A Differential Model Of The Spatial Spread Of Information

Agwu I. A, Abdulrahman S, Kalu A.U, Inyama S. C

Abstract: Sociologists recognize phenomenon called social diffusion, which is the spreading of a piece of information, a technological innovation or a cultural fad among a population . This paper, proposes a differential model of the spread of information among a population whose size is known. The members of the population $N(x,t)$ are grouped into two classes according to their information status: informed class $I(x,t)$ and uninformed class $U(x,t)$.We incorporated a diffusion term into the system as a tool for spreading information over significant distances. We investigate the spatial spread of an information into a uniform population of uninformed .Coupled conditions on threshold parameters which determine whether information will flow or not and condition for the existence of such travelling information wave and speed of propagation of information are determined. The system shows possibility of oscillatory behavior and stable dynamics

Index Terms: spatial spread, diffusion, information wave, informed-uninformed class, oscillatory behavior, linearize stability

1. INTRODUCTION

The study of the spread of information and propagation of ideas and influence in a social network has a long history in the Social Sciences (see [8], [3] and [5].) This network diffusion process has become an emerging research area in Computer Science and mathematics (see [13], [8], [5], [4] ,[9] and [7]) The research questions that can be answered include the understanding of the extent to which people are affected by friends, the extent to which word-of-mouth information and rumors spread and ultimately how the dynamics of adoption are likely to unfold within the underlying social network or system, which exhibits resemblance to epidemic processes [2]. [2] presented a model for the study of information diffusion over a realistic social network by introducing a dynamic interaction-based approach to describe the spread of information through conversations between pairs of individuals within a social network, or geographical area, capturing and utilizing the specific times of the interactions. They used a probabilistic model to decide whether two people will converse about a particular topic based on their similarity and familiarity and found that youths play a significant role in spreading information through a community rapidly, mainly through interactions in school and recreational activities. [13] studied the network diffusion process in the context of viral marketing to determine a cost-effective marketing plan.

- *Agwu I .A is with Department of Mathematics, Abia State Polytechnic, Aba, Nigeria (corresponding author, phone +234 803 338 9306, E-mail: uachilihu@yahoo.com)*
- *Abdulrahman. S is with Department of Mathematics /Statistics, Federal University of Technology, Minna, Nigeria (Phone +2348036896961 E-mail: sirajo.abdul@futminna.edu.ng)*
- *Kalu A. U is with Department of Mathematics, Abia State Polytechnic, Aba, Nigeria (Phone +2348033241846, e-mail: ugwason@yahoo.com)*
- *Inyama S.C is with Department of Mathematics, Federal University of Technology, Owerri, Imo State, Nigeria (+2348036661723 E-mail: scinyama2011@yahoo.com)*

They presented a probabilistic viral marketing model and apply the model on a knowledge sharing website. [8] studied the problem of [13] further and presented a greedy heuristics-based approximation algorithm with performance guarantee of 63% out of the optimal. [11] modeled rumor spreading where the population has been divided in three categories: Ignorant, spreaders and Stiflers using a model similar to an epidemiological one. Contact between an ignorant and a spreader could increase the number of spreaders and contact between a spread and a stifler increases the number of stifler, with some probabilities. They performed a systematic study of threshold properties of the diffusion process on different classes of static network. [12] presented a systematic analyses of all the processes related to the diffusion of information (or innovations) in a social network. He distinguishes the particular roles played by agents in the network according to their positions, elucidated the roles of homophily and heterophily and shows imitation as the principal mechanism for adoption [1] considered time-evolving networks and rumor spreading. In their model, the agents are in a square lattice (grid) and interact with each other through their random movements within the region. They showed that, starting from a single informed agent, the time it takes for information to reach the entire population has a power law distribution with respect to the size of the population and the size of the grid and that the average degree of information depends on the size of the grid. [4] studied the feedback effect of social ties and similarity and their interactions using data from two online communities. They also examined prediction of an individual's behavior using similarity and the social network formed base on previous interactions and observed that people encounter each other due to over-lap in their interests (similarity) they found that the consequences of these encounters can lead to further similar interests that are visible many months later. In this paper, we examine the spatial spread of an information wave from a population of informed to a population of uninformed individuals and determine a coupled threshold conditions which determine whether information will flow or not and conditions for the existence of such travelling information wave and the speed of propagation. The specific problem we investigate in this paper, is how the population of the uninform class can be reduced when initial homogeneous informed class I_o is introduced into the system.

2. DIFFERENTIAL MODEL

In this section, we formulate and analyze a differential equation model based on the sociological characteristics of the spatial spread of information. The members of the population $N(x,t)$ are grouped into two classes according to their information status: informed class $I(x,t)$ and uninformed class $U(x,t)$. Diffusion term is incorporated into the system as a tool for spreading information over significant distances. The following assumptions are considered in the formulation of the differential model. H_1: We assume that the population $I(x,t)$ and $U(x,t)$ are differentiable and continuous functions of space variable, x and time, t

H_2: The rate of diffusion of information is proportional to the number of informed individual times the number of uninformed individual.

H_3: We assume that once an individual is informed, he or she stays informed except through death.

H_4: We assume a population of known size.

H_5: A newly information individual can spread the information to other uninformed individual immediately.

H_6: Every uninformed individual would be willing to listen (and hence pass it on).

H_7: We assume that both the informed and uninformed have information induced mortality rate of ϕI and ϕU respectively.

H_8: Individual who are not collocated can communicate or interact. We consider the use of cell phone or other communication devices.

H_9: We assume that the process of the spread of information follows the mass-action law.(hence the term βUI corresponds to the direct spread of information by contact between I and U individuals)

Applying the assumptions, the differential model which describe the spatial spread of information is formulated

$$\frac{\partial U}{\partial t} = -\beta \cup I + \phi U + (\alpha - 1)\nabla^2 U$$

$$\frac{\partial I}{\partial t} = \beta \cup I - \phi I + \alpha \nabla^2 I \ (2.1)$$

where

$\beta =$ The rate at which the uninformed class receive information

$\phi =$ Death rates of information as a result of natural death of the informed and uninformed individual.

$\beta \cup =$ The number of uninformed who gets information from each informed individual.

$\alpha - 1 =$ Diffusion term for the uninformed individual

$\alpha =$ Proportion of newly informed individual from a previously informed individual (diffusion term for the informed individual)

$\frac{1}{\phi} =$ The life expectancy of information as a result of natural death of informed persons

$U(x,t) =$ Uninformed class

$I(x,t) =$ Informed class

$\nabla^2 =$ Laplace operator or Laplacian

2.1 NON-DIMENSIONALIZATION OF THE MODEL

In this section, we introduce a number of informed individuals into an uninformed population with initial homogeneous informed density I_0 and determine the geographical spread of the information. We non-dimensionalize the system (2.1) using the approach in [10] by writing

$$\lambda^0 = \frac{\phi}{\beta I_0}, I^* = \frac{I}{I_0}, U^* = \frac{I}{I_0} \ (2.2)$$

$$X_I{}^* = x\left(\frac{\beta I_0}{\alpha}\right)^{1/2}, X_u{}^* = x\left(\frac{\beta I_0}{\alpha - 1}\right)^{1/2}, t^* = \beta I_0 t$$

where I_0 is a representative informed population and on dropping the asterisks for notational simplicity the model (2.1) becomes

$$\frac{\partial U}{\partial t} = -UI + \lambda^0 U + \frac{\partial^2 U}{\partial x^2}$$

$$\frac{\partial I}{\partial t} = UI - \lambda^0 I + \frac{\partial^2 I}{\partial x^2} \qquad (2.3)$$

The basic reproduction rate of the information is $\frac{1}{\lambda^0}$, which is the number of previously informed individuals produced by one newly informed person in an uninformed population. The reproduction rate is also a measure of the two relevant time scale; that associated with contagious time of the information $\frac{1}{(\beta I_0)}$ and the life expectancy $\frac{1}{\phi}$ of an information as a result of natural death of informed individual.

2.2 INFORMATION WAVE SOLUTION

In this section, we look for travelling information wave solution using the approach of Murray [10].

We set $I(x,t) = I(z), U(x,t) = U(z), z = x - ct$ (2.4)

Equation (2.4) represents an information wave of constant shape travelling in the positive x direction, where c is the speed of the wave, which we have to determine Substituting (2.4) into (2.3) and applying method of separation of variable, we obtain a second order ordinary differential equation.

$$U'' + cU' - (1 - \lambda^0)U = 0 \qquad (2.5)$$

$$I'' + cI' + (U - \lambda^0)I = 0$$

where the prime denotes differentiation with respect to z. This eigenvalue problem consist of finding the values of λ^0 such that a solution exist with positive wave speed and non-negative I and U (that is $I(z)$ has to be bounded for all z and non-negative populations I and U) such that

$$U(-\infty) = U(\infty) = 0, 0 \le I(-\infty) < I(\infty) = 1 \qquad (2.6)$$

Then we have the following theorem.

Theorem 2.1. Consider (2.5) and assume that H1-H9 hold together with the boundary condition (2.6), then any positive solution of $U(x,t)$ exist for all $t > 0$ and is ultimately bounded, that is, there is $N_0 > 0$ such that $U(x,t) < N_0 \; for \; t \gg 1$

Proof:
Considering the first equation of (2.5), let $N_0 > 0$ be such that $z(x,t,m) \leq -\delta_0$, for some $\delta_0 > 0$ and any $m \geq N_0$. Then forr any $N \geq N_0$

$$L(x,t,N) = \begin{cases} N - \delta_0 N t, 0 \leq t < \frac{N-N_0}{\delta_0 N} \\ N_0, t \geq \frac{N-N_0}{\delta_0 N} \end{cases}$$

is a solution of the first equation of (2.5). Then there is $N \geq N_0$ such that, $U(x,0) < N$ for $x \varepsilon \mathfrak{R}^2$ it follows from the comparism principle of parabolic equation that

$$U(x,t) \leq L(x,t,N) \text{ for } t \geq 0$$

This implies that $U(x,t)$ exists for all $t \geq 0$ and $U(x,t) \leq N_0$ for $t \geq \frac{N-N_0}{\delta_0 N}$

with condition (2.6) we have

$$\underset{x \to \infty}{Lim \; sup} U(x,t) \leq N_0 = 1$$

and this completes the proof □ The prove of the positivity and boundedness of the solution of the second equation of (2.5) follows from Theorem 2.1

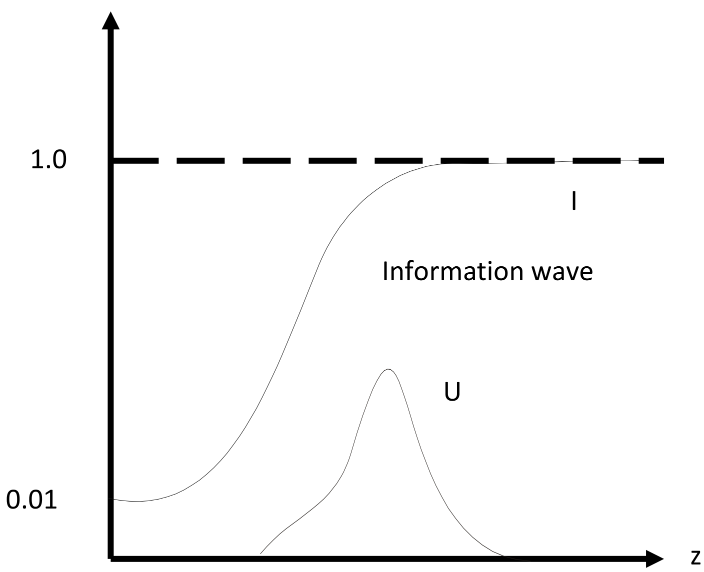

Fig.1 Shows the information wave of the informed and uninformed class

Using the technique in [10], we linearize the first equation of (2.5) near the leading edge of the wave where $I \to I$ and $U \to 0$, we get

$$U'' + cU' + (U - \lambda^0)U \approx 0 \qquad (2.7)$$

with solution

$$U(z) \propto \exp[-c \pm \{c^2 + 4(1-\lambda^0)\}^{1/2}] \qquad (2.8)$$

If a travelling wave speed exists, the wave speed and λ must satisfy

$$c \geq 2(\lambda^0 - 1)^{1/2}, \lambda^0 > 1 \; (2.9)$$

If $\lambda^0 < 1$, no wave speed exists, so this is the necessary threshold condition for the propagation of an information wave. From (2.2) in dimensional terms, the threshold condition is

$$\lambda^0 = \frac{\phi}{\beta I_0} \; (2.10)$$

The travelling weave solution of $I(z)$ cannot have a local minimum since its linearization takes place at $I = 1 \; and \; U = 0$, which implies that $I' = 0$ and substituting into the second equation of (2.5) we have

$$I'' = \lambda^0 I > 0 \qquad (2.11)$$

which implies a local minimum. So $I(z)$ is a monotonic increasing function of z. By linearizing the second equation of (2.5) as $z \to \infty$ when $I = 1$ and we have

$$I'' + cI' - \lambda^0 I = 0$$

$$\lambda^2 + c\lambda - \lambda^0 = 0$$

with solution

$$I(z) \propto exp \frac{[-c \pm \{C^2+4\}^{1/2}]}{2} \qquad (2.12)$$

with $c \geq 2(-\lambda^0)^{1/2}, \lambda^0 < 0$ (2.13)

(2.13) violates λ^0 as a positive parameter and so as $z \to \infty, I(z) \to 1$ exponentially The threshold result in (2.10) has some important implications. We see that there is a minimum critical informed population density $I_c = \frac{\phi}{\beta}$ that must be exceeded for an information wave to occur.

$$\lambda^0 = \frac{I_c}{I_o} > 1, (I_c > I_0) \qquad (2.14)$$

On the other hand, for a given population I_o and mortality rate ϕ, there is a critical transmission coefficient $\beta_c = \frac{\phi}{I_o}$ which if not exceeded, prevents the spread of information with a given transmission coefficient β and an uninformed population, we also get a threshold mortality rate $\phi_c = \beta I_o$ which if exceeded, prevents an information spread. So, if we can by any measure increase transmission coefficient β_c, condition (2.10) would not be violated and hence increase the spread of an information. Finally, with $\frac{\phi}{\beta I_o} > 1$ as threshold criterion, a sudden influx of informed individual into the system can raise I_o below I_c and hence decreases the population of the uninformed class and increases the population of the informed class.

3. STABILITY ANALYSIS OF THE DIFFERENTIAL MODEL

The analysis of (2.5) involves the study of a four-dimensional phase space. The system (2.5) posses two steady states (*0,0*) and (λ^0, λ^0). In this study, we shall consider a simpler case; that in which the diffusion of the uninformed $(\alpha - 1)$ is very much smaller than that of the informed. (α) and so as first approximation we take $\alpha - 1 = \frac{\alpha-1}{\alpha} = 0$. We might reasonably expect the qualitative behavior of the solutions of the system (2.5) with $\alpha - 1 \neq 0$ to be more or less similar to those with $\alpha - 1 = 0$, [6]. With $\alpha - 1 = 0$ in (2.1) and non-dimensionalizing as in (2.5), we write system (2.1) as a set of first order ordinary differential equation,

$$U' = \frac{(1-\lambda^0)U}{c} = F_1$$

Let $I' = V = F_2$ (3.1)

$$V' = -cv - uI + \lambda^0 I = F_3$$

In the (U, I, V) phase space there are two steady states (*0,0,0*) and $(\lambda^0, \lambda^0, 0)$ with the boundary conditions (2.6) and $U(-\infty) = 0, I(-\infty) = 0, U(\infty) = I(\infty) = \lambda^0$ (3.2) We linearized equation (3.1) about the point(*0,0,0*), that is the steady state, $U = 0, I = 0$, and determine the eigenvalues,λ.

$$J = \begin{pmatrix} \frac{\partial F_1}{\partial U} & \frac{\partial F_1}{\partial I} & \frac{\partial F_1}{\partial V} \\ \frac{\partial F_2}{\partial U} & \frac{\partial F_2}{\partial I} & \frac{\partial F_2}{\partial V} \\ \frac{\partial F_3}{\partial U} & \frac{\partial F_3}{\partial I} & \frac{\partial F_3}{\partial V} \end{pmatrix}$$

$$J = \begin{pmatrix} \frac{I-\lambda^0}{C} & \frac{U}{C} & 0 \\ 0 & 0 & 1 \\ -I & -U+\lambda^0 & -C \end{pmatrix}$$

$$J_{(0,0,0)} = \begin{pmatrix} \frac{-\lambda^0}{C} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & \lambda^0 & -C \end{pmatrix}$$

The characteristic polynominal of $J_{(0,0,0)}$ is $\left(\frac{-\lambda^0}{C} - \lambda\right)[\lambda^2 + c\lambda - \lambda^0] = 0$

The eigenvalues of $J_{(0,0,0)}$ are

$$\lambda_1 = \frac{-\lambda^0}{C} \qquad (3.3)$$

$$\lambda_2, \lambda_3 = \frac{[-c \pm (C^2 + 4\lambda^0)^{1/2}]}{2} \Rightarrow saddle\ point$$

The steady state (*0,0,0*) is unstable and we consider the second steady state$(\lambda^0, \lambda^0, 0)$, that is the steady state $U = \lambda^0$ and $I = \lambda^0$ The following theorem is immediate.

Theorem 3.1. If λ_c^0 is the critical value of λ_0 and the local extrema of the characteristic polynomial of $J_{(\lambda^0,\lambda^0,0)}$ are independent of λ^0 .The information wave solution of (U, I) of (3.1) with boundary condition (3.2) approach the steady state (λ^0, λ^0) in an oscillatory manner provided $\lambda^0 = 0$ and $(\lambda^0, \lambda^0, 0)$ is a non-hyperbolic point. roof: Linearized system (3.1) about the point $(\lambda^0, \lambda^0, 0)$, that is, the steady state $U = \lambda^0$ and $I = \lambda^0$ gives

$$J_{(\lambda^0,\lambda^0,0)} = \begin{pmatrix} 0 & \frac{\lambda^0}{C} & 0 \\ 0 & 0 & 1 \\ -\lambda^0 & 0 & -1 \end{pmatrix}$$

The characteristic polynomial of $J_{(\lambda^0,\lambda^0,0)}$ is

$$f(\lambda) = \lambda^3 + c\lambda^2 + \frac{(\lambda^0)^2}{c} \qquad (3.4)$$

To see how the solution of this polynomial behave as the parameters vary, we consider the plot of $f(\lambda)$ for real λ and see where it crosses $f(\lambda) = 0$

Differentiating $f(\lambda)$, the local maximum and minimum are at

$$\lambda_{max} = \frac{-2C}{3} \text{ and } \lambda_{min} = 0 \qquad (3.5)$$

Since the local extrema are independent of λ^0, we assume there exist a critical value of λ^0, that is λ_c^0 such that $\lambda^0 > \lambda_c^0$ .For $\lambda^0 = 0$, the characteristic equation (3.5) become

$$\lambda^3 + c\lambda^2 = 0, \qquad (3.6)$$

The eigenvalues are

$$\lambda_1 = 0, \lambda\lambda_2 = 0, \lambda_3 = -cc < 0 \qquad (3.7)$$

The linearized system about the steady state (*0,0,0*) has eigenvalues with zero real parts $(\lambda_1 = 0, \lambda_2 = 0)$ and $\lambda_3 < 0$. Thus, (*0,0 ,0*) is a non-hyperbolic point and the dynamical behavior near it can be stable, periodic or even chaotic. The information wave solution of (3.1) with boundary condition (3.2) approach the steady state (*0,0*) in an oscillatory manner,[10] and this completes the proof □.

4. DISCUSSION

In this paper, we have been able to formulate a differential equation model for the spread of information among a population of known size. We divided the population into two classes, according to their information status: informed class and uninformed class. We incorporated a diffusion term into the system as a tool for spreading the information over significant distances. With the diffusion term equal to zero ($\propto -1 = 0$) and nondimensionalizing the system, the qualitative behavior of the solution of the system is examined. They system posses two steady states: (*0,0*) and (λ^0, λ^0) in the (U, I) phase apace, in (U, I, V) phase space we have (*0,0 ,0*) and $(\lambda^0, \lambda^0, 0)$.

The steady state $(0, 0, 0)$ is unstable with eigenvalues

$$\lambda_1 = \frac{-\lambda}{c}$$

$$\lambda_2, \lambda_3 = \frac{[-c \pm (c^2+4)^{1/2}]}{2}$$

The linearized system about the steady state $(\lambda^0, \lambda^0, 0)$, that is steady state $U = \lambda^0, I = \lambda^0$ has eigenvalues with zero real parts $(\lambda_1 = 0, \lambda_2 = 0)$ and $\lambda_3 < 0$. Thus, it is a non-hyperbolic point and the dynamical behavior near it can be stable, periodic or even chaotic. The linearized system (3.1) about the point $(\lambda^0, \lambda^0, 0)$ gives the polynomial

$$f(\lambda) = \lambda^3 + c\lambda^2 + \frac{(\lambda^0)^2}{c}$$

The plot of $f(\lambda)$ for real λ shows how the solution of the polynomial behave and where it crosses $f(\lambda) = 0. Differentiating\ f(\lambda)$, we obtained a local maximum and minimum at $\lambda_{max} = \frac{-2C}{3}$ and $\lambda_{min} = 0$ Since λ_{max} and λ_{min} are independent of λ^0, we assume there exist a critical value of the parameter λ_c^0, such that when $\lambda^0 > \lambda_c^0$, the wave solution of (3.1) with boundary condition (3.2) approaches the steady state (λ^0, λ^0) in an oscillatory manner. Using the technique in [10] we linearized the first equation of (2.5) near the leading edge of the wave where $I \to 1$ and $U \to 0$ and the solution of the linearized differential equation is

$$U(z) \propto exp \frac{\left[-c \pm \{c^2+4(1-\lambda^0)\}^{1/2}\right]}{2}.$$

We found that the travelling wave speed exist provided $c \geq 2(\lambda^0 - 1)^{1/2}$ for $\lambda^0 > 1$. The travelling wave solution of $I(z)$ cannot have a local maximum since $I^1 = 0$ and the second equation of (2.5) shows that $I'' = \lambda^0 I > 0$, which implies a local minimum. So $I(z)$ is a monotonic increasing function of z. By linearizing the second equation of (2.5) as $z \to \infty$, when $I = 1$ and $U = 0$ we have $I'' + cI' - \lambda^0 I = 0$ whose auxiliary equation is $\lambda^2 + c\lambda - \lambda^0 = 0$ with solution $I(z) \propto exp \frac{\left[-c \pm \{c^2+4\lambda^0\}^{1/2}\right]}{2}$ and wave speed $c \geq 2(-\lambda^0), \lambda^0 < 0$ $\lambda^0 < 0$ violates λ^0 as positive parameter and as $z \to \infty, I(z) \to 1$ exponentially In dimensional terms, the threshold condition for the propagation of information is $\lambda^0 = \frac{\phi}{\beta I_0} > 1$ In addition to the threshold condition, we have that:

(1) For a given mortality rate ϕ, information spread coefficient β, there is a minimum critical informed population density $I_c = \frac{\phi}{\beta}$ that must be exceeded for an information wave to occur.
(2) For a given information population, I_0, and mortality rate, ϕ, there is a critical information spread coefficient $\beta_c = \frac{\phi}{I_0}$ which if not exceeded prevents the spread of information.
(3) For a given information spread coefficient β and informed population I_0, there is a threshold mortality rate $\phi_c = \beta I_0$ which if exceeded prevents the spread of information.

5. CONCLUSION

In this paper, we have formulated and analysed a differential model based on the sociological characteristics of the spatial spread of information. We say that $\lambda^0 = \frac{\phi}{\beta I_0}$ as the basic reproduction rate of information and with $\frac{\phi}{\beta I_0} > 1$ as threshold criterion, a sudden influx of informed individuals can less I_0 below I_c and hence decreases the population of the uninformed class and increases the population of the informed class. This means that it is easy to achieve more number of informed persons, which is an indication of the spread of information using a system similar to epidemiological one. Furthermore, the model presented in this paper is quite different from the previous approaches, as explained in section one. However, there is still some work to do in this area. It is beyong doubt that in reality, the effect of time delay due to the time required in passing information from informed individual to uninformed individual has to be taken into consideration. We would be interested in studying the spatial spread of information by incorporating a time-delay on the informed class. Scholars are encouraged to delve into this area for further study.

REFERENCES

[1]. E. Aghari, R. Burioni, D. Cassi. And F.M Neri 'Efficiency of information spreading in a population of diffusing agents' Physical Review E. vol.73, no.4, pp 46138, 2006.

[2]. A. Apolloni, K.Channakes, D. Lisa, K. Maleg, K. Chris, L. Bryan, and S. Samarth 'A study of information Diffusion over a realistic social network model',2003.

[3]. J. Brown and Reinegen 'Social ties and word–of–mouth referral behavior' Journal of Consumer Research, Vol.14, no. 3, pp 350-362, 1987.

[4]. D .Crandall, J. Cosley, D. Huttenlocher, D. P.Kleinberg, J.M and S.Suri 'feedback effects between similarity and social influence in online communities, in proceedings of the 14th ACM SIGKDD international conference on knowledge Discovery and data mining', pp160-168, 2008

[5]. P. Domingos 'Mining social networks for viral marketing IEEE intelligent system'. Vol 20 No.1, pp 80-82, 2005.

[6]. S. R. Dunbar 'Travelling wave solution of diffusive Lotka-Volterra equations a Leteroclinic connection in R^4', Trans.amer.math,soc. Vol.268, pp 557-594, 1984.

[7]. R.G. Frank, D.W.Maurice and P. F William 'A First Course in Mathematical Modeling' Thomson Books/Cole C.A, USA., 2003

[8]. D. Kempe, J.M Kleinberg and E. Tardos,'Maximizing the spread of influence through social network in proceeding of the 9th ACM SIGKDD international conference on knowledge Discovering and data mining', pp 137-147, 2003.

[9]. Kempe D, Kleinberg J. M and Tardos E. 'Influential nodes in a diffusion model for social networks'. In ICALP; pp 1127-1138, 2005..

[10]. J. D.Murrary Mathematical Biology, Springer-Verlag. New York' 1990..

[11]. M. Nekovee, Y. Moreno, G. Bianconi and M. Marsili. 'Theory of rumour spreading in Complex social network. Physical A; Statistical mechanics and its application vol.374 No.1, pp 457 – 470, 2007.

[12]. E.M Rogers,'Diffusion of innovation. New York free press of Glencoe', 1962.

[13]. M. Richardson. and P. Domingos 'Mining knowledge-sharing sites for viral marketing. in proceeding of the 8th ACM SIGKDD international conference on knowledge Discovery and data mining, pp 61 – 70, 2002.