New Results In Production Theory By Applying Goal Programming

B.Venkateswarlu B. Mahaboob, C. Subbarami Reddy, C. Narayana

Abstract: In this research article postulates of Data Envelope Analysis (DEA) and factor minimal cost function and its properties are mentioned because they are prerequisites to understand the goal programming estimation of stochastic cost function. The LPP by which the factor minimum cost is obtained has been proposed here. This research paper includes the two stages of estimation of stochastic cost function. The properties satisfied by factor minimal cost function are stated and a diagram by which the concepts of technical, allocated and overall production efficiencies can be understood is proposed. The GP/CR model with single priority is specified and the GPP by which the estimation of full frontier can be estimated is presented. The LPP by which the output technical efficiency is estimated has been specified here. The goal programming problem by which DA can be formulated is derived and DEA-additive model is proposed. Furthermore some applications of goal programming in portfolio management have been proposed.

Keywords: GP-CR, stochastic frontier, DEA, factor minimal cost function, DMU production function, technical efficiency, and full frontier cost function.

1. INTRODUCTION:

IN management and production economics, it is often assumed that the producer is cost minimizer or revenue maximizer or profit maximize. However, these assumptions are not always realistic, especially in a competitive situation where different producers employ different techniques. For example, to produce a homogeneous product, the producer may have multiple goals instead of a single goal such as cost minimization. The various diversified goals may be, maintenance of sable profits and prices, improving market share and so on. Goal programming paves a way of striving toward several such multiple objectives simultaneously. Goal programming provides an objective function for each objective and consequently finds a solution that minimizes the weighted sum of deviations of these objective functions from their respective goals in their research article estimated stochastic frontier cost function using DEA. In 2007, Stephen C.H. Leung et.al, in their research article developed a pre-emptive goal programming model to solve aggregate production planning for perishable products in which three objects are optimized hierarchically. In 2008, Ademola Adeyeye et.al. In their research article developed a GP modelfor production planning in a tooth paste factory and in their paper two objectives were distinguished.

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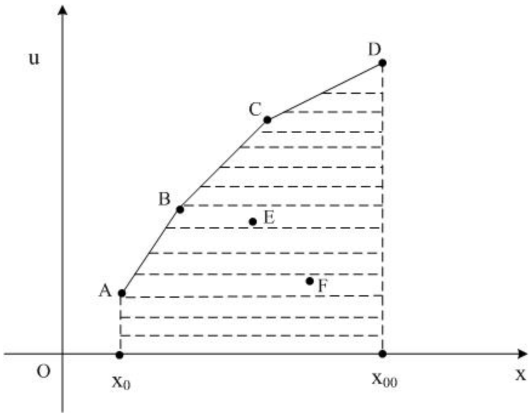
One can come across three possible types of goals (i) A lower, one sided goal which sets a lower limit that we do not fall under (ii) An upper, one sided goal that sets an upper limit which we are not allowed to exceed (iii) A two sided goal that specifies a lower limit that we do not fall under limit that we do not want to exceed. A goal programming problem may be pre-emptive. Pre-emptive goal programming is also called priority goal programming problem. Pre-emptive goal programming begins with the most important goal and continues until the achievement of a less important goal would cause management to fail to achieve a more important one. In the final solution, all the goals may not be fulfilled to the fullest extent. All the goals are equally important in non-pre-emptive goal programming. Further, a goal programming problem it is classified as linear or non-linear. Goal programming has applications in the theory of production. Goal programming can be used to estimate frontier full and stochastic cost functions of a production unit in a competitive environment. In 1991, Sueyyoshiet.al, minimization of processing cost and maximization of the capacity utilization of production facilities. In 2009, Mouna Mezghani et.al. In their paper, proposed an effective method to elaborate an aggregate plan which takes into account the manager preferences and established a goal programming approach with satisfaction functions. A. K. Bhargava et.al., in 2015,in their paper presented the production planning problem in industry with different operational constraints including strategic aim of the company, profit goal, limit on finishing and furnace hours needed, cups manufactured with target values being imprecise in nature.

2. DEA postulates production function and factor minimum cost

To understand goal programming estimation of stochastic cost function, it is necessary to understand the postulates of Data Envelopment Analysis (DEA) [2], factor minimal cost function and its properties. DEA is based upon production possibility sets determined by the input and output vectors of production units, called the Decision Making Units (DMU), which compete with each other in a competitive environment.

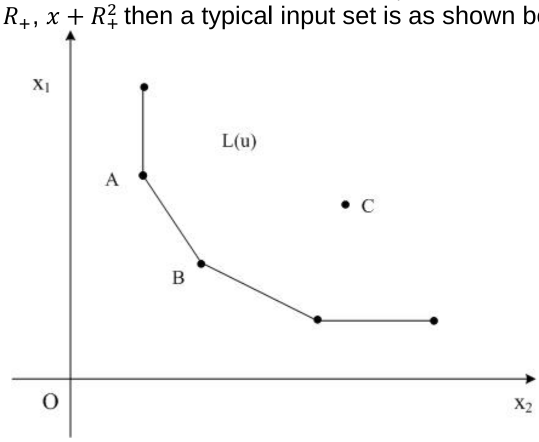
For example $\mathbf{X}_i$: Input vector of i^{th}production unit$x_i + R_+^n and u_i$: Output vectori^{th}production unit$u_i + R_+^m, p_i$: Input price vector $p_i + R_+^n - \{O\}$, Where O is null vector. It is hypothesised that producers are not equally efficient, since different producers employ different techniques even if they employ the same technique of production, they may differ in

terms of managerial efficiency. Thus, one of the postulates of DEA is 'inefficiency'. The following are DEA postulates [3].Let T be the production possibility set, which is defined as
$T = \{(x, u): x\ produces\ u\}$ (1)
(i) Convexity: (x_i, u_i) +T, (i =1, 2… k)
$(\sum_{i=1}^{k} \lambda_i x_i, \sum_{i=1}^{k} \lambda_i u_i)$ + T where $\lambda_i \geq 0, \sum_{i=1}^{k} \lambda_i = 1$ (2)
(ii) Inefficiency:$(\bar{x},\ \bar{u}) + T \Rightarrow \bar{x}\ produces\ \bar{u}$ and
where $x \geq \bar{x},\ u \leq \bar{u}$ (3)
(iii) Minimum extrapolation: T is the intersection of all input sets which contain(x_i, u_i)
For i = 1, 2,………..,k and $(T = \cap T_\alpha)$ (4)
The production possibility set consistent with the DEA postulates may expressed as
$T = \{(x, u): \sum \lambda_i x_i \leq x, \sum \lambda_i u_i \geq u, \lambda_i \geq 0, \sum \lambda_i = 1\ \}$ (5)
If$x_i$and$u_i$are scalar valued, then Tmay be graphically expressed as



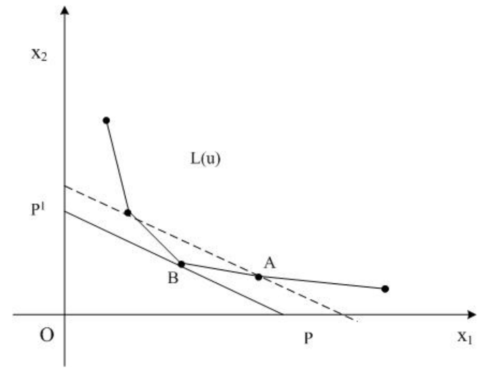
(Fig.1)

The dotted region bound by the line segments AB, BC, CD and ordinates atx_0 andx_{00} is the production possibility set. The production units which operate at A, B, C and D determine the production frontier. These are technically efficient DMUs. The production units that operate at E and F are inefficient. Production technology may be examined in terms of its inputs sets $L(u)$, where$L(u)$=$\{x: x\ produces\ u\}$. In addition, if the production technology is piece wise linear, as seen above, the input sets are expressed as [4,5]
$L(u) = \{x : \sum \lambda_i x_i \leq x, \sum \lambda_i u_i \geq u, \lambda_i \geq 0, \sum_{i=1}^{n} \lambda_i = 1\ \}$ (6)
Let$u + R_+$, $x + R_+^2$ then a typical input set is as shown below:



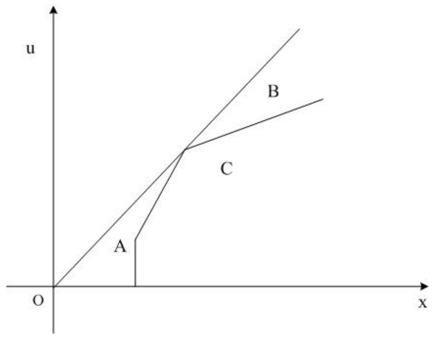
(Fig. 2)

The production units A and B which fall on the isoquant of $L(u)$ are technically efficient. But, the DMU-C is technically inefficient. The DMU's A, B and C produce the same output quantity.



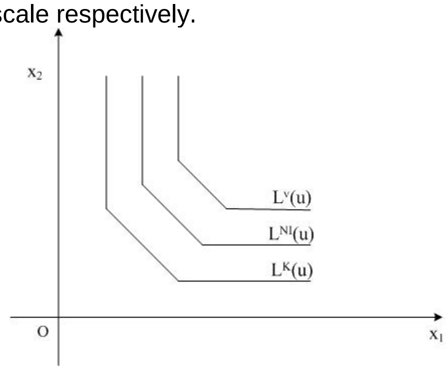
(Fig. 3)

The producer who operates at A is technically efficient.
PP′ is the cost line,C^n=$p_1 x_1^n$+$p_2 x_2^n$ (7)
Cost at 'A' is the higher than the cost at B. Therefore, producer-A is cost inefficient. For production unit that employs the input vector $x_0$and produces the output vectoru_0, factor minimal cost can be obtained by solving the problem.
$Q(u_0, p)$ = $\mathrm{Min}_x\ px$
Subject to $x + L(u_0)$ (8)
In terms of piecewise linear programming technology, the factor minimal cost is obtained by solving the linear programming problem$Q(u_0, p)$ = $\mathrm{Min}_x\ px$
Subject to$\sum \lambda_i x_i \leq x$,$\sum \lambda_i u_i \geq u_0$,$\lambda_i \geq 0\ and\ \sum \lambda_i = 1$ (9)
Production function:
A production function is a technical relationship between inputs and outputs. It has nothing to do with prices of inputs and outputs. For example, for scalar output $u$and input vector x, a production function may be expressed as $u = \emptyset(x)$
(10)
The production function and input sets determine each other completely [6],
$\emptyset(x)$=$Max\{u: x + L(u)\}$ (11)
$L(u) = \{x: \emptyset(x) \geq u\}$ (12)
A production function is said to be homogeneous,
if $\emptyset(\lambda x)$=$\lambda^\theta \emptyset(x)$
In addition suppose that≥ 1 .
$\theta < 1 \Rightarrow$Returns to scale are decreasing,
$\theta > 1 \Rightarrow$ Returns to scale are increasing and
$\theta = 1 \Rightarrow$ Returns to scale are constant
For a non-homogeneous production function returns to scale are defined as [7]
$$RTS = \frac{x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2}}{u} \quad (13)$$
This definition holds good for homogeneous production functions too. For piecewise linear technology the production possibility sets consistent with constant, non-increasing and variable returns to scale are respectively [8]
$T(k)$=$\{(x, u): \sum \lambda_i x_i \leq x, \sum \lambda_i u_i \geq u, \lambda_i \geq 0\}$
$T(NI) = \{(x, u): \sum \lambda_i x_i \leq x, \sum \lambda_i u_i \geq u, \lambda_i \geq 0, \sum \lambda_i \geq 1\}$ (14)
$T(v) = \{(x, u): \sum \lambda_i x_i \leq x, \sum \lambda_i u_i \geq u, \lambda_i \geq 0, \sum \lambda_i = 1\}$
Where$T(v)$=C, $T(NI) = A \cup C$and $T(k)$= $A \cup B \cup C$



(Fig. 4)

The production possibility set consistent with variable returns to scale is region C. The PP set that admits non-increasing returns to scale is union of the regions A and C. The ray that emanates from the origin is constant returns to scale frontier. The PP set that admits only constant returns to scale is union of the sets A, B and C. In terms of level sets, we have $L^v(u) \subseteq L^{NI}(u) \subseteq L^k(u)$.Where $L^v(u)$,$L^{NI}(u)$ and $L^K(u)$ are input level sets consistent with variable, non-increasing and constant returns to scale respectively.



(Fig. 5)

Factor minimal cost:

Let $Q(u,p)$be the factor minimal cost function. It satisfies the following properties.

P.1 $Q(u, \lambda p) = \lambda Q(u,p)$

p.2 $p_1 \geq p_2 Q(u,p_1) \geq Q(u,p_2)$

p.3 $Q[u, \lambda p_1 + (1-\lambda)p_2] \geq \lambda Q(u,p_1)$+$(1-\lambda)Q(u,p_2)$ for $0 \leq \lambda \leq 1$

p.4 $Q(o,p) = 0$

To produce null output vector the factor minimal cost is zero.

p.5 $Q(u.p) > 0, if\ u > 0. p \geq 0$

The factor minimal cost to produce a positive output is positive if at least one input is not a free good. The restrictions on the cost function imply restrictions on the first order conditions of the optimization problem is,

$Min\{px: \emptyset(x) = u\}$ (15)

The most commonly used form of the first order conditions is the system of input demand equations. If we introduce the axiom of inefficiency, then the factor minimal cost function may be expressed as the following optimization problem

$Q(u,p) = Min\{px: x + L(u)\}$ (16)

If output is scalar valued, equivalently we write

$Q(u,p) = Min\{px: \emptyset(x) \geq u\}$.

Where $\emptyset(x)$=Max$\{u: x + L(u)\}$. For piecewise linear technology, factor minimal cost is obtained by solving the following linear programming problem

$$Q^k(u,p) = Min\ px$$

Subject to $\sum \lambda_i x_i \leq x, \sum \lambda_i u_i \geq u$ and $\lambda_i \geq 0$ (17)

$Q^k(u,p)$ is factor minimal cost consistent with constant returns to scale. Let $Q(u,p)$be the factor minimal cost function and C be the observed cost.

Define C = $Q(u,p) + u + v$ (18)

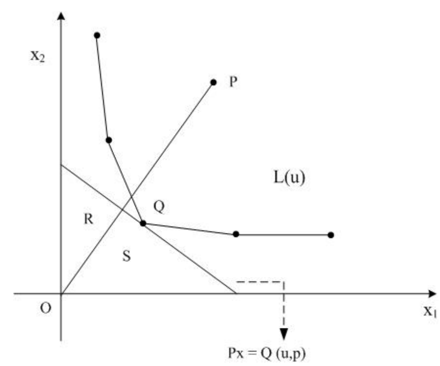
where u is one sided error term. Failure to achieve cost efficiency which can be decomposed into the product of technical and allocative efficiencies leads to the presence of one sided error component$'u'$.v is two sided statistical error term. For example, one may assume $'u'$ follows half normal and $'v'$follows normal distribution [9, 10]. $v = 0 \Longrightarrow Q(u,p)$ is full frontier. Observed costs fall on or above the full frontier. Estimator of stochastic cost function involves two stages. In the first stage, the managerial errors (u) are computed for the sake of which the following linear programming problem is solved for each DMU.

$Q(u,p) = Min\{px: x + L(u)\} and\ u = C - Q(u,p)$, Where C is observed cost of production

$u = C\left[1 - \frac{Q(u,p)}{C}\right]$ = C$[1 - OE]$ (19)

Where OE is over all productive efficiency.

The concepts of technical, allocative and overall production efficiencies can be best understood by means of the following diagram.



(Fig .6)

The producer who operates at P is technically inefficient. By reducing his inputs radically to the point Q he attains technical efficiency. Consequently, technical efficiency is defined as [11].$TE = \frac{OQ}{OP}$. Overall productive efficiency is defined as the ratio of factor minimal cost to actual cost of production $u = C[1 - OE]$. The stochastic factor minimal cost function may be expressed as,

C^*= Exp $[f(p,y) + v] \Longrightarrow log(C^*) = [f(p,y) + v]$ (20)

$C^*\ is$ Obtained by the adjustment and $C^* = C - u$

Thus, knowledge of actual cost and cost efficiency leads to $\{C^*\}$ series. Before we estimate a stochastic cost frontier, a priori a functional form should be assigned to $f(x,y), where\ p$ is the vector of input prices and y is the observed output. Let this functional form be approximated by a trans log cost function which is a second order Taylor series approximation of an arbitrary functional form.

$$f(p,y) = \alpha_0 + \sum \alpha_i \ln p_i + \frac{1}{2}\sum\sum \gamma_{iq} \ln p_i \ln p_q + \frac{1}{2}\sum\sum \delta_{rl} \ln y_r \ln y_l + \sum\sum \xi_{ir}\, lnp_i lny_r$$

(21)

This function should satisfy certain regularity conditions [12, 13].

Restriction 1: It should be linear in input prices, since if all prices increase by λ and output is constant, the factor minimal cost should also increase by λ.This requires imposing the following parametric restrictions.

$\sum_i \lambda_i$=1,$\sum_t tq = 0, \sum_q \gamma_{iq} = 0, \sum_t \xi_{tr} = 0, \sum_r \xi_{tr} = 0$

Restriction 2: Parameters satisfy the symmetry condition $\gamma_{i\theta} = \gamma_{\theta i}$ and $\xi_{ir} = \xi_{ri}$

Restriction 3: Monotonicity of cost shares require non-positive own elasticity's $\gamma_u \leq 0$.But these are only necessary conditions.

Parameters of Trans log stochastic cost function are estimated by linear goal programming approach. For j^{th} production unit, the logarithmic stochastic cost function may be expressed as,

$\ln C_j^* = f_j(p,u) + v_j$ and let $v_j = d_j^+ - d_j^-$, where $d_j^+ = Max\{0, v_j\}$ and $d_j^- = Min\{0, v_j\}$and

$d_j^+, d_j^- \geq 0$, i^{th} cost share equation of j^{th} production unit, $g_{ij}(p,y) + d_{ij}^+ - d_{ij}^- = S_{ij}^*$ and $d_{ij}^+, d_{ij}^- \geq 0$

The Goal Programming, Constrained Regression (GP-CR) model is [14]

$Minimize\ \sum(d_j^+ + d_j^-) + \sum\sum(d_{ij}^+ - d_{ij}^-)$

Subject to $f_j\ (p,y) + d_j^+ - d_j^- = \ln C_j^*$, $g_{ij}(p,u) + d_{ij}^+ - d_{ij}^- = S_{ij}^*$ and $d_j^+, d_j^-, d_{ij}^+, d_{ij}^- \geq 0$

(22)

This is a goal programming problem with single priority. It belongs to robust estimation, since its estimators are less influenced by outliers than the least squares estimators. The resultant estimators are equivalent to maximum likelihood estimators under the assumption of Laplace error distribution. Augmentation of cost share equations in estimation the estimating procedure has the effect of adding many additional degrees of freedom without including any regression co-efficient.

3.Full frontier cost function and its estimation:

Full frontier cost function [15] is associated with the following specification

$C = exp[f(p,u) + u]$ (23)

Estimation of full frontier requires solving the following goal programming problem:

$Minimize\ \sum\delta_j^* + \sum\sum(d_{ij}^+ - d_{ij}^-)$

Subject to $f_j\ (p,y) + \delta_j^+ = \ln C_j$, $g_{ij}(p,u) + d_{ij}^+ - d_{ij}^- = S_{ij}$and $\delta_j^+, d_{ij}^+, d_{ij}^- \geq 0$

(24)

There are two differences between the stochastic and full cost function frontiers.

(a) Full frontier uses observed C_j and S_{ij} as dependent variables, while stochastic frontier cost function uses $C_j^*\ and\ S_{ij}^*$.

(b) Full frontier maintains sloley δ_j^+ in the objective function so as to yield the full frontier cost function. Stochastic frontier includes both $\delta_j^+ and\ \delta_j^-$ in the objective function so that it can estimate parameters of the stochastic frontier cost function, using observed cost values. Linear programming approach is used to estimate stochastic production functions also [16].

Let Y_j be observed or transformed output of j^{th} production unit, X_j be the vector of inputs of j^{th} production unit. We postulate the following linear regression equation:

$Y_j = X_j\text{-}\ \beta - u_j + v_j \quad for\ (j = 1,2, \dots \dots .n)$

As an example consider the Cobb-Douglas production frontier [17] of j^{th} production unit:

$Y_j = A\ \prod_{i=1}^{m} x_{ij}^{\alpha_i}\ e^{-u_j + v_j}$ (25)

where x_{ij}: $i^{th}\ input\ employed\ by\ the\ j^{th}$ production unit and $A > 0, 0 \leq \alpha_i \leq 1$

$\ln y_i = \ln A + \sum_{i=1}^{m} \alpha_i \ln x_{ij} - u_j + v_j$ and

$Y_j = X_j\text{-}\ \beta - u_j + v_j$ (26)

Where Y_j transformed output and $\beta = \begin{bmatrix} a \\ \alpha_1 \\ \cdot \\ \cdot \\ \alpha_m \end{bmatrix}$,

$X = [x_{1j}, x_{2j}, \dots \dots \dots x_{mj}]$

$u_j(> 0)$ is one sided disturbance term.v_j is Stochastic distribution term.

The managerial error $u_j$is interpreted as deviation from frontier production function. δ is output technical efficiency, where $\delta \geq 1$.

Let y be observed output, then we have,

$\delta y = f(x) \Rightarrow y = \delta^{-1} f(x)$

$\ln y = \ln f(x) - \ln \delta \Rightarrow Y = X\beta - u, where\ u = \ln \delta$ (27)

$f(x)$ is full frontier, in the sense that the outputs of all production units fall on or below the frontier production function. The method of estimation is called Data Envelopment Analysis (DEA) and Least Absolute Deviation (LAV) method.

Stochastic frontier function [18]

$Y_j = X_j\beta - u_j + v_j$ Where $X_j\beta = full\ frontier,$

$X_j\beta + v_j$ = Average frontier

DEA/ LAV estimation requires first to estimate technical efficiency component, which can lead to the computation of $\{\{\hat{Y}_j\}\}$ series, where for j^{th} production unit DEA estimate of u_j is obtained by solving the linear programming problem:

$Max\ \delta$

$subject\ to\ \ \sum\lambda_i x_i \leq x_j and\ \ \sum\lambda_i y_i \leq \delta y_j$,

$\lambda_i \geq 0$ (28)

where $x_j, y_j$are input vector and output of j^{th} production unit respectively. $\delta_j$is the technical efficiency of j^{th} production unit.

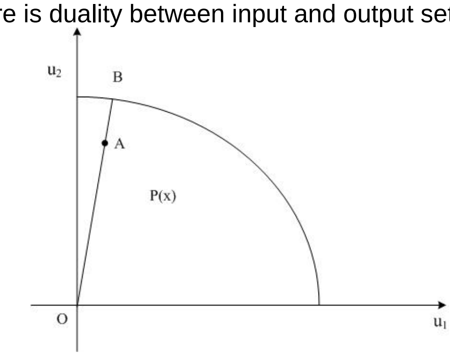
Let δ^*= $Max\ \delta$

$\theta^* = \frac{1}{\delta^*} and 0 \leq \theta^* \leq 1$

To understand the above linear programming problem one should have knowledge of output sets. An output set may be designated [19] as $P(x)$. $P(x) = \{u{:}\, x\ produces\ u\} = \{u{:}\, x \neg L(u)\}$

Conversely, $L(u) = \{u{:}\, x \neg P(x)\}$ (29)

Thus, there is duality between input and output sets.



(Fig. 7)

The output set $P(x)$ in the above figure belongs to one input and two output production technology.The production unit that operates at A is output technical inefficient. By radially expanding its outputs the producer attains output technical efficiency if he operates at B. Thus, output technical efficiency may be defined as $OTE = \frac{OB}{OA}(\geq 1)$. OTE is viewed as the optimization problem

$Maximize\ \delta$, Subject to $\delta u \neg P(x)$

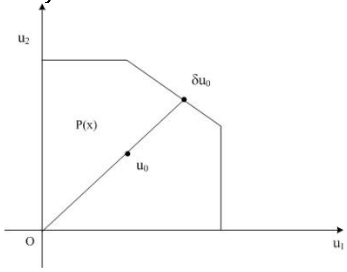
Further, if the production technology is piecewise linear, we can express $P(x)$ as,

$P(x) = \{u{:} \sum\lambda_i u_i \geq u, \sum\lambda_i x_i \geq x, \lambda_i \geq 0\ \}$

Output technical efficiency is estimated, solving the following linear programming problem [20]

$Max\ \delta$, Subject to $\sum \lambda_i x_i \leq x_0, \sum \lambda_i u_i \leq \delta u_0$ and $\lambda_i \geq 0$ (30)

Where, x_0, u_0 are inputs and output vectors of the production unit whose efficiency is under evaluation



(Fig .8)

For each production unit compute δ_j^* and u_j, where $u_j = \ln \delta_j^*$. To estimate stochastic frontier production function compute the series $\{\widehat{Y}_j\}$ where $\widehat{Y}_j = Y_j + u_j$.

The LAV estimation is carried out to obtain parametric vector estimates, the linear regression equation being $\widehat{Y}_j = X_j\beta + v_j$ define $v_j = v_j^+ + v_j^-$, for $j = 1,2,\dots\dots\dots.n$

The LAV, linear goal programming problem is $Minimize\ \prod = \sum_{j=1}^{n}(v_j^+ + v_j^-)$

Subject to $X_j\beta + v_j^+ + v_j^- = \widehat{Y}_j$ and $v_j^+, v_j^- \geq 0$ (31)

Let $\hat{\beta}$ be the LAV estimator of β. Some of the properties of LAV estimator are as follows

The goal programming problem is [21]

$Min\ 0\beta_1 + \dots\dots\dots + 0\beta_m + \eta_1^+ + \eta_1^- + \dots\dots\dots\dots\dots + \eta_n^+ + \eta_n^-$

$$\begin{bmatrix} x_{11}\ x_{21} \dots x_{m1} & 1 & -1 & 0 & 0 & \dots & 0 & 0 \\ x_{12}\ x_{22} \dots x_{m2} & 0 & 0 & 1 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x_{1n}\ x_{2n} \dots x_{mn} & 0 & 0 & 0 & 0 & \dots & -1 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \dots \\ \beta_m \\ \eta_1^+ \\ \eta_1^- \\ \dots \\ \dots \\ \eta_m^+ \\ \eta_m^- \end{bmatrix} = \begin{bmatrix} \hat{Y}_1 \\ \hat{Y}_2 \\ \dots \\ \dots \\ \dots \\ \hat{Y}_n \end{bmatrix}$$

$and\ \eta_n^+, \eta_n^- \geq 0$

The dual of this problem is

$Maximize\ (Z) = w_1\hat{Y}_1 + \dots\dots\dots\dots + w_n\hat{Y}_n$

Subject to $\sum_{j=1}^{n} w_i x_{ij} = 0, for\ i = 1,2,\dots\dots\dots.m\ and\ -1 \leq w_j \leq 1,\ j = 1,2,\dots\dots.n$

(32)

Where w_j is the dual variable associated with the j^{th} constraint

4.Goal programming in discriminant analysis

Goal programming has applications in discriminant analysis [22] (DA). DA is a statistical technique used to predict group membership. In the DA approach, a group of observations whose memberships are already identified are used for the measurement of a set of estimates by minimizing incorrect group classification. DA provides a set of estimates (weights), consequently yielding an evaluation score, that is compared with a threshold value for determining group membership. DA can be formulated by the following model:

$Min\ \sum_{j\in G_1} S_j^+ + \sum_{j\in G_2} S_j^-$

Subject to $\sum_{i=1}^{k} \lambda_i z_{ij} + S_j^+ \geq d, for\ j \in G_1$ and $\sum_{i=1}^{k} \lambda_i z_{ij} - S_j^- \leq d, for\ j \in G_2$

(33)

$S_j^+, S_j^- \geq 0\ , d\ and\ \lambda_i\ are\ unrestrited\ sign.$

This is a goal programming problem. There are n decision making units (DMU) with each DMU there are k characteristic measurements. It is a priori known that n_1 DMU's belongs to first group (G_1) and n_2 DMU's belongs to second group (G_2). The threshold value may be unknown. This DA model produces an optimum set of weights (λ^*) which define a hyper plane for separating the two groups. Suppose a newly sampled DMU possesses the measurements $\{z_{ij}^0\}$.To classifies this DMU into $G_1\ or\ G_2$.We combine $\lambda^*\ and\ z_{ij}^0$, we compute $\sum \lambda^* z_{ij}^0$ .we classify the DMU into $G_1 or\ G_2$ depending upon the value $\sum \lambda^* z_{ij}^0$ falls below or above the estimated threshold valued d^*. To avoid a trivial solution, $\lambda_i = 0,\ \forall i\ and\ d = 0$, take a more clear separation between $G_1 and\ G_2$, we reformulate the previous goal programming problem as,

$Min\ \sum_{j\in G_1} S_j^+ + \sum_{j\in G_1} S_j^-$

Subject to $\sum \lambda_i z_{ij} + S_j^+ \geq d, j \in G_1,$

$\sum \lambda_i z_{ij} - S_j^- \leq d - \eta, j \in G_2$ (34)

$S_j^+, S_j^- \geq 0$ and d, α_i unrestricted for sign.

η is a small number to impose a small gap between the two groups. Using additional slacks we reformulate the problem as, $Min\ \sum_{j\in G_1} S_j^+ + \sum_{j\in G_1} S_j^-$

Subject to $\sum \lambda_i z_{ij} + S_j^+ - S_j^- = d,\ j \in G_1\ and$

$\sum \lambda_i z_{ij} + S_j^+ + S_j^- d - \eta,\ j \in G_1$ (35)

$S_j^+, S_j^- \geq 0 and d, \alpha_i are$ Unrestricted for sign.

For a newly sampled DMU the classified into $G_1 or\ G_1$ is as follows:

(i) $S_j^- > 0 \Rightarrow S_j^+ = 0\ then\ \sum \lambda_i z_{ij} - S_j^- = d \Rightarrow \sum \lambda_i z_{ij} > d$ For $j \in G_1$

(ii) $S_j^+ > 0 \Rightarrow S_j^- = 0\ then\ \sum \lambda_i z_{ij} + S_j^+ = d - \eta \Rightarrow \sum \lambda_i z_{ij} < d - \eta$ For $j \in G_2$

(iii) The non-negative slacks $S_j^+ and\ S_j^-$ may be interpreted as errors due to misclassifications.

$j \in G_1\ , but\ suppose\ \ S_{1j}^+ > 0 \Rightarrow S_{1j}^- = 0\ then\ \sum \lambda_i z_{ij} < d$ For $j \in G_2$

Thus, S_{1j}^+ is an error due to mis-classifications.

Similarly, $j \in G_2. But\ S_{2j}^- > 0 \Rightarrow S_{2j}^+ = 0 \Rightarrow \sum \lambda_i z_{ij} > d - \eta$ for $j \in G_1\ where\ S_{2j}^+$ due to misclassifications

DEA additive model:

$Min\ \sum_{i=1}^{k} S_i^+ + \sum_{r=1}^{\beta} S_r^-$

Subject to $\sum_{j=1}^{n} x_{ij}\,\lambda_j + S_j^+ = x_{i0}$, for $i = 1,2,\dots\dots\dots\dots..,k$

$\sum_{j=1}^{n} y_{rj}\,\lambda_j + S_r^- = y_{r0}$, for $r = 1,2,\dots\dots\dots\dots..,s$ (36)

$\sum_{j=1}^{n} \lambda_j = 1$ and $S_j^+, S_r^-, \lambda_j \geq 0$

The various features of DEA problem are as follows:

(i) n Producers are in competition (ii) k inputs are combined to produce s outputs.

$(iii) x_{ij}$: i^{th} Input of j^{th} production unit

(IV) y_{rj}: r^{th} output of j^{th} production unit

$(v)\ x_{i0}, y_{r0}$ are i^{th} Input and r^{th} output of the production unit whose efficiency is under evaluation.

$(vi)\lambda_j$, $(j = 1,2,\dots\dots\dots\dots n)$ are the non-negative intensity parameters.

By combining additive DEA and DA the following goal programming is formulated:

$Min\ \sum_{j\in G_1} S_{1j}^+ + \sum_{j\in G_1} S_{2j}^-$

Subject to $\sum_{j=1}^{k} \alpha_i z_{ij}+S_{1j}^+ - S_{1j}^- = d$, for $j \in G_1$

$\sum_{j=1}^{k} \beta_i z_{ij}+S_{2j}^+-S_{2j}^-= d-\eta$, for $j \in G_2$

(37)

$\sum \alpha_i = 1\ and\ \sum \beta_i = 1$. All slacks are non-negative and $\alpha_i, \beta_i \geq 0, where\ d\ is\ unrestricted\ sign$

5.Goal Programming in portfolio management

Goal programming has applications in portfolio management [22]. In portfolio management the deviation of realized total return on all assets from expected total return on all assets subject to relevant linear constraints. An individual invests his monitory resource on n assets, namely$S_1, S_2, \dots\dots\dots\dots\dots, S_n$.Let R_j be rate of return on S_j . R_j: $r_j, r_{2j}, \dots\dots\dots\dots\dots, r_{Tj}$.where $R_J = r_{1j}$ is the rate of returns on S_j for the period t. x_j is the investment of S_j.

$\sum_{i=1}^{n} x_j = M_0$ (38)

Let u_j be maximum amount allowed for investment inS_j. $0 \leq x_j \leq u_j$

Define mean rate of return on $S_j\ as\ E(R_i) = \frac{\sum_{t=1}^{T} r_{tj}}{T} = r_j$ (39)

(i) Return on j^{th} asset: $r_j\, x_j$.Let $\rho\ be$ the minimum rate of return. Minimum return on

Total investment: ρM_0

Rate of the return constraint $\sum_{j=1}^{n} r_j x_j \geq \rho M_0$ (40)

(ii) Total investment on all the assets: $\sum_{j=1}^{n} x_j$ and investment constraint: $\sum x_j = M_0$

(iii) Investment constraint on j^{th} asset: $0 \leq x_j \leq u_j$

The following linear programming problem is postulated:

$$Minimize\ \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{j=1}^{n} a_{tj}x_j\right|$$

Subject to $\sum_{j=1}^{n} r_t x_j \geq \rho M_0, \sum_{j=1}^{n} x_j = M_0, 0 \leq x_j \leq u_j$ (41)

The objective function is the effective between realized total return on all assets, and expected total return on all assets. Consider

$\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{n} a_{tj}x_j = 0$ and $\frac{1}{T}\sum\sum(r_{tj} - r_j)x_j = 0$

$\frac{1}{T}\{\sum\sum r_{tj}x_j - T\sum_{j=1}^{n} r_j x_j\} = 0 \Rightarrow \frac{\sum\sum r_{tj}x_j}{T} = \sum_{j=1}^{n} r_j x_j$

The above linear programming problem can be represented as a goal programming problem. Let

$\sum a_{tj}x_j = v_t - w_t\ and\ v_t \geq 0, w_t \geq 0\ and\ v_t, w_t = 0, \forall t$

$\left|\sum a_{tj}x_j\right| = v_t + w_t$

$$Min\ \frac{1}{T}\sum_{t=1}^{T}(v_t + w_t)$$

Subject to $\sum_{j=1}^{n} a_{tj}x_j = v_t - w_t, for\ t = 1,2, \dots\dots.., T$

$\sum r_j x_j \geq \rho M_0 and\ \sum x_j = M_0$ (42)

Where$0 \leq x_j \leq u_j$, $for\ j = 1,2, \dots\dots\dots.., n$

$v_t, w_t \geq 0$, $for\ j = 1,2, \dots\dots\dots\dots\dots, T and v_t, w_t = 0$, $for\ j = 1,2, \dots\dots\dots\dots, T$

Conclusion and Future Research:

In the above research study the LAV linear goal programming problem is derived some properties of LAV estimator are stated. The model by which DA can be formulated is specified. Finally by combining additive DEA and DA, a GPP is formulated and an application of goal programming in portfolio management has been discussed. In the context of future research

i) One can frame some envelopment problems to identify extremely efficient DMUs

ii) One can estimate the slack based measure of efficiency of a DMU

iii) One can investigate how robust is the efficiency of an extremely efficient DMU.

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