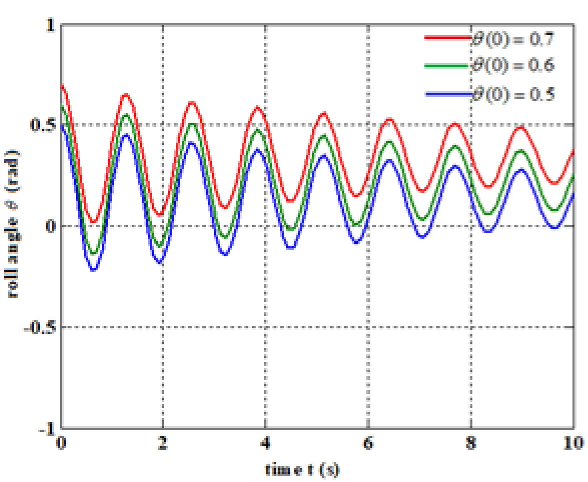


Fig. 6a *various initial conditions roll decay curve vs time for linear plus cubic damping.*

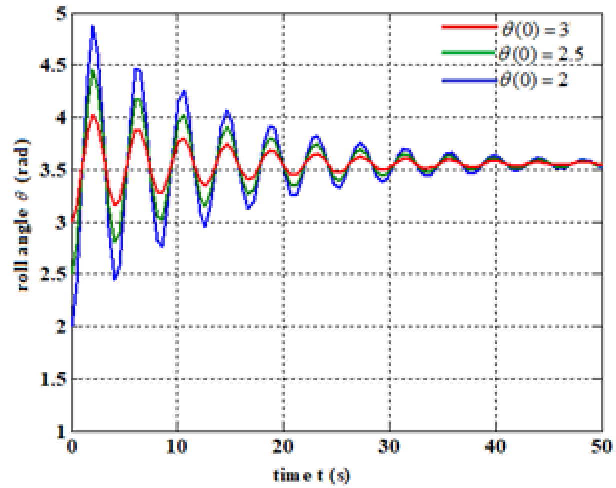


Fig. 6b *various initial conditions roll decay curve vs time for linear quadratic damping.*

The restoring and damping moment steadily decreases when time increases. It shows a nonlinear effect on the roll moment and stress around the hull by the width of the bilge keel. From the Figures, it is prominent that the peak amplitude and period of restoring moments are always higher than the damping moment. The peak amplitude in restoring moment is approximately 20 times higher than a damping moment for cubic damping and about ten times higher than a damping moment for quadratic damping.

Also the total energy is calculated using the expression

$$E_{tot} = E_{kin} + E_{pot} = \frac{1}{2} I \dot{\phi}^2 + \int_0^{\phi} \Delta \overline{GM} \phi \, d\phi \qquad (37)$$

The first term of Eq. (37) reflects the kinetic energy depending on the angular velocity, whereas the second term refers to the potential energy contained in the ship's inclination against the moment of restoration. It is necessary to determine the moment of restoration and other ship parameters to categorize the variations in the hull shape and to produce data sheets.

5. Conclusions

In this paper, the nonlinear equations in the roll motion of ships are solved by HPM. And an analytical result is compared with the numerical solution by using MATLAB. The results have demonstrated the efficiency and reliability of the proposed algorithm. The extension of this procedure is possible for the roll motion of ships for three or multiple degrees of freedom. The advantage of the method is illustrated that it does not need a small parameter in the system, and it is shown that the broad applications of nonlinear wave equations. In this paper, the given solution is appropriate for further treatment to analyze the nonlinear damping and restoring moment.

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